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..... COLUMNS, BEAMS, AND BEAM-COLUMNS."  
.....  
.....

DEGREE FOR WHICH THESIS WAS PRESENTED ..... PhD .....

YEAR THIS DEGREE GRANTED ..... 1980 .....

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LOCAL BUCKLING  
OF W SHAPES USED AS  
COLUMNS, BEAMS, AND BEAM-COLUMNS

by



JOHN L. DAWE

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF DOCTOR OF PHILOSOPHY  
IN CIVIL ENGINEERING

EDMONTON, ALBERTA

FALL, 1980





THE UNIVERSITY OF ALBERTA  
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "LOCAL BUCKLING OF W SHAPES USED AS COLUMNS, BEAMS, AND BEAM-COLUMNS" submitted by JOHN L. DAWE in partial fulfilment of the requirements for the degree of Doctor of Philosophy in CIVIL ENGINEERING.





## ABSTRACT

An analytical model, based on an extended Rayleigh-Ritz technique, has been developed for the purpose of predicting local buckling behaviour of columns, beams, or beam-columns composed of W shape sections. A tangent modulus buckling theory is used and a tri-linear stress - strain curve for an elastic - plastic - strain-hardening material is assumed. The effects of residual stresses as well as the interactive effects of web - flange restraints are included directly in the formulation. The flexibility of the analytical model allows for the possibility of separate flange or web buckling as well as simultaneous buckling of the web and flanges. An elaborate formulation of plate component stiffness matrices permits the use of varying material properties for longitudinal strips of a member as yielding and strain-hardening progress during loading.

A computer program based on the analytical model was verified by an extensive comparison of results with available classical results for elastic local buckling of plates. The validity of the local buckling analysis beyond the elastic range was well-established by comparison of computer results with the results of 57 tests conducted by various investigators using column, beam, and beam-column specimens.

Having thus been verified, the computer program was used to conduct an exhaustive study of the effect of various parameters





which were expected to have an important effect on local buckling behaviour. As a result of this study, various modifications to existing web and flange slenderness limitations for columns, beams and beam-columns are indicated. Further research in the form of full-size laboratory specimen tests is recommended and various suggestions are made with regard to future testing.



## ACKNOWLEDGEMENTS

This study was carried out under the supervision of Dr. G.L. Kulak of the Department of Civil Engineering at the University of Alberta. It forms part of a continuing research investigation into the local buckling behaviour of W shape sections. The project is funded by the Canadian Steel Industries Construction Council.

The author also wishes to thank Dr. T.M. Hrudehy and Dr. D.W. Murray for their helpful suggestions during the early stages of the theoretical development. The many words of encouragement from my friend and fellow student, Dr. Michael Hatzinikolas, are also greatly appreciated. I am especially grateful to Mrs. Gladys Stephens who so painstakingly prepared the final typewritten manuscript.





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## LIST OF SYMBOLS

The following is a list of the more commonly used symbols which are presented here for ease of reference. The meanings of all symbols used in the text are defined where they first appear. The meanings of symbolic names used in the computer program are explained within the program listing presented in Appendix C.

### Dimensions and Displacements

$\alpha_t, \alpha'_t$	=	Locations of material boundaries (Compression flange)
$\alpha_b, \alpha'_b$	=	Locations of material boundaries (Tension flange)
$\beta_1, \beta_2, \beta'_2, \beta_3, \beta'_3$	=	Locations of material boundaries (Web)
$b$	=	Flange width
$h$	=	Web height
$L$	=	Length of a specimen
$t_1$	=	Tension flange thickness
$t_u$	=	Compression flange thickness
$t_w$	=	Web thickness
$y_1$	=	Distance to neutral axis from mid- height of web
$u, v, w$	=	x-, y-, and z- Components of displacement
$u_i, u_j, u_k$	=	Subscript notation for displacements





$w_0$  = Displacement due to initial imperfection

$w_1$  = Displacement beyond  $w_0$

### Forces and Moments

$M$  = Applied end moment

$M_{cr}$  = Critical local buckling moment

$M_p$  = Plastic moment

$M_{pc}$  = Plastic moment reduced for compressive load

$M_y$  = Yield moment

$M_{yc}$  = Yield moment reduced for compressive load

$M_x, M_{xy}, M_y$  = Plate bending moments per unit width

$N_x, N_{xy}, N_y$  = In-plane plate forces per unit width

$P$  = Compressive axial force

$P_{cr}$  = Critical local buckling axial force

$P_y$  = Column yield load

### Geometric Properties

$A_u$  = Area of compression flange

$A_l$  = Area of tension flange

$A_w$  = Area of web

$b\sqrt{F_y}/2t$  = Flange slenderness term

$f$  = A general function

$F_i$  = Buckling factors based on longitudinal shapes



$h\sqrt{F_y}/w$	=	Web slenderness term
$I$	=	Moment of inertia of a unit strip of plate
$m$	=	Number of longitudinal half sinewaves
$\pi$	=	Ratio of circumference to diameter of a circle
$S(\zeta)$	=	Flange shape function at a cross-section
$x, y, z$	=	Rectangular coordinates in three dimensions
$\eta, \zeta, \xi$	=	Natural coordinates
$P_o(\zeta)$	=	Longitudinal shape envelope

#### Material Properties

$D_x, D_y, D_{xy}, D_{yx}$	=	Plate bending rigidity factors
$E$	=	Modulus of elasticity
$E_o$	=	Slope of yield plateau of stress - strain curve
$E_{st}$	=	Strain-hardening modulus
$E_t$	=	Tangent modulus
$E_x, E_y, E_z$	=	Material moduli of anisotropic materials
$G$	=	Shear modulus
$G_t$	=	Tangent shear modulus
$\nu$	=	Poisson's ratio

#### Matrices and Vectors

$[C]$	=	Matrix of shape function coefficients
$\{F_o\}$	=	Equivalent lateral load vector



$\{H\}$	=	Vector of polynomial terms
$[I]$	=	Identity matrix
$[K]$	=	Bending stiffness matrix
$[K_G]$	=	Geometric stiffness matrix
$[K_{G_o}]$	=	Intermediate geometric stiffness matrix
$[K_1]$	=	Stiffness matrix of tension flange
$[K_u]$	=	Stiffness matrix of compression flange
$[K_w]$	=	Stiffness matrix of web
$[S]$	=	Sweeping matrix
$[\Phi_i]$	=	Component stiffness matrices
$\langle \phi \rangle$	=	Transpose of shape function vector
$\{\phi\}_i$	=	Eigenvectors
$\{\theta\}$	=	Vector of coordinate displacements
$\{\theta\}_o$	=	Vector of initial coordinate imperfections

### Strains

$\epsilon_a$	=	Applied axial strain
$\epsilon_b$	=	Applied bending strain in compression
$\epsilon'_b$	=	Applied bending strain in tension
$\epsilon_c$	=	Strain at mid-height of web
$\epsilon_{rc}$	=	Residual compressive strain
$\epsilon_i (i = 1,2,3,4,5)$	=	Residual strains on a W shape section
$\epsilon_{b_{cr}}$	=	Critical compressive bending strain
$\epsilon_y$	=	Yield strain





$\bar{\epsilon}^{(P)}$	=	Effective_plastic strain
$\epsilon_{st}$	=	Strain at onset of strain-hardening
$e_x, e_y, e_z$	=	Strains in the x-, y-, and z - coordinate directions
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	=	Shear Strains

### Stresses

$F_y$	=	Specified minimum yield stress
$s_i (i = 1,2,3,4,5,6)$	=	Piecewise linear stress components
$\sigma_i (i = 1,2,3,4,5)$	=	Residual stresses on a W shape section
$\sigma_y$	=	Yield stress
$\sigma_{rc}$	=	Residual compressive stress
$\sigma_x, \sigma_y, \sigma_z$	=	Stress in the x-, y-, and z- coordinate directions
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	=	Shear stresses

### Tensors

$C_{ijkl}$	=	Fourth-order tensor of material constants
$e_{ij}$	=	Strain tensor
$e_{ij}^{(P)}$	=	Plastic strain tensor
$S_{ij}$	=	Stress deviator tensor
$\delta_{ij}$	=	Kronecker delta
$\sigma_{ij}$	=	Stress tensor

### Miscellaneous

$d$	=	Differential operator (eg. $dS$ )
$\delta$	=	Variational operator (eg. $\delta w$ )



$H'$	= Slope of effective stress - strain curve
$k^2$	= Constant defining a von Mises yield surface
$J_2$	= Loading function used in von Mises criterion
$\lambda$	= An eigenvalue
$\lambda_0$	= An intermediate eigenvalue
$\lambda'$	= An eigenvalue strain
$\lambda''$	= An eigenvalue difference ( $= \lambda_0 - \lambda'$ )
$\mu$	= Eigenvalue shift
$\omega$	= Smallest eigenvalue of a system ( $= 1/\lambda$ )
$\omega_s$	= Eigenvalue after shift applied



## Chapter 1

### INTRODUCTION

#### 1.1 General

A large proportion of the members used in present-day steel structures are uniform throughout their lengths and have I-shaped cross-sections<sup>1</sup>. Such members are referred to as W shapes and are particularly efficient when used as beams for transferring bending moments within a structure. W shapes are also used as columns for transferring pure axial compression and as beam-columns for transferring combined axial compression and bending. Depending on the width-to-thickness ratios of their flanges, W shapes are classified as Class 1, Class 2, Class 3, and Class 4 sections<sup>1</sup>. Because of their thin-walled characteristics these members are particularly susceptible to local buckling<sup>1,2,3</sup> of their component plates and this local instability limits the load-carrying capacity of the members. Thus, in limit states design<sup>1,4</sup> local buckling of component plates of W shapes is one limit which must be met.

#### 1.2 Local Buckling

In the present study the limit state of local buckling is isolated from other limit states such as those related to overall instability, material strength and excessive deflections<sup>1</sup>. This



procedure coincides with present design philosophies which require that local buckling be prevented prior to the attainment of the maximum strength of a member<sup>1,3,5</sup>. Since local buckling is isolated, a fundamental assumption is that all members are fully braced against overall instability. Local buckling of W shapes may be defined as a bifurcation phenomenon<sup>5</sup> whereby a component plate subjected to in-plane stresses may be in equilibrium in its original planar configuration or in a neighboring deflected configuration. This critical state can occur in the elastic or inelastic regions of material response depending on the level of yield stress, the plate width-to-thickness ratio, and plate boundary conditions<sup>2,3,6,7</sup>. Because of the nature of buckling, it is more precisely defined by a mathematical formulation. In the present study such a formulation is based on the principle of virtual work<sup>8</sup> and is presented in Appendix A.

### 1.3 Design Considerations

As already established by previous investigators<sup>2,3,6,7</sup>, a significant parameter affecting the stability of a plate is its width-to-thickness ratio. Therefore, a designer must consider the width-to-thickness ratios of flanges and webs when determining the local buckling resistance of W shapes. Other factors which affect plate stability in W shapes are the type of plate edge supports at the ends of a member, the type of stress distribution on a member cross-section, and the material properties of the steel<sup>2,7</sup>. Because of the significant effect that the yield stress level has on plate buckling<sup>7</sup>, the present design code<sup>4</sup> specifies limitations on the width-to-thickness ratios of webs and flanges multiplied by the square root of the yield stress.





### 1.3.1 Current Requirements for Flanges and Webs

The current Canadian design standard used for steel<sup>4</sup>, specifies width-to-thickness terms for four classes of W shape beam-columns. It is required that Class 1 sections permit the attainment of the reduced plastic moment and also allow for sufficient rotation capacity for subsequent redistribution of load before local buckling occurs. A Class 2 section is required to permit the attainment of the reduced plastic moment capacity with no provision for the requirement of subsequent load redistribution. A Class 3 section must permit the attainment of the reduced yield moment and for a Class 4 section the limit state of structural capacity is local buckling of elements in compression. Class 4 sections are light-gauge cold-formed sections and their behaviour does not form a part of this study. The present design limitations for Class 1, 2, and 3 sections are presented in detail in Chapter 6. These limitations are such that for a given class of section, local buckling of the component plates must not occur until the section has satisfied the minimum requirements for its classification. Since columns are not designed according to a required bending moment capacity, they are not classified as above. However, it is usual to use the same limitation for column flanges as that specified for Class 3 sections, while column webs are limited by a maximum width-to-thickness term<sup>4</sup>.

### 1.3.2 Previous Requirements for Flanges and Webs

Previous design codes<sup>9</sup> have presented width-to-thickness ratios for flanges and webs of W shapes subjected to axial compression, bending, and axial compression and bending combined. As will be examined further in Chapter 2, these values are based on the results of an investigation



by Haaijer and Thurlimann<sup>7</sup>. Recently, however, studies performed at the University of Alberta by Kulak et al<sup>10,11,12,13</sup> have shown that previously specified width-to-thickness limitations were overly conservative. As a result of these studies, the present code<sup>4</sup> uses values that supersede previously specified width-to-thickness limitations for component plates of columns, beams, and beam-columns.

#### 1.4 Objectives

The investigations conducted at the University of Alberta have been largely experimental and empirical in nature. The work has resulted in new design equations and graphs which are presently in use<sup>4</sup>. The present study is a continuation of this work with emphasis on the theoretical nature of plate buckling as it relates to W shapes subjected to axial compression, bending, and axial compression and bending combined.

The objectives of the present study are as follows:

1. to establish an idealized mathematical model to study local buckling of W shapes subjected to axial compression, bending, or to axial compression and bending combined.
2. to establish the validity of the mathematical technique by comparing analytical results with test results.
3. to present design equations and graphs for a broad spectrum of practical cases for which test results are not available.
4. to suggest, where appropriate, additional revisions



to presently specified width-to-thickness limitations for component plates of W shapes.

### 1.5 Scope

The mathematical formulation presented herein permits an analysis for local buckling of W shapes of various cross-sectional dimensions and material properties. The analysis is performed for axial compression, bending, and combined axial compression and bending applied at the member ends. The effects of residual stresses are included and web, flange, or combined web and flange buckling is predicted in the elastic or inelastic ranges. The restraint interaction of flanges and web is also accounted for by the mathematical model.

In Chapter 2 a review of the available literature that relates to the present study is outlined. In Chapter 3 the general mathematical formulation technique is discussed in detail and the application of this technique to the general case of combined axial compression and bending is presented in Chapter 4. In Chapter 5 the analytical technique is applied to the investigation of local buckling of 57 specimens that have been tested by various investigators. Theoretical and test results are compared. Analytical results for a wide range of Class 1, 2, and 3 columns, beams, and beam-columns are presented in Chapter 6, and in addition these theoretical results are interpreted for application to design. Also presented in Chapter 6 is a study of the parameters considered important in the analysis. The conclusions of the present study and the resulting design recommendations are presented in Chapter 7.

The mathematical formulation of plate buckling based on the





principle of virtual work<sup>8</sup> is presented in Appendix A and the material properties are discussed in Appendix B. Because of the nature of the mathematical formulation which employs the use of matrix algebra as well as the use of iterative techniques for the inelastic cases, hand computation is highly impractical. Therefore, a computer program coded in Fortran IV and suitable for use with an Amdahl 470V/6 or an IBM 3032 computer was used for the computations. A listing of the program with explanations of the subroutines and typical input data and corresponding output are presented in Appendices C and D.



## Chapter 2

### LITERATURE SURVEY

A review of the available literature indicates that the problem of elastic plate buckling has been thoroughly investigated since the publication in 1891 of the original work done in this area<sup>14</sup>. The theoretical investigations include the closed-form solutions of single plates having regular geometric shapes and various stress and displacement boundary conditions. These solutions have been well-documented and are readily available in the literature<sup>2,3,6,15,16</sup>.

It was not until the 1920's that the problem of inelastic plate buckling first began to receive some attention. In much the same way as for columns, inelastic buckling of rectangular plates was first treated by replacing the elastic modulus by a reduced modulus or a tangent modulus above the proportional limit. Bleich<sup>6</sup>, for example, assumed that above the proportional limit, the reduced modulus would be effective in the direction of uniaxial stress while the elastic modulus remained effective in the transverse direction. It was further assumed that an effective shear modulus equal to the geometric average of the elastic and reduced moduli would be applicable. Alternatively, Ros and Eichinger<sup>17</sup>, assuming isotropy in the inelastic range, suggested using the reduced modulus in all directions. Of the two, it was found that Bleich's assumptions led to values in closer agreement with test results.

Other investigators have attempted to improve upon the



existing knowledge in this area by including the total elastic-plastic stress-strain relationships in the plate buckling analysis. Prominent among those investigators were Bijlaard<sup>18</sup>, Ilyushin<sup>19</sup>, and Stowell<sup>20</sup> who used the deformation theory of plasticity<sup>8</sup> while Onat and Drucker<sup>21</sup> and Handelman and Prager<sup>22</sup> promoted the use of the incremental theory<sup>8</sup> in the plastic analysis of plate buckling. While the former method showed much better agreement with test results, it has been recognized that the latter method is the mathematically correct one<sup>7,21,23,24</sup>. The main reason for this paradox appears to be the prediction by the incremental theory of a value of the shear modulus in the inelastic range equal to its value in the elastic range<sup>7,21,23,25</sup>. It has been shown that a reduction in this value leads to much better correlation between theory and test results<sup>7,25</sup>.

Early investigations of web and flange stability for W shape members were based on several simplifying assumptions. For both elastic and inelastic buckling it was assumed that the plate components at web-to-flange junctions were either rigidly supported or simply supported and the effects of residual stresses were neglected. Using these assumptions as well as Bleich's assumption of anisotropy in the inelastic range it was possible to arrive at closed-form solutions for several types of in-plane loadings<sup>2,6,26</sup>.

In the late 1950's Haaijer and Thurlimann published the results of an investigation of inelastic local buckling in steel<sup>7</sup>. The main purpose of this investigation was to determine maximum plate width-to-thickness ratios for W shapes suitable for plastic design. An incremental theory<sup>22</sup> was used in the analysis and the inelastic value of the shear modulus was reduced by 80 per cent. (The investigators



attributed this reduction to the effects of initial imperfections. However, in a more recent investigation<sup>25</sup>, Lay discounted this effect of initial imperfections and arrived at a similar reduction in the shear modulus by applying slip-line field theory).

In their investigations, Haaijer and Thurlimann presented analytical solutions for single web or flange plates assuming either simple support or fully rigid support at the web-to-flange junctions. The effects of residual stresses were not included directly in the analysis. However, for the buckling strength of columns and plates in uniaxial compression, an empirical transition curve was suggested for use above the proportional limit. The experimental investigation of W shapes included six axial specimens and six flexural specimens. The present code limitations for flanges and webs of Class 1 W shapes are based on the results of these tests<sup>4</sup>.

Although Haaijer and Thurlimann did not test any specimens subjected to axial and flexural loadings combined, they used the results of a semi-empirical method to suggest plate width-to-thickness values for such members. In this method it was assumed that values obtained for axial specimens failing by web buckling would be applicable to an average strain in the compression zone of the webs of beam-columns. The present code limitations for plate components of Class 1 W shape beam-columns are based on the results of this semi-empirical method<sup>4</sup>.

In the years following the work of Haaijer and Thurlimann many investigators published results of research concerning the elastic and inelastic buckling strength of single plates. With the aid of discretization techniques and computer methods it has been possible to investigate many different cases<sup>27,28,29,30,31,32</sup>. However, although





useful, none of these works is concerned directly with the understanding of local buckling behaviour in W shape members. More to the point, other studies have been directed towards local buckling in W shape beams. The studies of Lay<sup>33</sup>, Culver<sup>34</sup>, and McDermott<sup>35</sup>, for example, have all indicated a flange width-to-thickness limitation of  $b \sqrt{F_y}/2t = 54$  for sections required to reach the strain-hardening strain. Results of tests performed by Lukey and Adams<sup>36</sup> have indicated that a flange width-to-thickness term of 64 can be used for sections required to develop the full plastic moment capacity. A study by Basler and Thurlimann<sup>37</sup> has indicated that webs of girders required to reach  $M_y$  can have a value of  $h\sqrt{F_y}/w$  as high as 980. More recent research by Croce<sup>38</sup> has indicated that  $h\sqrt{F_y}/w$  can have a value as high as 750 for beams used in plastic design.

In 1973 a study of coupled local buckling in beam-columns was presented by Rajasekaran and Murray<sup>39</sup>. The method of analysis was based on finite element techniques and could accommodate a large variety of boundary conditions. The method assumed linear elastic material response and did not include the effects of residual stresses. It was found that the analysis gave good results for flange local buckling but web buckling could not be accurately predicted.

In 1977, Akay, Johnson, and Will presented a study of lateral and local buckling of beams and frames<sup>40</sup>. A finite element technique using plate elements for webs and line elements for flanges was used. The buckling modes were restricted by the assumption that straight lines across the flanges and normal to the web remain straight during buckling. Although the method is quite general with regard to



plate geometry and boundary conditions it assumes linear elastic response and neglects the effects of residual stresses.

In 1978, a study of local, distortional, and lateral buckling of W shape beams was presented by Hancock<sup>41</sup>. A finite strip technique was used in the analysis and an elastic, linear material response was assumed. The effects of residual stresses were not included in the analysis. Other studies using finite strip techniques were presented in 1964 by Plank and Wittrick<sup>42</sup> and in 1974 by Goldberg, Bogdanoff, and Glauz<sup>43</sup>, who extended the work of Przemieniecki<sup>32</sup>. Plank and Wittrick suggested a method for analysing thin-walled sections for lateral-torsional buckling. Presumably local buckling could be predicted by iterating on the length parameter. Plate thickness as well as geometry, material properties, and loading could be varied for a given member. However the effects of residual stresses were not included and apparently the method was not suitable for local buckling of W shapes since it was neither used, nor recommended, for this purpose.

As mentioned above, Goldberg, Bogdanoff, and Glauz<sup>43</sup> extended the work of Przemieniecki<sup>32</sup> to include lateral buckling modes as well as more complicated states of stress. An elastic material response was assumed and the effects of residual stresses were not included in the analysis. Again, the method was neither applied to, nor recommended for, the analysis of local buckling of W shapes.

None of the above-mentioned techniques has been applied to an in-depth study of local buckling of Class 1, Class 2, and Class 3 W shape sections. In 1973, Kulak initiated such a study on an experimental basis<sup>10</sup>. A total of ten beams (eight Class 2 and two



Class 1 sections) were tested under equal third-point loadings. Based on the results of these tests it was concluded that the existing web width-to-thickness limitations for Class 2 sections were conservative and a need for additional tests on beams and beam-columns was indicated. Other tests followed, and in 1975 two Class 3 beams were tested and an increase in the existing web limitations for Class 3 beams was indicated<sup>11</sup>.

In 1974 Kulak and Perlynn published the results of a study in which nine Class 2 W shape beam-columns were tested under various amounts of axial load<sup>12</sup>. Again it was determined that the existing web limitations were too conservative for Class 2 beam-columns and a need for additional tests on Class 3 beam-columns were indicated. In 1976, the results of such a study were reported by Kulak and Nash<sup>13</sup>. It was indicated that web limitations for Class 3 beam-columns could also be somewhat relaxed. As a result of the work carried out by Kulak, et al. at the University of Alberta, significant changes in the web limitation requirements for W shapes have been implemented for Class 2 and Class 3 sections<sup>4</sup>.

The investigations carried out by Kulak, et al, were largely experimental although two semi-empirical methods for determining critical web width-to-thickness ratios were presented<sup>12</sup>. Method I was based directly on test results and Method II combined test results with a variation of the method used by Haaijer and Thurlimann<sup>7</sup>. Because of the limited number of test results used, the methods were valid only for sections that were similar to those tested. Furthermore, the methods did not allow for variations in flange sizes, lengths of specimens, residual stresses, and material properties. For the types of



specimens tested, the methods were valid only between the limits of  $P/P_y$  equal to 0.15 and 0.80 since these were the lower and upper limits used in the tests. A purely analytical method was not developed.





## Chapter 3

### GENERAL ANALYTICAL METHOD

#### 3.1 Introduction

The problem of plate buckling in the elastic range has been thoroughly investigated and solutions are available for many cases including various plate shapes and stress and displacement boundary conditions<sup>2,3,16,44</sup>. Solutions for the analysis of orthotropic plates and for buckling capacities of plates in the inelastic range have also been published<sup>2,6,18,44</sup>. The technique for obtaining a mathematical formulation for such problems is based on either an equilibrium method or an energy method<sup>2</sup>.

In the equilibrium method<sup>2,6</sup>, the equations of equilibrium are formulated on a deformed configuration of the plate. This configuration is compatible with the expected mode of buckling. Once the equations of equilibrium are solved simultaneously, the problem reduces to that of the solution of a biharmonic differential equation. This method makes use of the fact that, during buckling, a plate may be in equilibrium in its original planar configuration as well as in a neighboring buckled configuration; that is, the plate is at a point of bifurcation.

Two commonly used energy techniques for formulating plate buckling problems are the principle of minimum potential energy and the principle of virtual work<sup>8</sup>. Of these two methods, the principle of virtual work is a more general statement of the principle of the



conservation of energy. It does not require the assumption of the existence of a strain energy function and it can be applied to an elastic or an inelastic material<sup>8,45</sup>. In the present analysis the principle of virtual work is used for the formulation of a plate buckling condition. In applying this method, a buckled configuration is first assumed. Then, a virtual displacement<sup>46</sup> from a buckled configuration is postulated. By equating the internal work done by the equilibrium stress field existing in a plate during this virtual displacement to the work done by the external forces acting on the plate during the same displacement, an integral differential equation is obtained. Using a Rayleigh-Ritz technique<sup>5,46,47</sup> and a displacement field defined in terms of a set of nodal displacement coordinates, a matrix buckling condition is obtained and a standard eigenvalue problem results<sup>48,49,50</sup>. This technique is developed in detail in Appendix A.

### 3.2 Idealized Cross-section

The technique outlined above is applied to W shapes having idealized cross-sections such as that illustrated in Figure 3.1. All subsequent mathematical formulations are referred to the mid-planes of the component plates of a W shape. Thus, the assumed height of web extends into each flange by a distance equal to one-half the flange thickness. Consequently, the area corresponding to each projection is twice included in the calculations of axial loads and bending moments. The purpose of this is to recognize that fillets at web-to-flange junctions do exist and that these additional areas of projection do, to some extent, account for them.



The width-to-thickness ratio for a flange is obtained by dividing one half the flange width by the flange thickness. In determining the width-to-thickness ratio of a web it is assumed that web-to-flange fillets have leg lengths equal to one-half the flange thickness. Therefore, the length of the web is taken as the clear distance between flanges minus the fillet leg lengths. The purpose of this is to take into account, to some extent, the effects of the fillets in decreasing the effective buckling height of a web.

As shown in Figure 3.1, variations in flange dimensions are permitted by using different flange widths and thicknesses. In subsequent discussions, in addition to the notations shown in Figure 3.1,  $A_l$  represents the area of a lower flange,  $A_w$  represents the area of a web, and  $A_u$  represents the area of an upper flange. Furthermore, for clearness of discussion, it is assumed that an upper flange is one that will normally be in compression and a lower flange will normally be in tension under an applied bending moment. The subscripts, l and u, are used to refer to the lower and upper flanges respectively.

### 3.3. Material Properties

In applying any mathematical technique to the prediction of plate buckling capacities it is necessary to have an accurate evaluation of the material properties to be used. In the method presented herein, a uniaxially stressed longitudinal fibre of a W shape section is assumed to have an idealized tri-linear tensile stress - strain response such as that shown in Figure 3.2. It is further assumed that this stress - strain response also applies to a fibre in compression. At the point where yielding occurs in a fibre, the strain



is designated by  $\epsilon_y$  and the corresponding yield stress is designated by  $\sigma_y$ . For values of strain less than the yield strain, the fibre has an elastic modulus (also called Young's modulus) designated by  $E$ . At the point where strain-hardening of a fibre begins, the strain is represented by  $\epsilon_{st}$ . For strains larger than this, the fibre displays an increased resistance to further straining. This fibre stiffness is evaluated as the strain-hardening modulus,  $E_{st}$ .

In the intermediate range of strain between  $\epsilon_y$  and  $\epsilon_{st}$ , the fibre yields and a yield modulus,  $E_0$ , represents the slope of this portion of the curve. If the value of  $E_0$  is zero there exists no explicit and definable relationship between stress and strain. That is, at the yield stress level, a fibre can assume an arbitrary value of strain and, if the direction of loading is such that strains tend to increase, it is likely that the strain-hardening modulus will govern the behaviour of a fibre that has yielded. The assumption that strain-hardening material properties govern buckling behaviour for strains above the yield strain has been successfully used by several investigators<sup>25,34,51,52,53,54</sup> and this assumption is also made in the present investigation.

### 3.4 Analytical Technique

As explained in section 3.1 a plate buckling condition is derived using the principle of virtual work. This derivation is presented in detail in Appendix A and it results in an integral differential equation for the buckling condition. For a uniaxially stressed orthotropic plate, the buckling condition is as follows:





$$\int_x \int_y (D_x w,_{xx} \delta w,_{xx} + D_y w,_{yy} \delta w,_{yy} + D_{xy} w,_{yy} \delta w,_{xx} + D_{yx} w,_{xx} \delta w,_{yy} + 4G_t I w,_{xy} \delta w,_{xy}) dx dy - \int_x \int_y N_x w,_{x} \delta w,_{x} dx dy = 0$$

(A-35)

where  $D_x$ ,  $D_y$ ,  $D_{xy}$ , and  $D_{yx}$  are plate bending rigidities,  $G_t$  is the tangent shear modulus, and  $I$  is the moment of inertia per unit length of plate. These properties are further discussed in Appendix B. Also in Equation A-35,  $w$  represents the deflection of a point normal to the middle plane of the plate,  $\delta w$  represents a virtual deflection in this direction,  $x$  and  $y$  represent Cartesian coordinate directions, and differentiation is indicated using comma notation<sup>47</sup>.

In Equation A-35 the first integral represents the virtual work resulting from the strain energy of plate bending while the second integral represents the virtual work done by the in-plane stresses which act during buckling.

Once a plate buckling condition has been formulated into a mathematical expression using an energy method, one may try to obtain an exact mathematical solution or an approximate solution. Because of the complexity of the present formulation, solutions are obtained using the Rayleigh-Ritz technique<sup>48</sup>. This method has also been applied quite successfully by other investigators<sup>4,47,48,49,50</sup>.

A basic assumption of this technique is that the displacement field describing a buckled shape can be expressed in terms of a set of assumed shape functions and corresponding coordinate displacements<sup>55</sup>.



Together these must form a set of kinematically admissible generalized coordinate displacements which requires that the assumed displacement function satisfy the boundary conditions of the physical problem. The development of appropriate shape functions is discussed in a later section. However, it is appropriate here to state that a displacement function can be defined as follows:

$$w = f\langle\phi\rangle\{\theta\} \quad (3.1)$$

where  $\{\theta\}$  is a vector of displacement coordinates defined at distinct nodal points in a cross-section (Figure 3.4) and  $\langle\phi\rangle$  is a set of shape functions which interpolate the coordinate displacements over a cross-section. The function,  $f$ , is a shape function which interpolates a set of cross-sectional displacements over the length of a member. By substituting this assumed displacement function into Equation A-35, the problem is reduced from a continuum problem, with an infinite number of degrees of freedom, to a problem with a finite number of degrees of freedom equal to the total number of coordinate displacements defined at the nodes. As a result, a system of algebraic equations, rather than a partial differential equation, must be solved. This procedure is carried out in detail in Appendix A where it is shown that the problem of local buckling reduces to the form:

$$\left[ [K] - \lambda [K_G] \right] \{\theta\} = \{0\} \quad (A-51)$$

where  $[K]$  is a bending stiffness matrix depending on material properties and plate dimensions,  $[K_G]$  is a geometric stiffness matrix depending on the distribution and magnitude of applied in-plane stresses, and  $\lambda$  is a



multiple of the applied loading which causes buckling.

The expression in brackets represents the reduced stiffness of a plate which buckles when, depending upon the value of  $\lambda$ , the determinant of the reduced stiffness matrix becomes zero. The corresponding values of  $\lambda$  and  $\{\theta\}$  are referred to as the eigenvalue and eigenvector, respectively. The eigenvalue in this case is the critical stress at which a plate buckles and the eigenvector defines the critical buckled shape.

The solution to the buckling problem as presented herein reduces to the problem of extracting the lowest eigenvalue from the system of equations expressed by Equation A-51. If, in this equation,  $[K]$  and  $[K_G]$  are each of dimension  $n \times n$ , then the characteristic equation<sup>50</sup> of the reduced stiffness matrix will have  $n$  roots, each one corresponding to an eigenvalue of the system. However, in problems concerning statical stability only the lowest of these roots is of interest. A method that is commonly used to find the smallest eigenvalue of a system, and also one that is readily adaptable to electronic computation, is that of inverse matrix iteration<sup>50</sup>.

Using this technique, Equation A-51 is first rewritten as:

$$[K]\{\theta\} = \lambda[k_G]\{\theta\} \quad (3.2)$$

Multiplying both sides of Equation 3.2 by the inverse of  $[K]$  and dividing by  $\lambda$  result in:

$$[K]^{-1}[K_G]\{\theta\} = \frac{1}{\lambda}\{\theta\} \quad (3.3)$$



or:

$$[E]\{\theta\} = \omega\{\theta\} \quad (3.4)$$

where,

$$[E] = [K]^{-1}[K_G] \quad (3.5)$$

and,

$$\omega = \frac{1}{\lambda}. \quad (3.6)$$

In the solution of Equation 3.4, an initial shape vector,  $\{\theta\}_0$ , is assumed and this vector is multiplied by matrix  $[E]$ . The resulting vector,  $\{\theta\}_1$ , is then normalized by dividing by the highest-valued element of the vector. The process is then repeated with this new normalized vector. It has been shown<sup>50</sup> that, after several cycles of iteration,  $\omega$  converges to the highest eigenvalue of the system. As a result, the lowest critical value of  $\lambda = 1/\omega$  is obtained for the original system, Equation 3.2, and the corresponding eigenvector gives the critical buckled shape.

In certain eigensystems the smallest eigenvalue may appear as a positive value or a negative value. For example, the bending stress required to cause local buckling in a doubly-symmetric W shape may occur as a positive or negative eigenvalue of the associated eigensystem. This result is due solely to the symmetry of the system and the positivity or negativity of the eigenvalue has no physical significance. It does however result in considerable mathematical difficulty in achieving convergence during matrix iteration. This is because the rate of convergence is proportional to the ratio of the





lowest eigenvalue to the next higher value<sup>50</sup>, and for values of this ratio approaching 1.0 the rate of convergence is very slow. For problems of the type described above, the first two eigenvalues of the system are equal but of opposite sign; that is,  $\omega_1 = -\omega_2$ . Therefore, the rate of convergence is  $\omega_1/\omega_2 = -1.0$  and the technique will not converge.

To avoid this problem, a constant shift,  $\mu$ , is applied to all of the eigenvalues of the system, in which case the rate of convergence is proportional to  $(\omega_1 - \mu)/(\omega_1 + \mu) < 1.0$  and the method will converge. Applying a shift,  $\mu$ , to the system, Equation 3.4 becomes:

$$[E]\{\theta\} - \mu\{\theta\} = \omega\{\theta\} - \mu\{\theta\} \quad (3.7)$$

or,

$$[[E] - \mu[I]]\{\theta\} = \omega_s\{\theta\} \quad (3.8)$$

where  $\omega_s = \omega - \mu$  and  $[I]$  is an identity matrix of the same dimension as  $[E]$ . It has been shown<sup>50</sup> that the eigensystem defined by Equation 3.8 has the same eigenvectors as those defined by the original system described by Equation 3.4. The relationship between the eigenvalues of the two systems is given by:

$$\omega - \mu = \omega_s \quad (3.9)$$

or,

$$\omega = \frac{1}{\lambda} = \omega_s + \mu \quad (3.10)$$

In subsequent sections, the eigenvalue problem is formulated in such a way that only the smallest positive eigenvalue is



required. Although the eigenvalue shift technique increases the rate of convergence in most cases, it does not guarantee the convergence will be to the lowest positive root. In some cases it is found that the process converges to the lowest negative root. It is desirable therefore to have a method for eliminating certain eigenpairs from the system so that inverse iteration can be repeated to determine the lowest positive root. The method used to do this in the present analysis is referred to as a sweeping technique<sup>48,50</sup>.

Since the eigenvectors,  $\{\phi\}_i$ , of an eigensystem are linearly independent<sup>50</sup>, they provide a vector basis for the system, and any starting vector,  $\{\theta\}$ , may be expressed as a linear combination of these eigenvectors, that is:

$$\{\theta\} = \sum_{i=1}^n c_i \{\phi\}_i \quad (3.11)$$

If it is desired to remove the  $k$ th eigenmode from the system, a starting vector should be selected as follows:

$$\{\theta\}_o = \{\theta\} - c_k \{\phi\}_k \quad (3.12)$$

The value of  $c_k$  is found by pre-multiplying Equation 3.11 by  $\langle \phi \rangle [K_G]$  and using the orthogonality properties<sup>50</sup> of eigenvectors. Doing so, results in:

$$c_k = \frac{\langle \phi \rangle_k [K_G] \{\theta\}}{\langle \phi \rangle_k [K_G] \{\phi\}_k} \quad (3.13)$$

Substituting this value into Equation 3.12 gives:



$$\begin{aligned}
\{\theta\}_o &= \left[ [I] - \frac{\{\phi\}_k \langle \phi \rangle_k [K_G]}{\langle \phi \rangle_k [K_G] \{\phi\}_k} \right] \{\theta\} \\
&= [S]_k \{\theta\}
\end{aligned} \tag{3.14}$$

where  $[S]_k$  is the desired sweeping matrix. Using this value of  $\{\theta\}_o$  as a starting value, Equation 3.8 becomes:

$$\left[ [E] - \mu [I] \right] [S]_k \{\theta\} = \omega_s \{\theta\} \tag{3.15}$$

which represents an eigensystem with the  $k^{\text{th}}$  mode removed. In the analytical technique presented herein this method is used to remove negative eigenvalues and corresponding eigenvectors from a system. It has been found that only one application of a sweeping matrix is required for the majority of cases where the first eigenvalue calculated is negative.

### 3.5 Effects of Initial Imperfections

As described previously, the present analysis uses a precise mathematical formulation to describe the local buckling condition. This implies that the maximum strength of a plate or system of plates is limited only by critical local buckling and this is the basis on which the formulation is made. The assessment of the effects of initial imperfections on the buckling strength of plates is also based on this type of formulation.

It is assumed for this purpose that  $\{\theta_o\}$  is a set of initial



coordinate displacements defined at the nodes of a plate system. Using the same interpolation functions as those in Equation 3.1, initial deflections may be described by:

$$w_o = f\langle\phi\rangle\{\theta_o\} \quad (3.16)$$

and additional deflections due to applied in-plane loads are given by:

$$w_1 = f\langle\phi\rangle\{\theta\} \quad (3.17)$$

Therefore, the total out-of-plane deflection at any point is given by:

$$\begin{aligned} w &= w_o + w_1 \\ &= f\langle\phi\rangle\{\theta_o\} + f\langle\phi\rangle\{\theta\} \end{aligned} \quad (3.18)$$

To evaluate the effects of initial imperfections, the appropriate values of deflection are substituted into the buckling condition, Equation A-35. Since the first integral in Equation A-35 represents the work done by bending of a plate from its initially deflected position to its buckled shape, the net deflection,  $w - w_o$ , must be used to evaluate this integral<sup>2</sup>. The second integral in Equation A-35 represents the work done by the in-plane forces during buckling of a plate. It can be evaluated by calculating the work done by the in-plane forces acting through displacements caused by initial imperfections only. The net work is then obtained by subtracting this value from the work done by the in-plane forces acting through displacements caused by the total deflection of a plate. By proceeding in this manner, and using the concepts of bending and geometric stiffness matrices, as used in Section 3.4, Equation A-35 results in the following set of equations:





$$\left[ [K] - \lambda [K_G] \right] \{\theta\} = \lambda [K_G] \{\theta_o\} \quad (3.19)$$

From this relationship it can be seen that the effect of initial imperfections is to simulate an equivalent lateral load of:

$$\{F_o\} = \lambda [K_G] \{\theta_o\} \quad (3.20)$$

The effect of this equivalent lateral load is to cause deflections to increase gradually as the in-plane loads are increased. However, the mathematical definition of buckling can be expressed by setting the determinant of the reduced stiffness matrix equal to zero. Since the reduced stiffness matrix formulated with initial imperfections included is identical to that for the case of a perfectly straight plate, initial imperfections do not affect the value of the critical buckling load as defined herein. As the in-plane loads increase, lateral deflections increase gradually and as the critical buckling load is approached the lateral deflections become asymptotic to the critical buckled shape.

### 3.6 Coordinate Systems

In the formulation of the theory for local buckling, the shape functions are referred to a system of natural coordinates<sup>50,55</sup>. Figure 3.3. illustrates the relationship between a natural coordinate system for a plate of length  $\ell$ , and a Cartesian coordinate system. Points 1, 2, and 3 in this figure are referred to as nodes at which the coordinate displacements comprising the vector,  $\{\theta\}$ , are defined. At these nodes the elements of  $\{\theta\}$  may represent translations,



rotations, curvatures, and higher order derivatives. Using these coordinates and an appropriate set of interpolating polynomials it is possible to define a set of shape functions at a cross-section for the flanges and web of a W shape section.

Local natural coordinate systems for the flanges and web of a W shape cross-section as well as the corresponding node numbering system are shown in Figure 3.4. Each coordinate displacement defined at a given node is interpolated by a polynomial function over the cross-sectional edge of a flange or a web. The order of each such interpolating polynomial is equal to the number of coordinate displacements that must be interpolated for a flange or web. Thus, for example, if a translation and a rotation are defined for each of three nodes of a flange, a quintic polynomial interpolating function is used for each nodal displacement.

### 3.7 Flange Shape Functions

Polynomial functions used to interpolate coordinate displacements along a plate edge at a cross-section may be obtained by a matrix technique<sup>55</sup> or by inspection. Since the latter method is used herein it is described in detail below. Quintic polynomials are used to interpolate the nodal displacements for flanges at a cross-section. At each of three nodes on a flange, a translation and a rotation are interpolated. These interpolation functions and their shapes are shown in Figure 3.5.

The method of inspection for obtaining shape functions is presented as an illustration for the particular case of interpolating a translation at node number one of Figure 3.4. Since a total of six



coordinate displacements are defined (a translation and a rotation at each of three nodes) a fifth order polynomial is first assumed as follows:

$$S(\zeta) = \zeta^2(\zeta-1)^2(a_0\zeta+b_0) \quad (3.21)$$

A shape function evaluated at the coordinate displacement being interpolated must have a value of 1.0 and must have a zero value when evaluated at all other coordinate displacements. The first factor of the above function ensures that the translation and rotation at  $\zeta=0$  are both zero. The second factor ensures that the translation and rotation at  $\zeta=1$  are zero. The third factor is chosen so that the function will be a fifth order polynomial. Also, the constants,  $a_0$  and  $b_0$ , can be determined so that at  $\zeta=-1$ , the rotation is zero and the translation is positive and unity. Evaluating  $a_0$  and  $b_0$  for these two conditions results in the function:

$$S(\zeta) = \frac{1}{4}\zeta^2(\zeta-1)^2(3\zeta+4) \quad (3.22)$$

The value of this expression is positive unity at the coordinate displacement being interpolated while at all other coordinate displacements its value is zero.

### 3.8 Web Shape Functions

Octic polynomials are used to interpolate nodal displacements for webs at a cross-section. For the purpose of studying local buckling of W shapes, it is assumed that the line of intersection of two plates at a flange-to-web junction does not translate during buckling. This line appears as a nodal point on a cross-section of a W shape and the



corresponding translations are not interpolated for the flange or web since they have zero value. Therefore, octic polynomials are used to interpolate rotations and curvatures of a web at its extremities and translation, rotation, and curvature at its center for a given cross-section. The interpolating polynomials for webs are obtained by the method of inspection as described previously for flanges. The polynomials and their corresponding shapes for coordinate displacements of a web at a cross-section are shown in Figure 3.6.

### 3.9 Longitudinal Shape Functions

A buckled shape which has been established at a cross-section of a W shape member must be interpolated over the length of the member. For plates simply supported at the loaded edges and subjected to uniaxial stresses, it has been determined<sup>2,6</sup> that the buckled shape in the direction of applied stress occurs in the form of a sine wave. This result is inherent in the nature of the solution to a partial differential equation which defines the plate buckling of a simply supported rectangular plate subjected to uniaxial stresses.

For this reason, and because the Rayleigh-Ritz solution is not very sensitive to the actual shapes used<sup>4,47,48,50</sup>, a sine shape is included as a principle component of longitudinal shape functions used in this study. As shown in Figure 3.7, the effect of various boundary conditions can be accounted for by multiplying a sine function by a polynomial function which adequately describes the boundary conditions. Essentially, this technique applies an envelope to a sine function.

The general form of a longitudinal shape function is  $P_0(\xi)\sin m\pi\xi$ . In this form  $P_0(\xi)$  represents a polynomial envelope of a





sine function where  $m$  is the number of half sinewaves that occurs along the length of a plate during buckling. The complete buckled shape of a plate component of a W shape is given by:

$$w = P_0(\xi) \sin m\pi \xi \langle \phi \rangle \{ \theta \} \quad (3.23)$$

As described previously,  $\langle \phi \rangle \{ \theta \}$  describes the buckled shape of a flange or a web at a cross-section. Each polynomial of  $\langle \phi \rangle$  is a function of  $\zeta$  when referring to a flange, and a function of  $\eta$  when referring to a web.  $\xi$  is the natural coordinate in the longitudinal direction of a member. The natural coordinates,  $\xi$ ,  $\zeta$ , and  $\eta$  and the corresponding natural coordinate systems are defined in Figure 3.8 for a W shape.

### 3.10 Integration of Cross-Sectional Shape Functions

As is evident from the formulation presented in Appendix A, extensive use of integration and differentiation of shape functions is required. Furthermore, shape functions expressed in terms of natural coordinates must be integrated and differentiated with respect to Cartesian coordinates. The function in Equation 3.22, for example, may be integrated as follows:

$$\int_0^{b/2} s(\zeta) dy = \frac{b}{2} \int_0^1 s(\zeta) d\zeta \quad (3.24)$$

In this operation, the limits of integration and the differential element  $dy$  in the Cartesian coordinate system were transformed to a natural coordinate system using the relationship,

$$\zeta = \frac{2y}{b} \quad (3.25)$$



as defined in Figure 3.5. Differentiation of the function in Equation 3.22 proceeds as follows:

$$\frac{\partial S(\zeta)}{\partial y} = \frac{\partial S}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial y} = \frac{2}{b} \frac{\partial S}{\partial \zeta} \quad (3.26)$$

where the chain rule of differentiation<sup>56</sup> and the relationship of Equation 3.25 were used.

As indicated by Equations A-42 to A-51, it is necessary to integrate many different functions and products of functions in order to obtain a solution for a given problem. These operations are best performed by computer if they are first expressed in terms of matrices. As an illustration of the technique, the following integral from Equation A-48 will be considered:

$$[\Phi_2] = \int_y \{\phi,_{yy}\} \langle \phi,_{yy} \rangle dy \quad \text{A-48}$$

A set of shape functions,  $\{\phi\}$ , may be expressed by:

$$\{\phi\} = [C]\{H\} \quad (3-27)$$

where  $[C]$  is a matrix of constant coefficients of shape functions and,

$$\{H\} = \left\{ \begin{array}{c} 1 \\ \zeta \\ \zeta^2 \\ \zeta^3 \\ \zeta^4 \\ \zeta^5 \end{array} \right\} \quad (3.28)$$

for a quintic polynomial. Differentiating Equation 3.27 twice with



respect to  $y$  gives:

$$\begin{aligned} \{\phi,_{yy}\} &= [C]\{H,_{yy}\} \\ &= \frac{4}{b^2} [C] \begin{Bmatrix} 0 \\ 0 \\ 2 \\ 6\zeta \\ 12\zeta^2 \\ 20\zeta^3 \end{Bmatrix} \end{aligned} \quad (3.29)$$

where the relationship given by Equation 3.25 was used.

Using the relationships of Equation 3.29, the integrand of Equation A-48 may be written as follows:

$$\begin{aligned} \{\phi,_{yy}\} \langle \phi,_{yy} \rangle &= [C]\{H,_{yy}\} \langle H,_{yy} \rangle [C]^T \\ &= \frac{16}{b^2} [C] \begin{Bmatrix} 0 \\ 0 \\ 2 \\ 6\zeta \\ 12\zeta^2 \\ 20\zeta^3 \end{Bmatrix} \langle 0 \ 0 \ 2 \ 6\zeta \ 12\zeta^2 \ 20\zeta^3 \rangle [C]^T \\ &= \frac{16}{b^2} [C] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 12\zeta & 24\zeta^2 & 40\zeta^3 \\ 0 & 0 & 12\zeta & 36\zeta^2 & 72\zeta^3 & 120\zeta^4 \\ 0 & 0 & 24\zeta^2 & 72\zeta^3 & 144\zeta^4 & 240\zeta^5 \\ 0 & 0 & 40\zeta^3 & 120\zeta^4 & 240\zeta^5 & 400\zeta^6 \end{bmatrix} [C]^T \end{aligned} \quad (3.30)$$



The integral of Equation A-48 may now be evaluated by integrating each term in the matrix of variables in Equation 3.30, to obtain the following:

$$\int_y \{\phi,_{yy}\} \langle \phi,_{yy} \rangle dy =$$

$$\frac{16}{b^2} [C] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4\zeta & 6\zeta^2 & 8\zeta^3 & 10\zeta^4 \\ 0 & 0 & 6\zeta^2 & 12\zeta^3 & 18\zeta^4 & 24\zeta^5 \\ 0 & 0 & 8\zeta^3 & 18\zeta^4 & 24\zeta^5 & 40\zeta^6 \\ 0 & 0 & 10\zeta^4 & 24\zeta^5 & 40\zeta^6 & (400/7)\zeta^7 \end{bmatrix}^{\ell_2}_{\ell_1} [C]^T \quad (3.31)$$

where the elements of the matrix of variables are to be evaluated for the integration limits,  $\ell_1$  and  $\ell_2$ . This method permits the use of computer techniques to obtain the integral of functions between any limits of integration. This is important because, depending on the level of stress and the stress distribution, longitudinal strips of a flange or a web may be elastic, yielded, or strain-hardened. Therefore the integration over a cross-section of a plate component is performed in a piecewise fashion between the limits of these zones so that the differing material properties may be included in the evaluation of a stiffness matrix. In this way also, abrupt changes in stress distribution may be accommodated in evaluating a geometric stiffness matrix.

### 3.11 Integration of Longitudinal Shape Functions

As shown in the formulation of the analytical technique in





Appendix A, extensive integration is also required for longitudinal shape functions. For example, factor  $F_1$  in Equation A-42 is expressed as:

$$F_1 = \int_x f_{,xx}^2 dx \quad (A-42)$$

As discussed in Section 3.9, the function,  $f(\xi)$ , may be of the form,

$$f(\xi) = P_o(\xi) \sin(m\pi\xi) \quad (3.32)$$

where  $P_o(\xi)$  is a polynomial function. As a result, the functions to be integrated over the length of a member may be quite complicated expressions depending on the order of the polynomial,  $P_o(\xi)$ , and therefore a numerical integration technique is used. The length of a member is divided into a number of equal intervals over which Gaussian quadrature<sup>50</sup> is used to integrate a function. The total integral over a length is obtained by adding together these sub-integrals. The number of intervals used for a given length is equal to the number of half sine wavelengths along a member and a 6-point Gauss integration<sup>50</sup> technique is performed for each interval.

### 3.12 General Procedure for W shapes

The procedure described in Section 3.4 is used to develop a bending stiffness matrix and a geometric stiffness matrix for the web and each flange of a W shape. These represent stiffness submatrices that must be assembled into overall stiffness matrices for an entire member. In assembling the matrices, compatibility between the flanges and web at a cross-section is maintained by enforcing zero



relative rotation between the plate components at a web-to-flange junction.

Figure 3.9 illustrates the node numbering and coordinate displacements for a typical W shape cross-section. The notations of this figure will be used in describing the assembly of a stiffness matrix. Since a bending stiffness matrix and a geometric stiffness matrix are both assembled in exactly the same manner the procedure is illustrated for a bending stiffness matrix assembly only. In Section 3.4 it was shown that a plate buckling problem reduced to an eigenvalue form as shown by Equation A-52 which can be rewritten as follows:

$$[K]\{\theta\} = \lambda[K_G]\{\theta\} \quad (3.33)$$

The bending strain energy of a component plate during buckling results in the term on the left hand side of this equation. Therefore, assembling the stiffness submatrices of the two flanges and the web of a W shape section to obtain the assembled stiffness matrix is, in effect, equivalent to adding together the plate bending strain energies of the individual component plates. Using the notation given in Figure 3.9, the left hand side of Equation 3.33 may be written as;

$$[K]\{\theta\} = [K_1] \begin{Bmatrix} u_1 \\ u_1' \\ u_2 \\ u_2' \\ u_3' \end{Bmatrix} \quad (3.34)$$



for a lower flange;

$$[K]\{\theta\} = [K_w] \begin{Bmatrix} u_3' \\ u_3'' \\ u_4' \\ u_4'' \\ u_5' \\ u_5'' \end{Bmatrix} \quad (3.35)$$

for a web; and

$$[K]\{\theta\} = [K_u] \begin{Bmatrix} u_5' \\ u_6 \\ u_6' \\ u_7 \\ u_7' \end{Bmatrix} \quad (3.36)$$

for an upper flange. The slopes  $u_3'$  and  $u_5'$  represent the rotations of the flanges and web at the web-to-flange junctions of a W shape cross-section. The fact that  $u_3'$  is common to Equations 3.34 and 3.35 and  $u_5'$  is common to Equations 3.35 and 3.36 ensures compatibility (in this case, rigidity of attachment) between the flanges and web. Therefore in assembling the stiffness submatrices,  $[K_l]$ ,  $[K_w]$ , and  $[K_u]$ , to obtain the total stiffness matrix of a W shape, stiffness elements of the submatrices corresponding to  $u_3'$  and  $u_5'$  must be added directly. This procedure is illustrated schematically in Figure 3.10.



### 3.13 Iteration on the Number of Wavelengths

The vector of coordinate displacements,  $\{\theta\}$ , represents the amplitudes of the shape functions defining the buckled shape of a cross-section. This vector is automatically obtained as a natural part of the process of matrix iteration<sup>50</sup> for each assumed value of  $m$ , where  $m$  is the number of half sine wavelengths of buckling along the length of a member. The correct value of  $m$  is that value for which the energy of a system is a minimum since this represents the lowest energy state of the buckled configuration<sup>2</sup>.

In an actual solution, a starting value of  $m = 1$  is assumed. This value is successively incremented in steps of unity and at each increment a matrix iteration is performed to determine an eigenvalue and the corresponding critical buckled shape of a cross-section. As  $m$  is incremented, the critical eigenvalue continues to decrease until a point of minimum potential energy of a system is reached. Thereafter, increases in  $m$  cause an increase in the potential energy. The correct value of  $m$  is that value for which the potential energy of the system is a minimum and the corresponding eigenpair gives the critical stress and the buckled shape of a cross-section.

### 3.14 Effect of Residual Stresses

Residual stresses acting on a W shape section alter the characteristics of a geometric stiffness matrix and therefore influence the value of a critical stress. In order to include this effect in the analysis, a residual strain pattern, as shown in Figure 3.11, is assumed. Such a pattern allows for a fairly general representation and has been used successfully by other investigators<sup>37,51,57,58,59</sup> in





assessing the effects of residual stresses. At the outset of a problem, values must be specified for  $\epsilon_2$ , the residual tension strain at the lower edge of a web,  $\epsilon_3$ , the residual compressive strain at mid-depth of a web, and  $\epsilon_4$ , the residual tension strain at the upper edge of a web. The values of  $\epsilon_1$  and  $\epsilon_5$ , the residual strains at the lower and upper flange tips respectively, are then determined using the following conditions of equilibrium for a cross-section subjected to residual stresses:

$$\Sigma F = 0 \quad (3.37)$$

$$\Sigma M = 0 \quad (3.38)$$

Equation 3.37 is an expression of translational equilibrium of a cross-section in the longitudinal direction of a member, and Equation 3.38 expresses equilibrium of a cross-section with respect to rotation about an axis perpendicular to the web.

The residual strain pattern of Figure 3.11 may be transformed into a stress pattern using a simple stress - strain relationship as shown in Figure 3.2. The forces and moments due to the residual stresses are obtained by integrating the effects of the stresses over the web and each flange of a W shape. Performing this integration, Equations 3.37 and 3.38 may be written as follows:

$$\Sigma F = 2A_u(\sigma_5 - \sigma_4) + 2A_1(\sigma_1 - \sigma_2) + A_w(2\sigma_3 - \sigma_4 - \sigma_1) = 0 \quad (3.39)$$

and,

$$\Sigma M = \frac{A_w h}{24} (6\sigma_3 - 5\sigma_4 - \sigma_2) + \frac{h}{A_2} (\sigma_5 - \sigma_4) = 0 \quad (3.40)$$



where,  $\sigma_i (i=1,2,3,4,5)$  are stresses corresponding to the strains  $\epsilon_i (i=1,2,3,4,5)$ ,  $A_l$  is the area of a lower flange,  $A_u$  is the area of an upper flange,  $A_w$  is the web area, and  $h$  is the web height.

Solving Equations 3.39 and 3.40 simultaneously and using Figure 3.2 to transform from stress to strains, results in the following values:

$$\epsilon_1 = \epsilon_4 - \frac{A_w}{12A_u} (\epsilon_2 - 6\epsilon_3 + 5\epsilon_4) \quad (3.41)$$

$$\epsilon_5 = \epsilon_2 - \frac{A_w}{12A_l} (5\epsilon_2 - 6\epsilon_3 + \epsilon_4) \quad (3.42)$$

In subsequent formulations, residual strains are added to applied strains so that their effects on yielding and strain-hardening as well as their effects on decreasing a geometric stiffness matrix are included directly in the analytical technique.



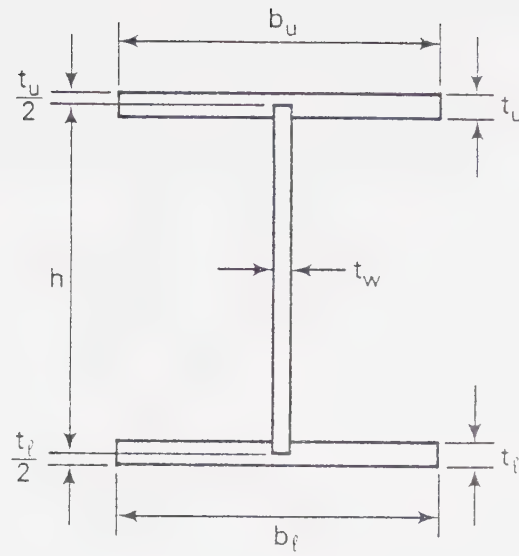


Figure 3.1 Idealized Cross-section

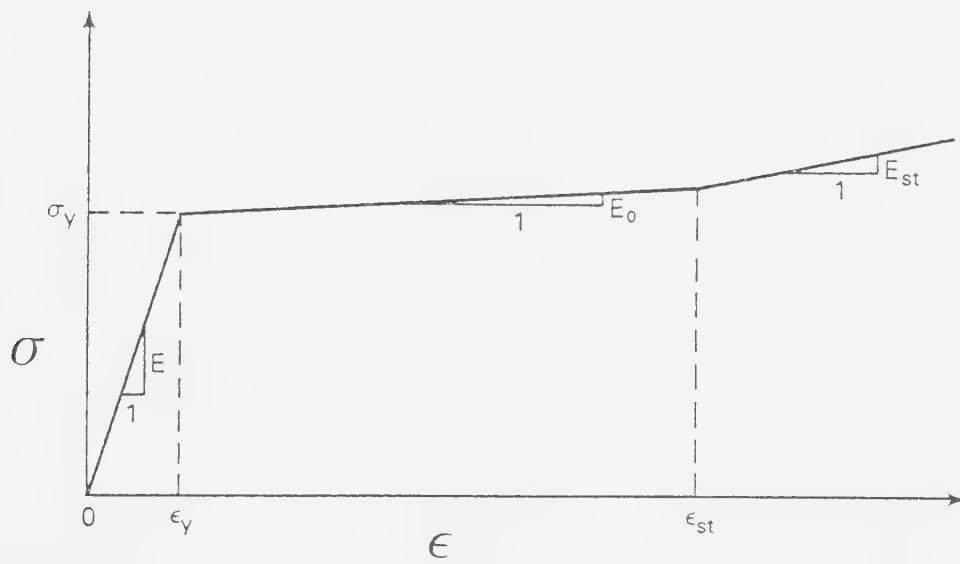


Figure 3.2 Idealized Tri-linear Stress-Strain Curve



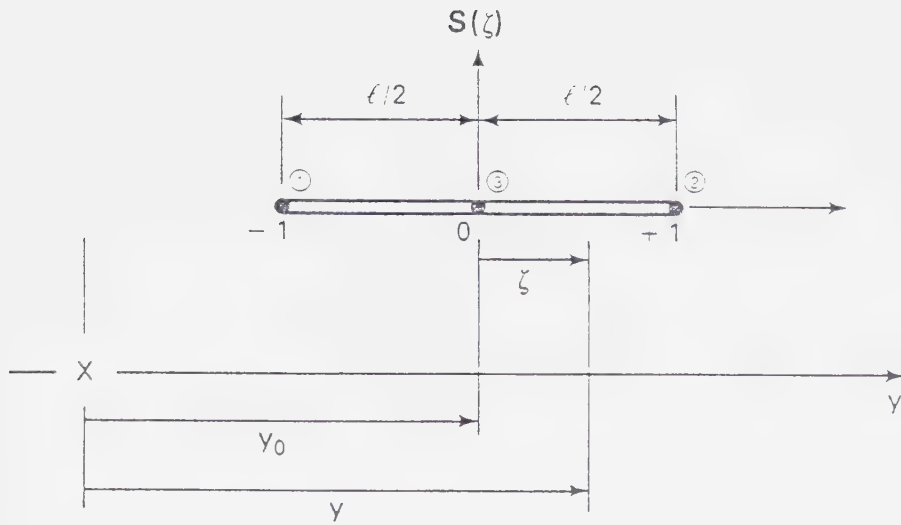


Figure 3.3 Relationship Between Cartesian and Natural Coordinates

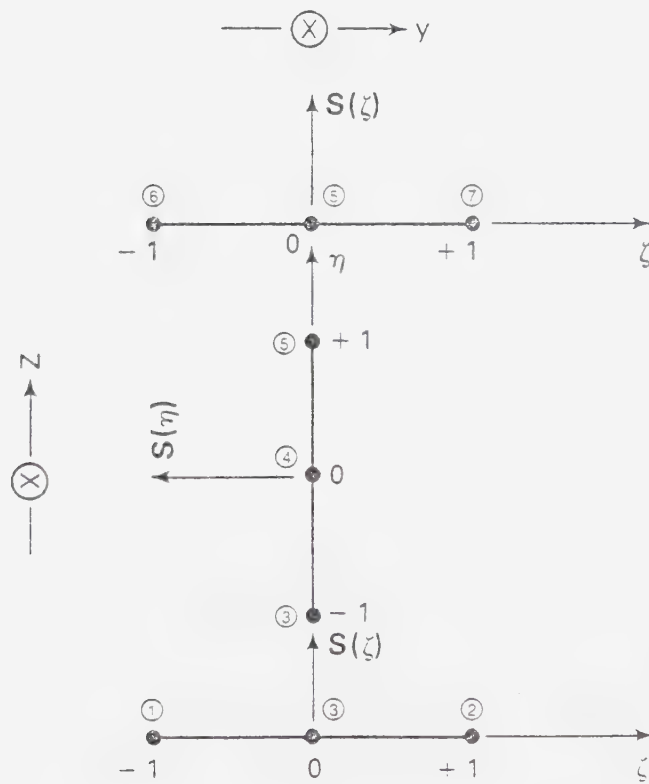


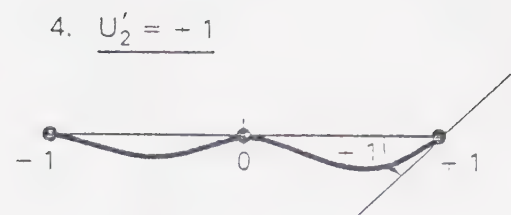
Figure 3.4 Local Coordinates and Numbering System



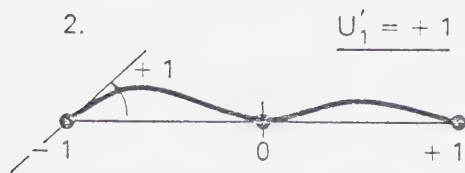




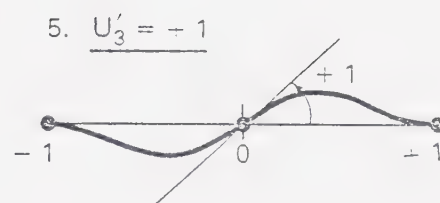
$$\phi_1 = \frac{1}{4} \zeta^2 (\zeta - 1)^2 (3\zeta + 4)$$



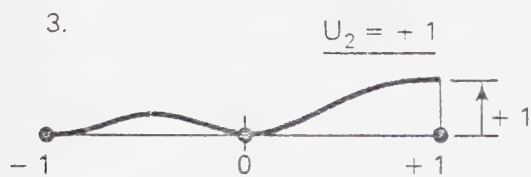
$$\phi_4 = \frac{b}{8} \zeta^2 (\zeta - 1) (\zeta + 1)^2$$



$$\phi_2 = \frac{b}{8} \zeta^2 (\zeta + 1) (\zeta - 1)^2$$



$$\phi_5 = \frac{b}{2} \zeta (\zeta^2 - 1)^2$$



$$\phi_3 = -\frac{\zeta^2}{4} (\zeta + 1)^2 (3\zeta - 4)$$

$$\zeta = \frac{2y}{b}$$

b – flange width

Figure 3.5 Shape Functions for Flanges



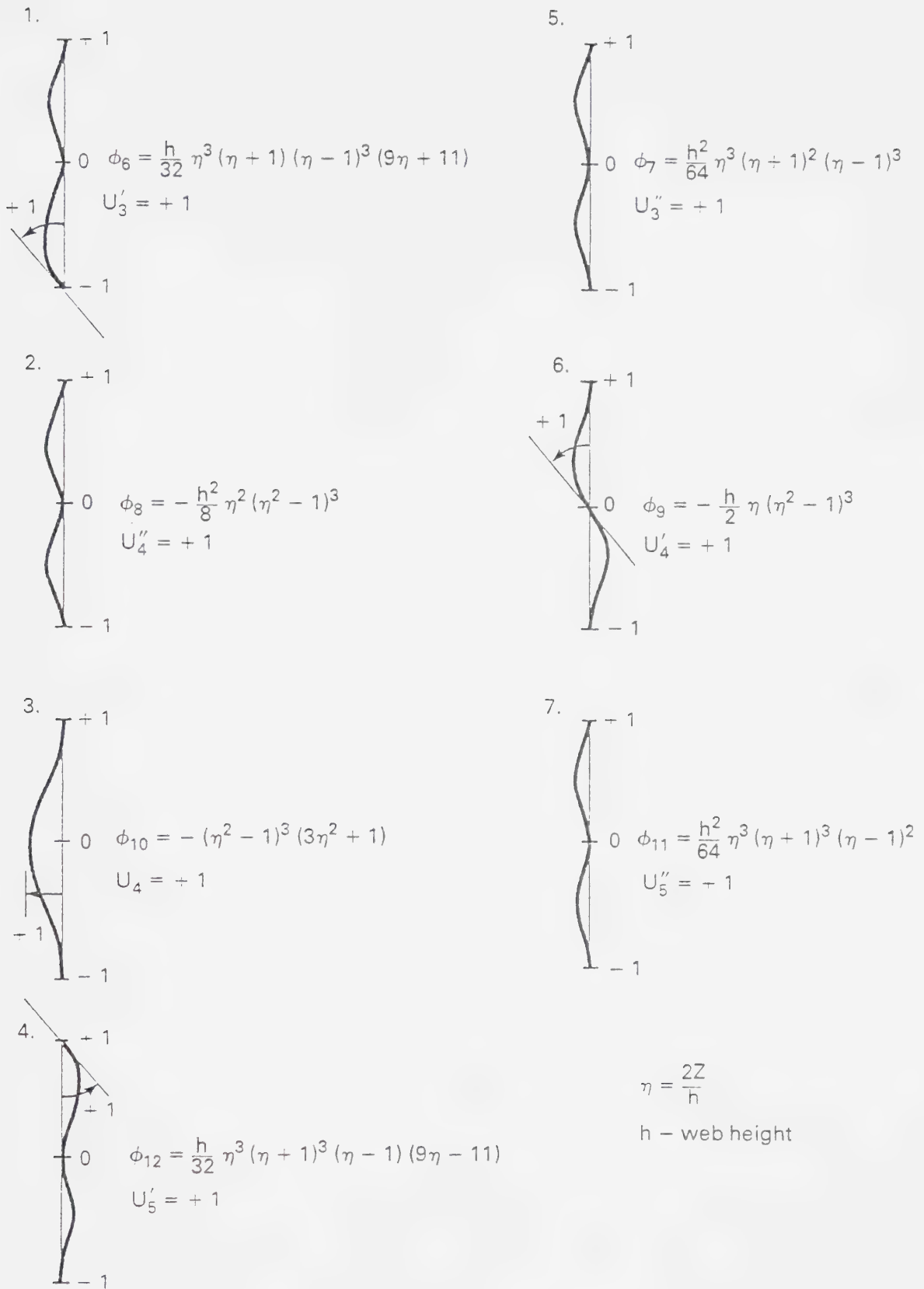
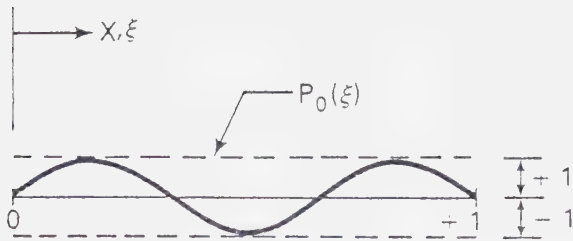


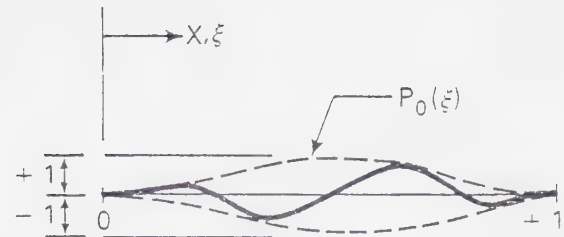
Figure 3.6 Shape Functions for Webs





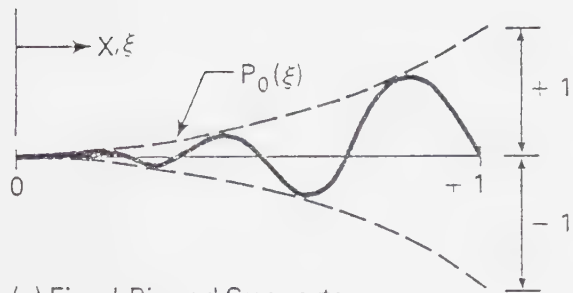
(a) Pinned Supports

$$P_0(\xi) = +1$$



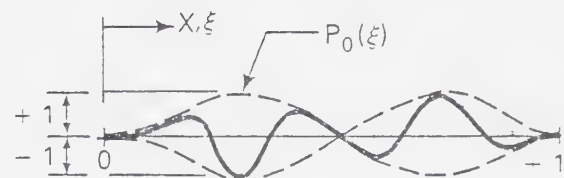
(b) Fixed Supports

$$P_0(\xi) = 16\xi^2(\xi - 1)^2$$



(c) Fixed-Pinned Supports

$$P_0(\xi) = \xi^4$$



(d) Fixed-Pinned-Fixed Supports

$$P_0(\xi) = 111.8(\xi^5 - 2.5\xi^4 + 2\xi^3 - \frac{\xi^2}{2})$$

$$\xi = \frac{X}{L}, \quad L = \text{member length}$$

Figure 3.7 Longitudinal Shape Functions



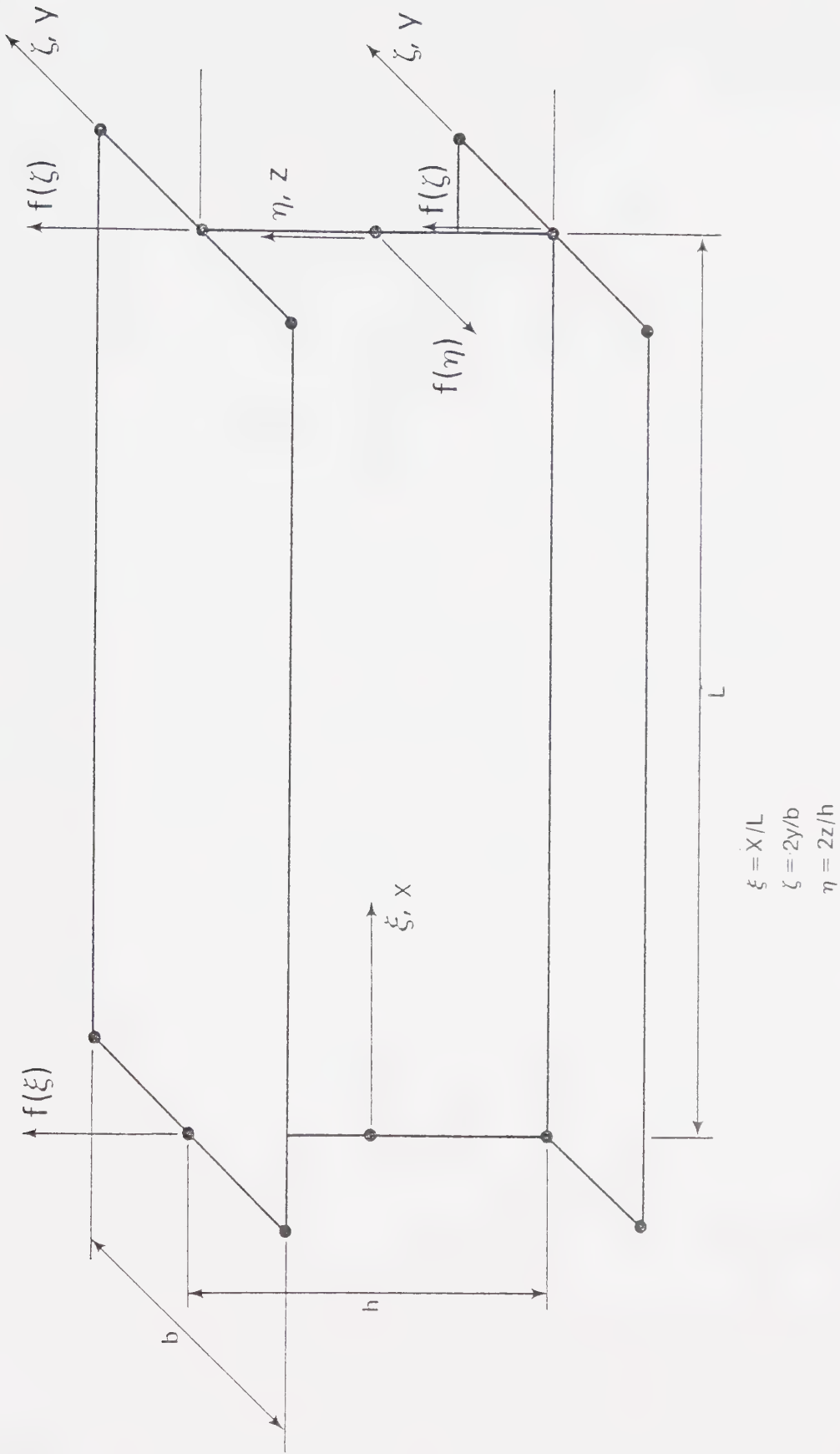


Figure 3.8 Natural Coordinate Systems for a W shape





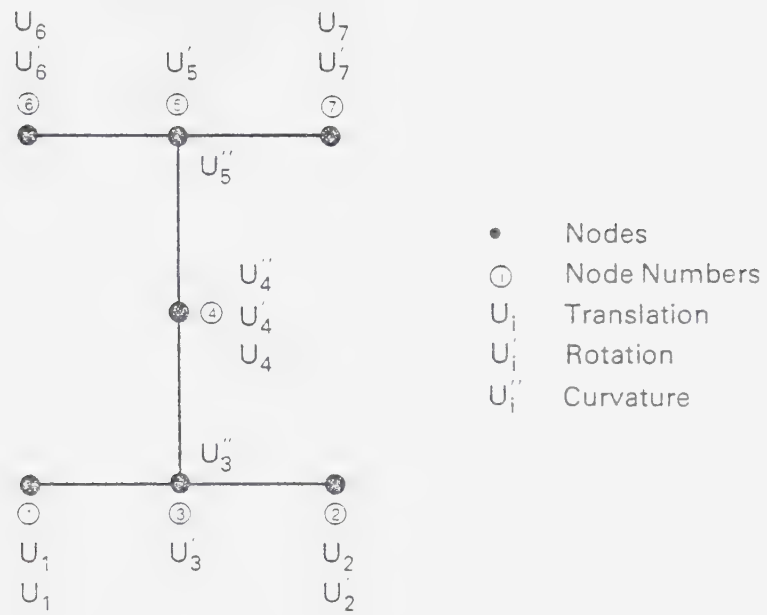
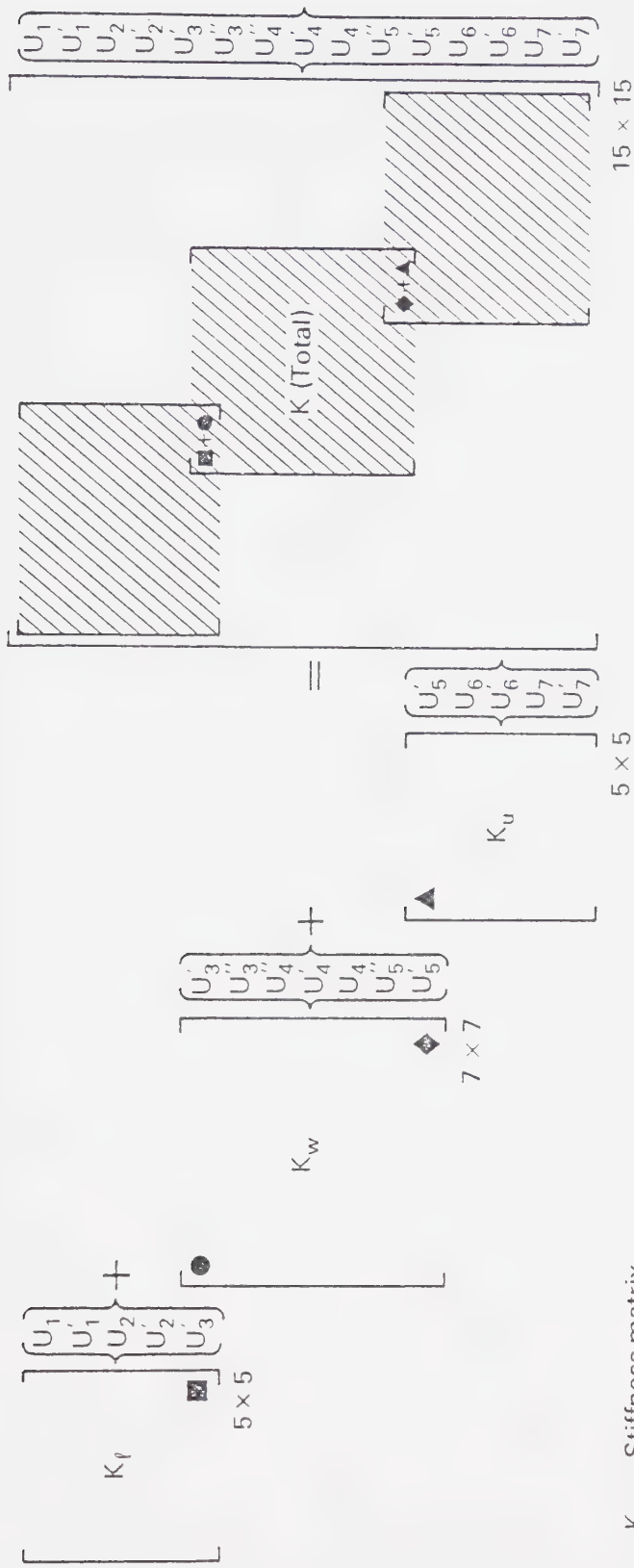


Figure 3.9 Node Numbering and Coordinate Displacements





- K — Stiffness matrix
- f — Lower flange
- w — Web
- u — Upper flange

Figure 3.10 Schematic Stiffness Assembly



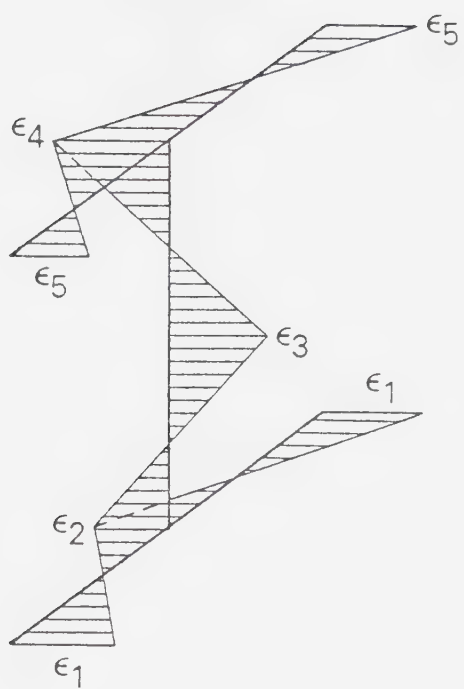


Figure 3.11 Residual Strain Distribution



## Chapter 4

### ANALYSIS FOR COMBINED AXIAL COMPRESSION AND BENDING

#### 4.1 Introduction

In Chapter 3 a general formulation was presented for the analysis of buckling of plates subjected to piecewise linearly varying uniaxial stresses. The inclusion of residual stresses was also discussed and it was stated that the method could be applied to buckling in the elastic as well as the inelastic range. A W shape subjected to combined axial compression and strong-axis bending is composed of three uniaxially stressed plates (two flanges and a web). Because of the presence of residual stresses, uniaxial stresses on a component plate are piecewise linear at a section. The problem of local buckling of a W shape section is formulated by combining the effects of the individual component plates to obtain the total stiffness matrices for a member. In this chapter the procedure is explained in detail for the general loading case of a W shape section subjected to axial compression and strong axis bending combined. The formulation for the general case may be applied to a particular case of pure axial load or pure bending by setting the applied bending moment or the applied axial load equal to zero, respectively.

#### 4.2 Assumptions

In Appendix A, a plate buckling condition is developed for a single uniaxially stressed rectangular plate. This buckling condition





is derived using the principle of virtual work and it is applicable to the elastic and inelastic ranges of stress. In addition to the usual assumptions of plate buckling presented in Appendix A, the following assumptions applicable to local buckling of a structural steel W shape section are made:

1. The member is loaded in such a way that all longitudinal fibres are subjected only to uniaxial stresses.
2. The idealized stress - strain response shown in Figure 3.2 applies for each longitudinal fibre in a cross-section.
3. The buckling condition expressed by Equation A-35 is applicable to material which is elastic, yielded, or strain-hardened.
4. Shape functions for a cross-section are continuous across boundaries between elastic and yielded material and between yielded and strain-hardened material.
5. Local buckling of component plates may occur when a cross-section is in any one of the following strain ranges:
  - (a) fully elastic range
  - (b) partly elastic and partly yielded range
  - (c) fully yielded range
  - (d) partly yielded and partly strain-hardened range
  - (e) fully strain-hardened range
6. Member failure occurs when a plate component of a cross-section buckles locally.



### 4.3 Stiffness Matrix Formulations

#### 4.3.1 Introduction

In the following sections, stiffness submatrices are formulated for individual plate components of a W shape. An applied uniform axial strain is superimposed on a general residual strain distribution such as that shown in Figure 4.1 where  $\epsilon_i$  ( $i=1,2,3,4,5$ ) are residual strains, and  $\epsilon_a$  is an axial strain. A bending moment strain,  $\epsilon_b$ , is then added. The resulting total strain distribution is used to determine the stresses on each component plate as well as the extent of yielded and strain-hardened regions within a plate. Stiffness submatrices are formulated for each plate component for a general case of material being partly elastic, partly yielded, and partly strain-hardened. The component plate stiffness submatrices are then combined as described in Chapter 3.

As previously mentioned, it is assumed for the purpose of clarity, that a section is oriented with its web in a vertical plane. Furthermore, it is assumed that the direction of an applied moment is such that it tends to place the upper flange in compression and the lower flange in tension. As a result of this assumption, the upper flange will always be in compression under the actions of the applied axial compression and bending loads combined. The lower flange, however, may be in tension when the axial load is low compared to the flexural load or in compression when the flexural load is low compared to the axial load. In the latter case, the general analysis of a lower flange is identical to that of an upper flange in compression.



### 4.3.2 Application of Incremental Bending Strains

In this analysis of local buckling of beam-columns, it is necessary to apply additional increments of bending strains to a cross-section. Before an increment is applied, a cross-section may be partially yielded or strain-hardened as a result of previously applied strains. Before additional bending strain increments can be applied it is therefore necessary to update the location of the neutral axis.

Figure 4.2(a) shows a cross-section which is partially elastic and partially yielded as a result of a total strain distribution such as that shown in Figure 4.1. The neutral axis is located at a distance,  $y_1$ , from the mid-depth of a web. This location is determined from the requirement that a cross-section must be in equilibrium under the action of applied loads. Once the neutral axis has been located the bending strains,  $\epsilon_c$ , at mid-depth of a web, and  $\epsilon'_b$ , at the lower edge of a web may be determined from the strain geometry.

Referring to Figure 4.2(b), the following expressions for  $\epsilon_c$  and  $\epsilon'_b$  may be obtained:

$$\epsilon_c = \left( \frac{2y_1}{2y_1 - h} \right) \epsilon_b \quad (4.1)$$

where  $h$  is the web depth and  $\epsilon_b$  is the applied compressive bending strain, and

$$\epsilon'_b = \left( \frac{h + 2y_1}{h - 2y_1} \right) \epsilon_b \quad (4.2)$$



In these relationships  $\epsilon_c$  and  $\epsilon_b'$  are given as functions of the applied compressive bending strain,  $\epsilon_b$ . Thus the distribution of incremental bending strains is completely specified when a value of the applied strain,  $\epsilon_b$ , is specified. In subsequent formulations an analytical technique is set up so that  $\epsilon_b$  is an eigenvalue which corresponds to a critical buckling strain.

#### 4.3.3 Stiffness Submatrices

In this section, bending and geometric stiffness submatrices are formulated for individual plate components of a W shape. The stiffness matrices,  $[K]$  and  $[K_G]$ , are formulated separately for a compression flange, a tension flange, and a web. For the purpose of analysis only, a web is considered to consist of two parts; the lower half of a web between nodes 3 and 4, and the upper half between nodes 4 and 5 as shown in Figure 3.9, Chapter 3. Because the origin of local coordinates is at node 4 of a web, this particular division simplifies the analysis somewhat with regard to integration of piecewise continuous functions along its height.

Equations A-53 and A-54 are expressions for bending and geometric plate stiffness matrices, respectively. These expressions are repeated below for ease of reference:

$$[K] = F_1[\Phi_1] \quad (A-53)$$

$$[K_G] = F_5[\Phi_5] \quad (A-54)$$

where  $i = 1, 2, 3, 4$  and repeated subscripts indicate summation in Equation A-53. In this expression,  $[\Phi_i]$  are integral matrices as





defined by Equations A-47 to A-50, and  $F_i$  are material constants as defined by Equations A-42 to A-45. The values of  $F_i$  depend on whether the material is elastic, yielded, or strain-hardened. In Equation A-54,  $F_5$  is a constant depending on the material thickness and is given by Equation A-46, and  $[\Phi_5]$  is an integral matrix as defined by Equation A-51.

Partial yielding or strain-hardening of a cross-section results in non-uniform material properties and stresses defined piecewise over a section. Therefore, the integral expressions of Equations A-53 and A-54 are also defined piecewise over a section. The limits of integration correspond to the locations of material boundaries between elastic and yielded material and between yielded and strain-hardened material within a plate cross-section. Therefore, in order to carry out the integration required to determine  $[K]$ , it is necessary to locate boundaries corresponding to material discontinuities within a plate cross-section. The evaluation of  $[K_G]$  can be made once the stress discontinuities and stress distributions are determined for a plate cross-section. In the following sections these quantities are evaluated and explicit values of  $[K]$  and  $[K_G]$  are determined for each plate component of a W shape.

#### 4.3.3.1 Compression Flange

The distributions of strain and stress for a compression flange subjected to combined residual, axial, and flexural stresses are shown in Figure 4.3. In Figure 4.3(a),  $\epsilon_a$  is an axial strain,  $\epsilon_b$  is a bending strain,  $\epsilon_4$  is a residual tensile strain,  $\epsilon_5$  is a residual compressive strain,  $\epsilon_y$  is a yield strain, and  $\epsilon_{st}$  is a strain-hardening



strain. The corresponding stresses shown in Figure 4.3(b) are derived from the strain diagram according to the stress - strain relationship defined in Figure 3.2. In Figure 4.3(b),  $s_i$  ( $i = 1, 2, 3, 4, 5, 6$ ) are stress components and  $\alpha_t$  and  $\alpha'_t$ , in natural coordinates, are the locations of the material boundaries between elastic and yielded and between yielded and strain-hardened material.

The values of  $\alpha_t$  and  $\alpha'_t$  are listed in Table 4.1(a) for various levels of strain. The second column of this table indicates the material condition for the corresponding range of strain indicated in the first column. For example, for the second strain range indicated the material is partially elastic (e) and partially yielded (y), and for the fifth strain range indicated the material is fully strain-hardened (s). The stresses defined piecewise for the various stress regions of Figure 4.3(b) are defined in Table 4.1(b). In this table,  $\zeta$  is a natural coordinate as indicated in Figure 4.3,  $E$  is the elastic modulus,  $E_o$  is the slope of the yield portion of a stress - strain curve, and  $E_{st}$  is the strain-hardening modulus.

Using the limits of integration,  $\alpha_t$  and  $\alpha'_t$ , as given in Table 4.1(a), and expanding the expression in Equation A-53 over the non-uniform material regions, the stiffness matrix for a flange in compression is given as:

$$\begin{aligned}
 [K] = & F_{ie} [\phi_i]_{-\alpha_t}^{\alpha_t} + F_{iy} \left[ [\phi_i]_{-\alpha'_t}^{-\alpha_t} + [\bar{\phi}_i]_{\alpha_t}^{\alpha'_t} \right] \\
 & + F_{is} \left[ [\bar{\phi}_i]_{-1}^{-\alpha'_t} + [\bar{\bar{\phi}}_i]_{\alpha'_t}^1 \right]
 \end{aligned} \tag{4.4}$$



where subscripts, e, y, and s, indicate that material constants,  $F_i$ , have elastic, yielded, and strain-hardened values, respectively. The limits of integration are shown as subscripts and superscripts on each integral matrix, and double subscripts, i, indicate summation.

The geometric stiffness matrix for a compression flange is obtained by substituting the stresses,  $S_i$ , from Table 4.1(b) into Equation A-54 and integrating between the appropriate limits defined in column one of Table 4.1(b). The following expression is obtained:

$$\begin{aligned}
 [K_G] = F_5 \left\{ s_1 \left[ [\Phi_5]_{-1}^{-\alpha'_t} + [\Phi_5]_{\alpha'_t}^1 \right] - s_2 \left[ [\zeta\Phi_5]_{-1}^{-\alpha'_t} - [\zeta\Phi_5]_{\alpha'_t}^1 \right] \right. \\
 + s_3 \left[ [\Phi_5]_{-\alpha'_t}^{-\alpha_t} + [\Phi_5]_{\alpha_t}^{\alpha'_t} \right] - s_4 \left[ [\zeta\Phi_5]_{-\alpha'_t}^{-\alpha_t} - [\zeta\Phi_5]_{\alpha_t}^{\alpha'_t} \right] \\
 \left. + s_5 [\Phi_5]_{-\alpha_t}^{\alpha_t} - s_6 \left[ [\zeta\Phi_5]_{-\alpha_t}^0 - [\zeta\Phi_5]_0^{\alpha_t} \right] \right\}
 \end{aligned} \tag{4.5}$$

where  $[\zeta\Phi_5]$  is used to indicate that matrix,  $[\Phi_5]$ , is multiplied by natural coordinate,  $\zeta$ , before integration is performed. This accounts for linearly varying stresses on a portion of a plate cross-section.

#### 4.3.3.2 Tension Flange

The strain and stress distributions for a flange in tension are shown in Figure 4.4. In addition to the previously explained symbols,  $\epsilon_1$  is the residual compressive stress at the flange tips and



$\epsilon_2$  is the residual tensile stress at the flange-to-web junction. The strain ranges and limits of integration corresponding to Figure 4.4(a) are listed in Table 4.2(a) and the corresponding stresses and stress regions illustrated in Figure 4.4(b) are listed in Table 4.2(b).

The stiffness matrix for a tension flange may now be obtained by performing the integration in Equation A-53 over the limits indicated by Figure 4.4 and Table 4.2(a) and using the appropriate material constants,  $F_i$ . The resulting expression is as follows:

$$\begin{aligned}
 [K] = & F_{ie} \left[ \begin{matrix} [\phi_i]^{-\alpha_b} + [\phi_i]^1 \\ -1 \quad \alpha_b \end{matrix} \right] + F_{iy} \left[ \begin{matrix} [\phi_i]^{-\alpha'_b} + [\phi_i]^{\alpha_b} \\ -\alpha'_b \quad \alpha'_b \end{matrix} \right] \\
 & + F_{is} \left[ \begin{matrix} [\phi_i]^{\alpha'_b} \\ -\alpha'_b \end{matrix} \right]
 \end{aligned} \tag{4.6}$$

The geometric stiffness matrix is obtained from Equation A-54 by substituting the stresses given in Table 4.2(b) and integrating over the appropriate limits as indicated in Figure 4.4 and Table 4.2(b). The resulting expression is as follows:

$$\begin{aligned}
 [K_G] = & F_5 \left\{ s_1 \left[ \begin{matrix} [\phi_5]^{-\alpha_b} + [\phi_5]^1 \\ -1 \quad \alpha_b \end{matrix} \right] - s_2 \left[ \begin{matrix} [\zeta\phi_5]^{-\alpha_b} - [\zeta\phi_5]^1 \\ -1 \quad \alpha_b \end{matrix} \right] \right. \\
 & + s_3 \left[ \begin{matrix} [\phi_5]^{-\alpha'_b} + [\phi_5]^{\alpha_b} \\ -\alpha'_b \quad \alpha'_b \end{matrix} \right] - s_4 \left[ \begin{matrix} [\zeta\phi_5]^{-\alpha'_b} - [\zeta\phi_5]^{\alpha_b} \\ -\alpha'_b \quad \alpha'_b \end{matrix} \right] \\
 & \left. + s_5 \left[ \begin{matrix} [\phi_5]^{\alpha'_b} \\ -\alpha'_b \end{matrix} \right] - s_6 \left[ \begin{matrix} [\zeta\phi_5]^0 \\ -\alpha'_b \quad 0 \end{matrix} \right] \right\}
 \end{aligned} \tag{4.7}$$





#### 4.3.3.3 Web - Tension Zone

The general strain and stress distributions for the lower half of a web (loaded in the orientation described previously) are shown in Figure 4.5. In this Figure,  $\beta_1$ ,  $\beta_1'$ ,  $\beta_3$ , and  $\beta_3'$ , in natural coordinates, locate the boundaries between elastic and yielded material and between yielded and strain-hardened material. These values are defined in Tables 4.3(a) and 4.3(b) for the various ranges of strain indicated. The stresses corresponding to the stress regions indicated, are listed in Table 4.3(c).

Referring to Figure 4.5 and Table 4.3 and proceeding in the manner described previously for the flanges, the bending stiffness and geometric stiffness matrices for the tension zone of a web are obtained as follows:

$$\begin{aligned}
 [K] = & F_{ie} [\Phi_i]_{-\beta_1}^{-\beta_3} + F_{iy} \left[ [\Phi_i]_{-\beta_1}^{-\beta_1'} + [\Phi_i]_{-\beta_3}^{-\beta_3'} \right] \\
 & + F_{is} \left[ [\Phi_i]_{-1}^{-\beta_1'} + [\Phi_i]_{-\beta_3'}^0 \right]
 \end{aligned} \tag{4.8}$$

and,

$$\begin{aligned}
 [K_G] = & F_5 \left\{ s_1 [\Phi_5]_{-1}^{-\beta_1'} + s_2 \left[ [\eta\Phi_5]_{-1}^{-\beta_1'} + [\eta\Phi_5]_{-\beta_3'}^0 \right] \right. \\
 & \left. + s_3 [\Phi_5]_{-\beta_1'}^{-\beta_1} + s_4 \left[ [\eta\Phi_5]_{-\beta_1'}^{-\beta_1} + [\eta\Phi_5]_{-\beta_3}^{-\beta_3'} \right] \right\}
 \end{aligned}$$

... continued



$$\begin{aligned}
& + s_5 [\Phi_5]_{-\beta_1}^{-\beta_3} + s_6 [\eta \Phi_5]_{-\beta_1}^{-\beta_3} + s_7 [\Phi_5]_{-\beta_3}^{-\beta_3'} \\
& + s_8 [\Phi_5]_{-\beta_3'}^0 \}
\end{aligned}
\tag{4.9}$$

#### 4.3.3.4 Web - Compression Zone

Because of the complex yield pattern possible under the actions of residual, axial, and flexural stresses combined, three cases must be considered for the compression zone of a web. These three cases correspond to the material condition at the center of the web which may be elastic, yielded, or strain-hardened at the time the incremental bending strains are applied. Each of the three cases is considered separately.

The strain and stress distributions for the compression zone of a web when the center of the web is elastic are shown in Figure 4.6. In this case the total strain at a web center must not be greater than the yield strain, and therefore,

$$\epsilon_a + \epsilon_c + \epsilon_3 \leq \epsilon_y
\tag{4.10}$$

The material boundaries corresponding to the various ranges of strain possible are given in Table 4.4(a) and the stresses corresponding to the material zones indicated in Figure 4.6(b) are listed in Table 4.4(b). Referring to Figure 4.6 and Table 4.4, and proceeding as described for the previous cases, the bending and



geometric stiffness matrices may be written as follows:

$$[K] = F_{ie} [\phi_i]_{\beta_2}^{\beta_2'} + F_{iy} [\phi_i]_{\beta_2}^{\beta_2'} + F_{is} [\phi_i]_{\beta_2}^1 \quad (4.11)$$

and,

$$[K_G] = F_5 \left\{ s_1 [\phi_5]_0^{\beta_2} + s_2 [\eta \phi_5]_0^{\beta_2} + s_3 [\phi_5]_{\beta_2}^{\beta_2'} + s_4 [\phi_5]_{\beta_2}^{\beta_2'} + s_5 [\phi_5]_{\beta_2}^1 + s_6 [\phi_5]_{\beta_2}^1 \right\} \quad (4.12)$$

The center of a web is yielded when,

$$\epsilon_y < \epsilon_a + \epsilon_c + \epsilon_3 \leq \epsilon_{st} \quad (4.13)$$

and the corresponding strain and stress distributions as well as the limits of integration and various material zones are described in Figure 4.7 and Table 4.5. Using the appropriate values obtained therefrom, the bending and geometric stiffness matrices for this case may be written as follows:

$$[K] = F_{ie} [\phi_i]_{\beta_2}^{\beta_2'} + F_{iy} [\phi_i]_{\beta_2}^{\beta_2'} + F_{is} [\phi_i]_{\beta_2}^1 \quad (4.14)$$

and,



$$\begin{aligned}
[K_G] = F_5 \left\{ s_1 [\Phi_5]_{\beta_2}^{\beta_2} + s_2 [\eta\Phi_5]_{\beta_2}^{\beta_2} + s_3 [\Phi_5]_{\beta_2}^{\beta_2'} \right. \\
\left. + s_4 [\eta\Phi_5]_{\beta_2}^{\beta_2'} + s_5 [\Phi_5]_{\beta_2}^1 + s_6 [\eta\Phi_5]_{\beta_2}^1 \right\}
\end{aligned}
\tag{4.15}$$

The third and final case which must be considered for the compression zone of a web is that corresponding to the center of the web being in the strain-hardened condition. In this case,

$$\epsilon_{st} < \epsilon_a + \epsilon_c + \epsilon_3 \tag{4.16}$$

The corresponding distributions of strain and stress are shown in Figure 4.8 and the material boundaries and stress components for each region are given in Table 4.6. The resulting expressions for the bending and geometric stiffness matrices are as follows:

$$[K] = F_{ie} [\Phi_i]_{\beta_2}^1 + F_{iy} [\Phi_i]_{\beta_2}^{\beta_2} + F_{is} [\Phi_i]_{\beta_2}^{\beta_2'} \tag{4.17}$$

and,





$$\begin{aligned}
[K_G] = F_5 \left\{ s_1 [\Phi_5]_{\beta_2'}^{\beta_2'} + s_2 [\eta\Phi_5]_{\beta_2'}^{\beta_2'} + s_3 [\Phi_5]_{\beta_2'}^{\beta_2} \right. \\
\left. + s_4 [\eta\Phi_5]_{\beta_2'}^{\beta_2} + s_5 [\Phi_5]_{\beta_2}^1 + s_6 [\eta\Phi_5]_{\beta_2}^1 \right\}
\end{aligned}
\tag{4.18}$$

#### 4.4 Iterative Technique

In the solution of a particular problem, bending stiffness and geometric stiffness submatrices for each flange and a web are formulated as described in the previous sections. The global stiffness matrices are then assembled as explained in Chapter 3. In any problem of axial, bending, or combined loading, the required eigenvalue will be either  $\epsilon_b$ , the bending strain, or  $\epsilon_a$  the axial strain. However, because of material and stress non-linearities over a cross-section, the bending stiffness matrix,  $[K]$ , and the geometric stiffness matrix,  $[K_G]$ , are implicit functions of the eigenvalue strain. Therefore at each successive value of an eigenvalue strain, it is necessary to reformulate  $[K]$  and  $[K_G]$  for that particular value of strain. Thus an iterative technique is required.

As described in Chapter 3, an eigenvalue problem reduces to the form:

$$\left[ [K] - [K_G] \right] \{\theta\} = \{0\}
\tag{4.19}$$

As stated previously,  $[K]$  and  $[K_G]$  are implicit functions of  $(\epsilon_b + \epsilon_a)$  when a material is non-linear. Equation 4.19 may be re-written as



follows:

$$\left[ [K] - \lambda_o [K_{G_o}] \right] \{\theta\} = \{0\} \quad (4.20)$$

where,

$$[K_{G_o}] = \frac{1}{\epsilon_b + \epsilon_a} [K_G] \quad (4.21)$$

In the solution of Equation 4.20, a value of  $(\epsilon_b + \epsilon_a)$  is assumed. Knowing this value, the elastic, yield, and strain-hardening material zones in a cross-section as well as the discontinuous stress distributions are fully defined. Therefore,  $[K]$  and  $[K_{G_o}]$  are completely determinable and matrix iteration may be performed to determine  $\lambda_o$ . The solution to Equation 4.19 will be obtained when,

$$\frac{\lambda_o}{\epsilon_b + \epsilon_a} = 1.0 \quad (4.22)$$

In general, this will require the determination of several values of  $\lambda_o$  by matrix iteration. An exact solution is obtained when the eigenvalue,  $\lambda_o$ , is equal to the assumed value of  $\epsilon_b + \epsilon_a$ . In general, however, this will not be true, and,

$$\lambda_o = \lambda' + \lambda'' \quad (4.23)$$

where,

$$\lambda' = \epsilon_b + \epsilon_a \quad (4.24)$$

and  $\lambda''$  is a residue which represents the amount by which  $\lambda_o$  deviates



from the exact value of  $(\epsilon_b + \epsilon_a)$ . Thus, the problem reduces to that of finding a value of  $\lambda_o$  such that,

$$|\lambda''| = |\lambda_o - \lambda'| \leq e \quad (4.25)$$

where  $e$ , is a small positive value which reflects the required precision of a solution.

For the purpose of illustration, Figure 4.9 shows a graph of  $(\lambda_o - \lambda')$  vs.  $(\epsilon_b + \epsilon_a)$ . In the iteration technique, an initial value of  $(\epsilon_b + \epsilon_a)$  is chosen so that  $(\lambda_o - \lambda')$  is positive. Another value of  $(\epsilon_b + \epsilon_a)$  is found so that the corresponding value of  $(\lambda_o - \lambda')$  is negative. Once these two starting values have been found (by trial and error, if necessary) the method of bisection<sup>31</sup> is used to determine a value of  $(\epsilon_b + \epsilon_a)$  for which  $|\lambda_o - \lambda'| \leq e$ . Once the convergence criterion is satisfied, the critical bending strain is given by:

$$\epsilon_{b_{cr}} = \lambda_o - \epsilon_a \quad (4.26)$$

This general technique may be used for pure bending when  $\epsilon_a = 0$ , or for pure axial load when  $\epsilon_b = 0$ . In the case of axial compression and bending combined,  $\epsilon_a$  is a constant value of axial strain depending on the magnitude of applied axial load.



Strain Range	Material Condition	Location of Material Boundaries	
		$\alpha_t$	$\alpha'_t$
$\epsilon_b + \epsilon_a \leq \epsilon_y - \epsilon_5$	(e)	1.0	1.0
$\epsilon_y - \epsilon_5 < \epsilon_b + \epsilon_a \leq \epsilon_y + \epsilon_4$	(e,y)	$\frac{\epsilon_y + \epsilon_4 - \epsilon_b - \epsilon_a}{\epsilon_4 + \epsilon_5}$	1.0
$\epsilon_y + \epsilon_4 < \epsilon_b + \epsilon_a \leq \epsilon_{st} - \epsilon_5$	(y)	0.0	1.0
$\epsilon_{st} - \epsilon_5 < \epsilon_b + \epsilon_a \leq \epsilon_{st} + \epsilon_4$	(y,s)	0.0	$\frac{\epsilon_{st} + \epsilon_4 - \epsilon_b - \epsilon_a}{\epsilon_4 + \epsilon_5}$
$\epsilon_{st} + \epsilon_4 < \epsilon_b + \epsilon_a$	(s)	0.0	0.0

## (a) Strains

Stress Region		Stress	Stress Components
$-1.0 \leq \zeta \leq -\alpha'_t$	(s)	$s_1 - s_2 \zeta$	$s_1 = \sigma_y + (\epsilon_{st} - \epsilon_y) E_o + (\epsilon_b + \epsilon_a - \epsilon_{st} - \epsilon_4) E_{st}$
$-\alpha'_t \leq \zeta \leq -\alpha_t$	(y)	$s_3 - s_4 \zeta$	$s_2 = (\epsilon_4 + \epsilon_5) E_{st}$
$-\alpha_t \leq \zeta \leq 0.0$	(e)	$s_5 - s_6 \zeta$	$s_3 = \sigma_y + (\epsilon_b + \epsilon_a - \epsilon_y - \epsilon_4) E_o$
$0.0 \leq \zeta \leq \alpha_t$	(e)	$s_5 + s_6 \zeta$	$s_4 = (\epsilon_4 + \epsilon_5) E_o$
$\alpha_t \leq \zeta \leq \alpha'_t$	(y)	$s_3 + s_4 \zeta$	$s_5 = (\epsilon_b + \epsilon_a - \epsilon_4) E$
$\alpha'_t \leq \zeta \leq 1.0$	(s)	$s_1 + s_2 \zeta$	$s_6 = (\epsilon_4 + \epsilon_5) E$

## (b) Stresses

Table 4.1 Stresses and Strains for a Compression Flange





Strain Range	Material Condition	Location of Material Boundaries	
		$\alpha_b$	$\alpha'_b$
$\epsilon'_b - \epsilon_a \leq \epsilon_y - \epsilon_2$	(e)	0.0	0.0
$\epsilon_y - \epsilon_2 < \epsilon'_b - \epsilon_a \leq \epsilon_y + \epsilon_1$	(e,y)	$\frac{\epsilon'_b - \epsilon_a - \epsilon_y + \epsilon_2}{\epsilon_1 + \epsilon_2}$	0.0
$\epsilon_y + \epsilon_1 < \epsilon'_b - \epsilon_a \leq \epsilon_{st} - \epsilon_2$	(y)	1.0	0.0
$\epsilon_{st} - \epsilon_2 < \epsilon'_b - \epsilon_a \leq \epsilon_{st} + \epsilon_1$	(y,s)	1.0	$\frac{\epsilon'_b - \epsilon_a - \epsilon_{st} + \epsilon_2}{\epsilon_1 + \epsilon_2}$
$\epsilon_{st} + \epsilon_1 < \epsilon'_b - \epsilon_a$	(s)	1.0	1.0

(a) Strains

Stress Region		Stresses	Stress Components
$-1.0 \leq \zeta \leq -\alpha_b$	(e)	$s_1 - s_2 \zeta$	$s_1 = (\epsilon_a - \epsilon'_b - \epsilon_2)E$
$-\alpha_b \leq \zeta \leq -\alpha'_b$	(y)	$s_3 - s_4 \zeta$	$s_2 = (\epsilon_1 + \epsilon_2)E$
$-\alpha'_b \leq \zeta \leq 0.0$	(s)	$s_5 - s_6 \zeta$	$s_3 = -\sigma_y + (\epsilon_y - \epsilon_2 + \epsilon_a - \epsilon'_b)E_o$
$0.0 \leq \zeta \leq \alpha'_b$	(s)	$s_5 + s_6 \zeta$	$s_4 = (\epsilon_1 + \epsilon_2)E_o$
$\alpha'_b \leq \zeta \leq \alpha_b$	(y)	$s_3 + s_4 \zeta$	$s_5 = -\sigma_y - (\epsilon_{st} - \epsilon_y)E_o + (\epsilon_{st} + \epsilon_a - \epsilon'_b - \epsilon_2)E_{st}$
$\alpha_b \leq \zeta \leq 1.0$	(e)	$s_1 + s_2 \zeta$	$s_6 = (\epsilon_1 + \epsilon_2)E_{st}$

(b) Stresses

Table 4.2 Stresses and Strains for a Tension Flange.



Strain Range	Material Condition	Location of Material Boundaries	
		$\beta_1$	$\beta'_1$
$\epsilon'_b - \epsilon_a \leq \epsilon_y - \epsilon_2$	(e)	1.0	1.0
$\epsilon_y - \epsilon_2 < \epsilon'_b - \epsilon_a \leq \epsilon_{st} - \epsilon_2$	(e,y)	$\frac{\epsilon_a + \epsilon_c + \epsilon_y + \epsilon_3}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$	1.0
$\epsilon_{st} - \epsilon_2 < \epsilon'_b - \epsilon_a$	(e,y,s)	$\frac{\epsilon_a + \epsilon_c + \epsilon_y + \epsilon_3}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$	$\frac{\epsilon_a + \epsilon_c + \epsilon_{st} + \epsilon_3}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$

(a) Strains Adjacent to Lower Edge of Web

Strain Range	Material Condition	Location of Material Boundaries	
		$\beta_3$	$\beta'_3$
$\epsilon_a + \epsilon_c + \epsilon_3 \leq \epsilon_y$	(e)	0.0	0.0
$\epsilon_y < \epsilon_a + \epsilon_c + \epsilon_3 \leq \epsilon_{st}$	(e,y)	$\frac{\epsilon_a + \epsilon_c + \epsilon_3 - \epsilon_y}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$	0.0
$\epsilon_{st} < \epsilon_a + \epsilon_c + \epsilon_3$	(e,y,s)	$\frac{\epsilon_a + \epsilon_c + \epsilon_3 - \epsilon_y}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$	$\frac{\epsilon_a + \epsilon_c + \epsilon_3 - \epsilon_{st}}{\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3}$

(b) Strains Adjacent to Middle of Web

Table 4.3 Stresses and Strains for the Tension Zone of a Web -

... cont'd.



Table 4.3 - continued

Stress Region		Stress	Stress Components
$-1 \leq \eta \leq -\beta'_1$	(s)	$s_1 + s_2 \eta$	$s_1 = -\sigma_y - (\epsilon_{st} - \epsilon_y) E_o + (\epsilon_a +$ $\epsilon_c + \epsilon_{st} + \epsilon_3) E_{st}$
$-\beta'_1 \leq \eta \leq -\beta_1$	(y)	$s_3 + s_4 \eta$	$s_2 = (\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3) E_{st}$
$-\beta_1 \leq \eta \leq -\beta_3$	(e)	$s_5 + s_6 \eta$	$s_3 = -\sigma_y + (\epsilon_a + \epsilon_c + \epsilon_y + \epsilon_3) E_o$
$-\beta_3 \leq \eta \leq -\beta'_3$	(y)	$s_7 + s_4 \eta$	$s_4 = (\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3) E_o$
$-\beta'_3 \leq \eta \leq 0$	(s)	$s_8 + s_2 \eta$	$s_5 = (\epsilon_a + \epsilon_c + \epsilon_3) E$ $s_6 = (\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3) E$ $s_7 = \sigma_y + (\epsilon_a + \epsilon_c + \epsilon_3 - \epsilon_y) E_o$ $s_8 = \sigma_y + (\epsilon_{st} - \epsilon_y) E_o + (\epsilon_a + \epsilon_c +$ $\epsilon_{st} + \epsilon_3) E_{st}$

(c) Stresses

Table 4.3 Stresses and Strains for the Tension Zone of a Web



Strain Range	Material Condition	Location of Material Boundaries	
		$\beta_2$	$\beta'_2$
$\epsilon_b + \epsilon_a \leq \epsilon_y + \epsilon_4$	(e)	1.0	1.0
$\epsilon_y + \epsilon_4 < \epsilon_b + \epsilon_a \leq \epsilon_{st} + \epsilon_4$	(e,y)	$\frac{\epsilon_y - \epsilon_3 - \epsilon_c - \epsilon_a}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$	1.0
$\epsilon_{st} + \epsilon_4 < \epsilon_b + \epsilon_a$	(e,y,s)	$\frac{\epsilon_y - \epsilon_3 - \epsilon_c - \epsilon_a}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$	$\frac{\epsilon_{st} - \epsilon_3 - \epsilon_c - \epsilon_a}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$

## (a) Strains

Stress Region		Stresses	Stress Components
$0.0 \leq \eta \leq \beta_2$	(e)	$s_1 + s_2 \eta$	$s_1 = (\epsilon_a + \epsilon_c + \epsilon_3)E$ $s_2 = \epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4$
$\beta_2 \leq \eta \leq \beta'_2$	(y)	$s_3 + s_4 \eta$	$s_3 = \sigma_y + (\epsilon_a + \epsilon_c - \epsilon_y + \epsilon_3)E_o$ $s_4 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4)E_o$
$\beta'_2 \leq \eta \leq 1.0$	(s)	$s_5 + s_6 \eta$	$s_5 = \sigma_y + (\epsilon_{st} - \epsilon_y)E_o + (\epsilon_a + \epsilon_c - \epsilon_{st} + \epsilon_3)E_{st}$ $s_6 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4)E_{st}$

## (b) Stresses

Table 4.4 Stresses and Strains in the Compression Zone of a Web (Case I - Center of Web Elastic)





Strain Range	Material Condition	Location of Material Boundaries	
		$\beta_2$	$\beta'_2$
$\epsilon_b + \epsilon_a \leq \epsilon_y + \epsilon_4$	(e)	$\frac{\epsilon_y - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$	1.0
$\epsilon_y + \epsilon_4 < \epsilon_b + \epsilon_a \leq \epsilon_{st} + \epsilon_4$	(e,y)	1.0	1.0
$\epsilon_{st} + \epsilon_4 < \epsilon_b + \epsilon_a$	(y,s)	$\frac{\epsilon_{st} - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$	$\frac{\epsilon_{st} - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$

## (a) Strains

Stress Region		Stress	Stress Components
$0.0 \leq \eta \leq \beta_2$	(y)	$s_1 + s_2 \eta$	$s_1 = \sigma_y + (\epsilon_a + \epsilon_c - \epsilon_y + \epsilon_3) E_o$ $s_2 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4) E_o$ $s_3 = (\epsilon_a + \epsilon_c + \epsilon_3) E$
$\beta_2 \leq \eta \leq \beta'_2$	(e)	$s_3 + s_4 \eta$	$s_4 = (\epsilon_b + \epsilon_c - \epsilon_3 - \epsilon_4) E$ $s_5 = \sigma_y + (\epsilon_s - \epsilon_y) E_o +$ $(\epsilon_a + \epsilon_c - \epsilon_{st} + \epsilon_3) E_{st}$
$\beta'_2 \leq \eta \leq 1.0$	(s)	$s_5 + s_6 \eta$	$s_6 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4) E_{st}$

## (b) Stresses

Table 4.5 Stresses and Strains for the Compression Zone of a Web  
(Case II - Center of Web Yielded)



Strain Range	Material Condition	Location of Material Boundaries	
		$\beta_2$	$\beta'_2$
$\epsilon_b + \epsilon_a \leq \epsilon_y + \epsilon_4$	(e,y,s)	$\frac{\epsilon_y - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$	$\frac{\epsilon_{st} - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$
$\epsilon_y + \epsilon_4 < \epsilon_b + \epsilon_a \leq \epsilon_{st} + \epsilon_4$	(y,s)	1.0	$\frac{\epsilon_{st} - \epsilon_3 - \epsilon_a - \epsilon_c}{\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4}$
$\epsilon_{st} - \epsilon_4 < \epsilon_b + \epsilon_a$	(s)	1.0	1.0

(a) Strains

Stress Region		Stress	Stress Components
$0.0 \leq \eta < \beta$	(s)	$s_1 + s_2 \eta$	$s_1 = \sigma_y + (\epsilon_{st} - \epsilon_y) E_o +$ $(\epsilon_a + \epsilon_c - \epsilon_{st} + \epsilon_3) E_{st}$ $s_2 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4) E_{st}$
$\beta'_2 \leq \eta \leq \beta_2$	(y)	$s_3 + s_4 \eta$	$s_3 = \sigma_y + (\epsilon_a + \epsilon_c - \epsilon_y + \epsilon_3) E_o$ $s_4 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4) E_o$
$\beta_2 \leq \eta \leq 1.0$	(e)	$s_5 + s_6 \eta$	$s_5 = (\epsilon_a + \epsilon_c + \epsilon_3) E$ $s_6 = (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4) E$

(b) Stresses

Table 4.6 Stresses and Strains for the Compression Zone of a Web  
(Case III - Center of Web Strain-hardened)



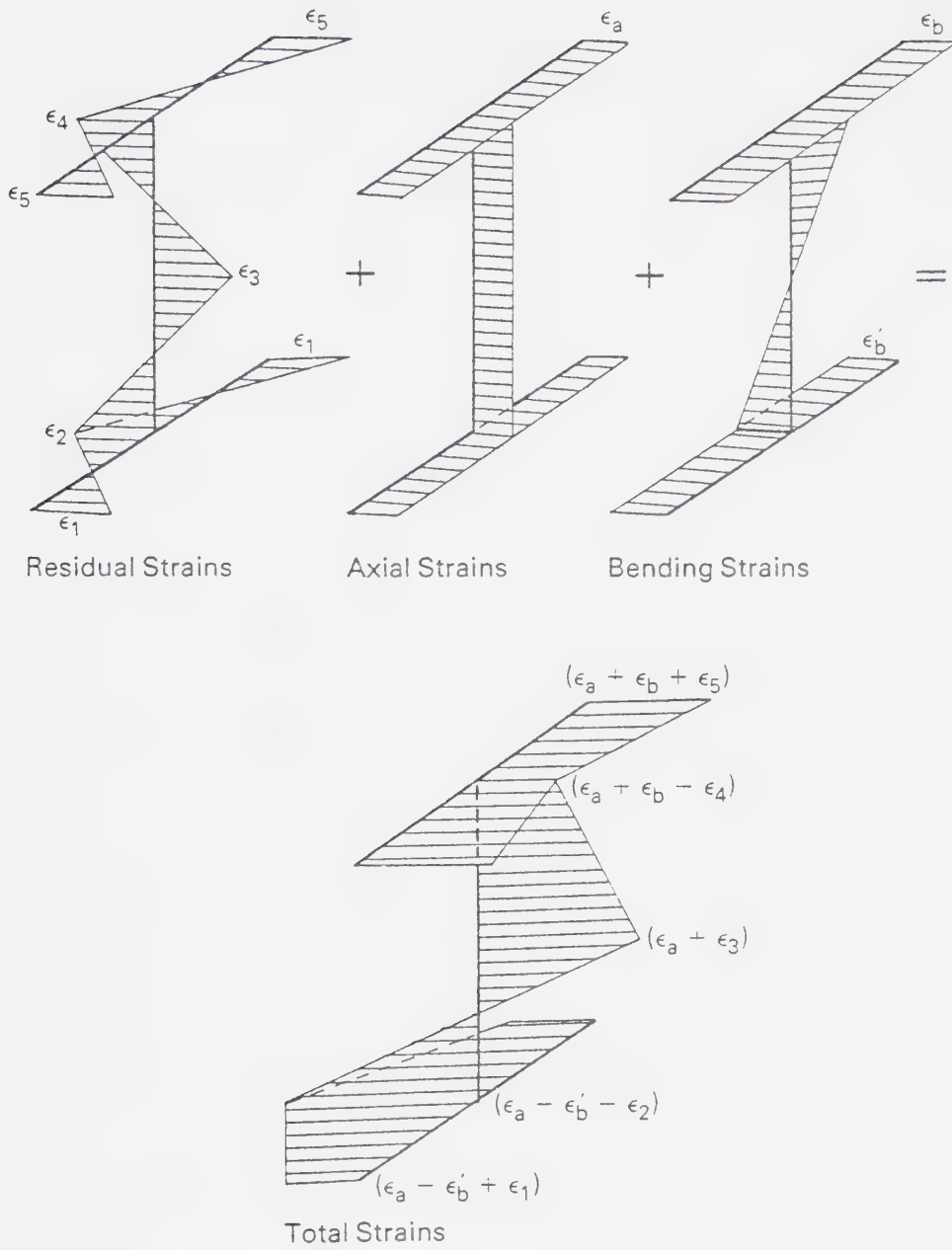


Figure 4.1 Superposition of Beam-Column Strains



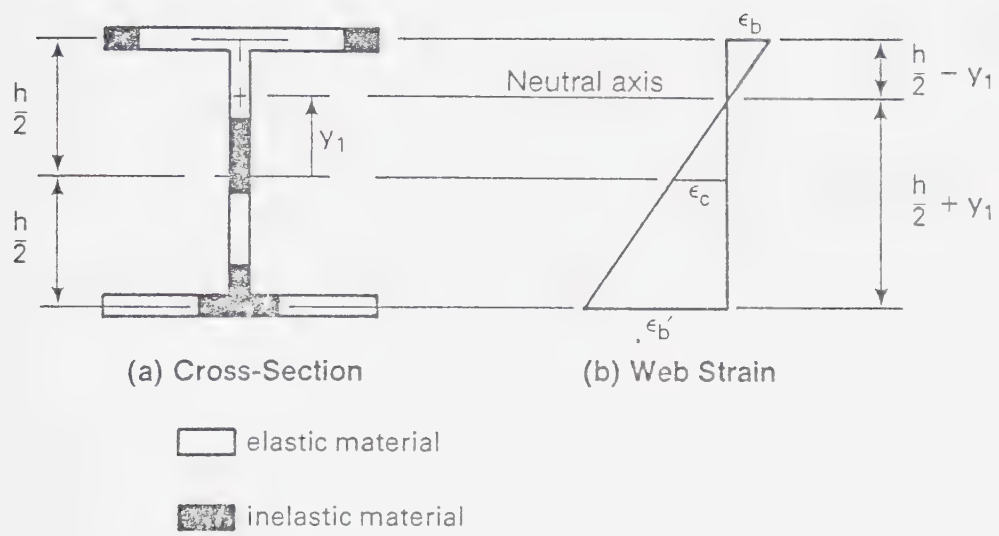
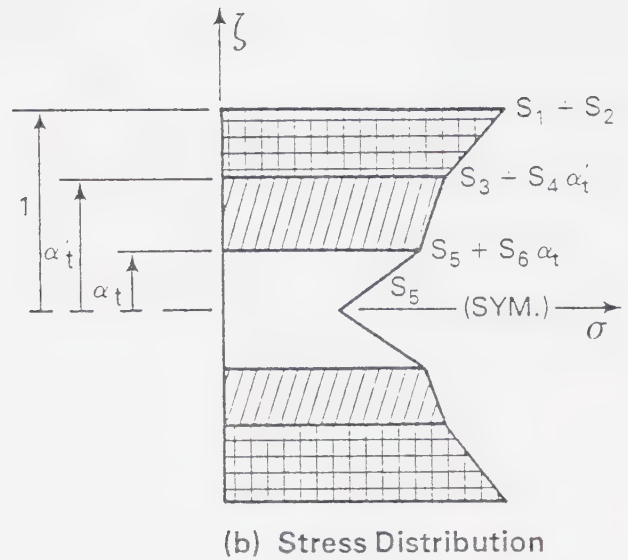
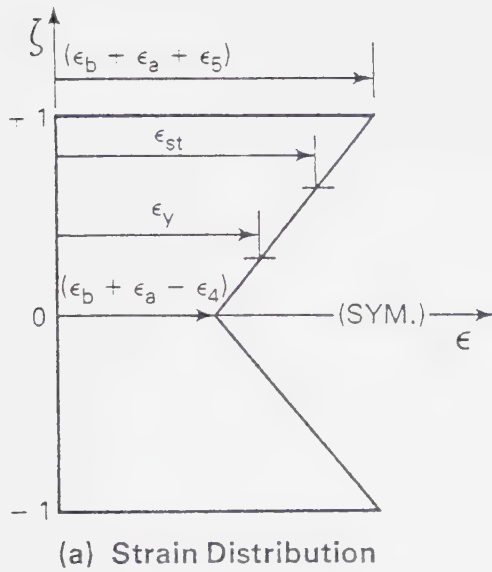


Figure 4.2 Flexural Strain on an Inelastic Section





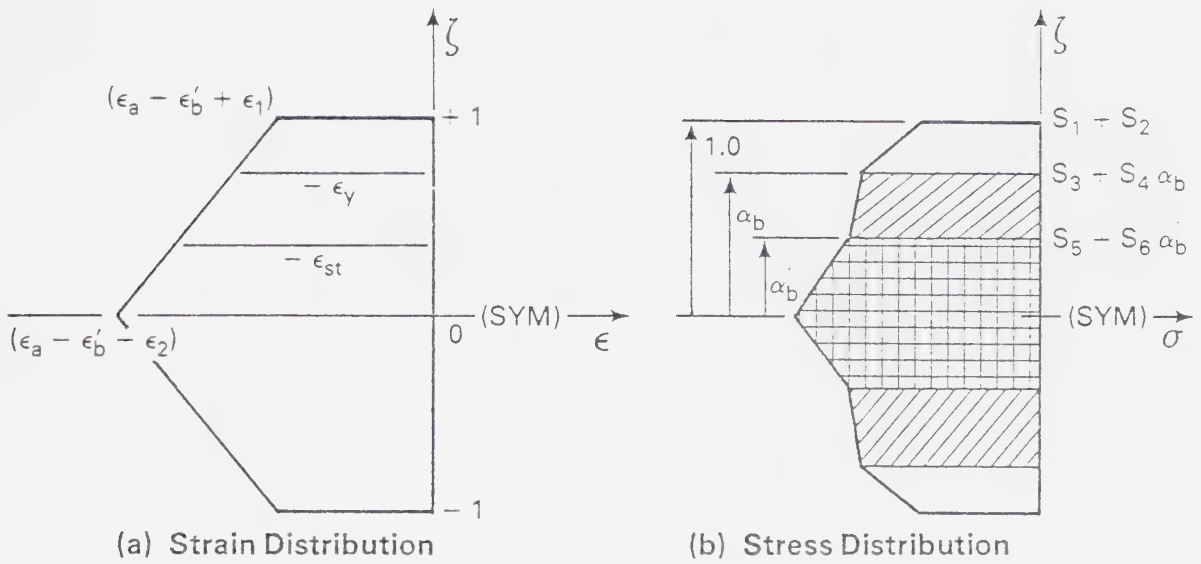


$$\begin{aligned}
 0 \leq \zeta \leq +1 \\
 \epsilon &= \epsilon_b + \epsilon_a - \epsilon_4 + (\epsilon_4 + \epsilon_5) \zeta \\
 -1 \leq \zeta \leq 0 \\
 \epsilon &= \epsilon_b + \epsilon_a - \epsilon_4 - (\epsilon_4 + \epsilon_5) \zeta
 \end{aligned}$$



Figure 4.3 Strain and Stress Distributions for a Flange in Compression.





$$-1 \leq \zeta \leq 0$$

$$\epsilon = -\epsilon'_b + \epsilon_a - \epsilon_2 - (\epsilon_1 + \epsilon_2) \zeta$$

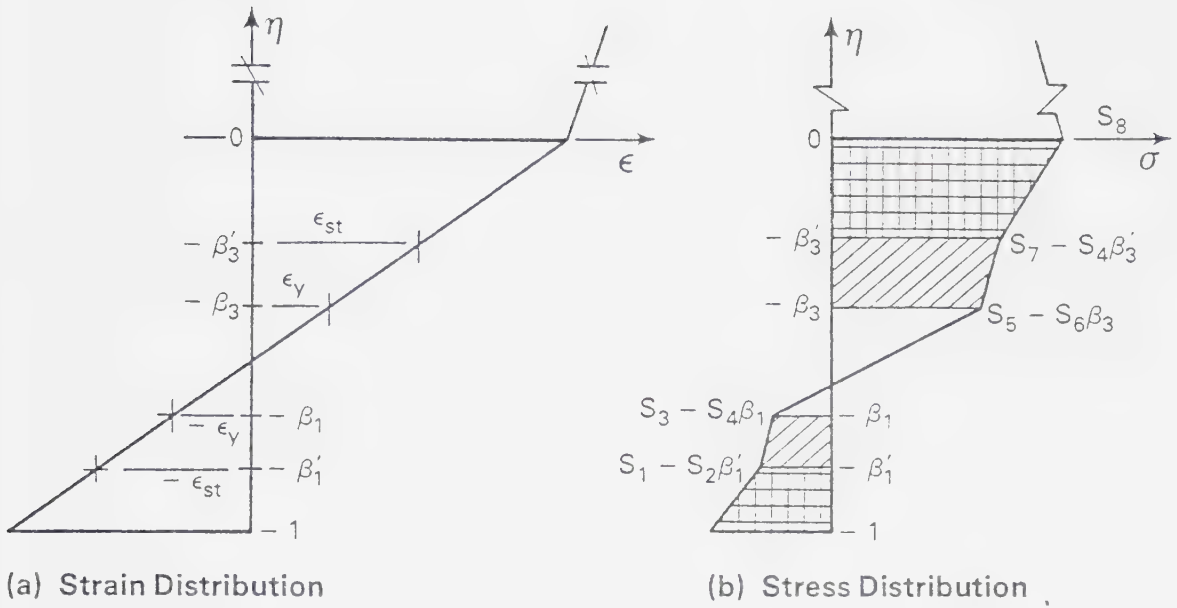
$$0 \leq \zeta \leq 1$$

$$\epsilon = -\epsilon_b + \epsilon_a - \epsilon_2 + (\epsilon_1 + \epsilon_2) \zeta$$

- Elastic (e)
- Yielding (y)
- Strain Hardened (s)

Figure 4.4 Strain and Stress Distributions for a Flange in Tension.





$$-1 \leq \eta \leq 0$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3 + (\epsilon'_b + \epsilon_c + \epsilon_2 + \epsilon_3) \eta$$

$$\eta = -1$$

$$\epsilon = \epsilon_a - \epsilon'_b - \epsilon_2$$

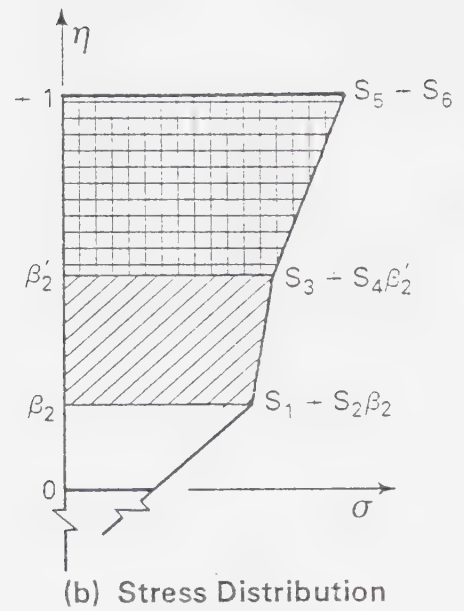
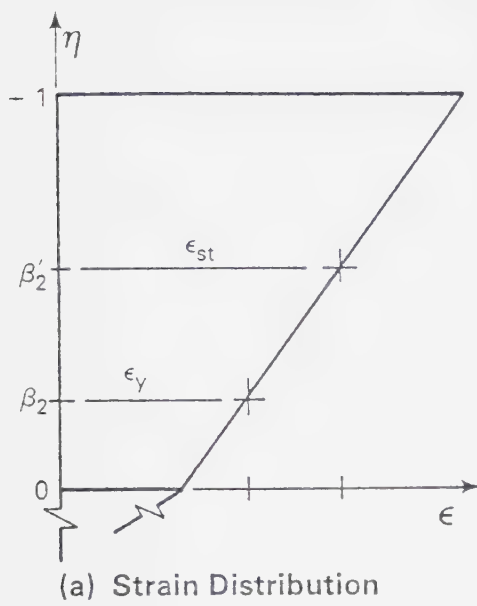
$$\eta = 0$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3$$



Figure 4.5 Strain and Stress Distributions for Tension Zone of a Web.





$$0 \leq \eta \leq +1$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3 + (\epsilon_b - \epsilon_c - \epsilon_3 - \epsilon_4)\eta$$

$$\eta = 0$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3$$

$$\eta = +1$$

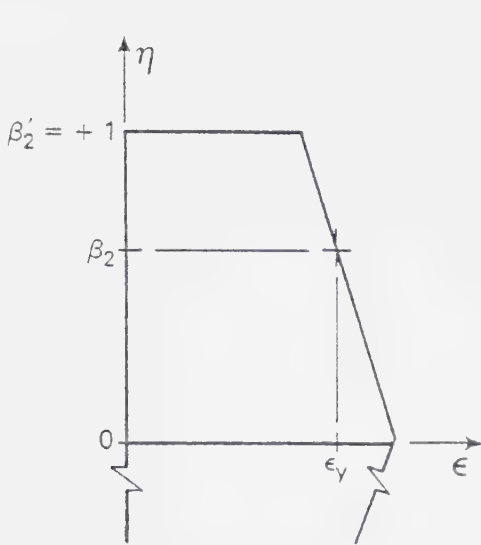
$$\epsilon = \epsilon_b + \epsilon_a - \epsilon_4$$



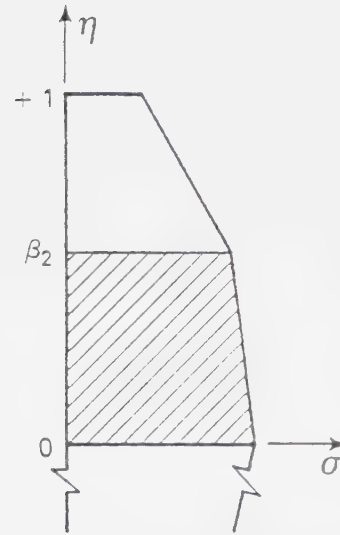
Figure 4.6 Strain and Stress Distributions for Compression Zone of Web —  
(Case I - Center of Web Elastic)





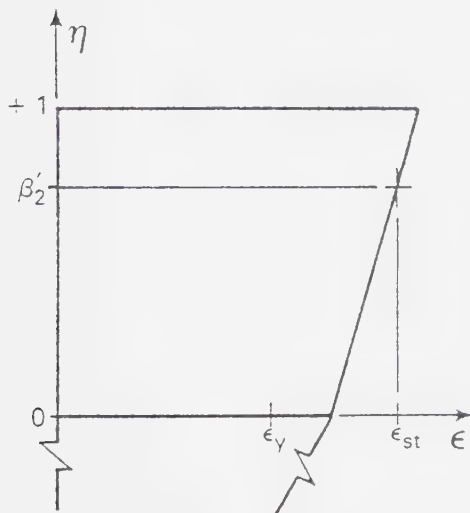


(a) Strain Distribution

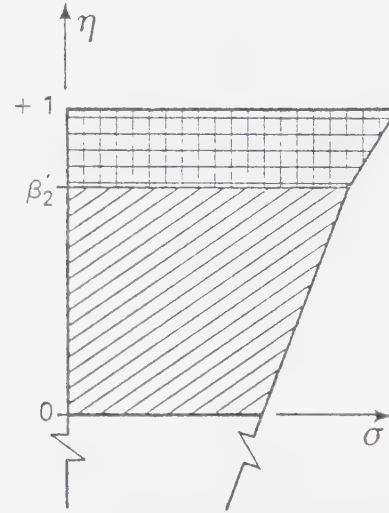


(b) Stress Distribution

Case (i) Strain at  $\eta = +1 \leq$  Strain at  $\eta = 0$



(a) Strain Distribution



(b) Stress Distribution

Case (ii) Strain at  $\eta = 1 >$  Strain at  $\eta = 0$

$$0 \leq \eta \leq 1$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3 + (\epsilon_b - \epsilon_c - \epsilon_4 - \epsilon_3)\eta$$

$$\eta = 0$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3$$

$$\eta = 1$$

$$\epsilon = \epsilon_b + \epsilon_a - \epsilon_4$$



Elastic (e)



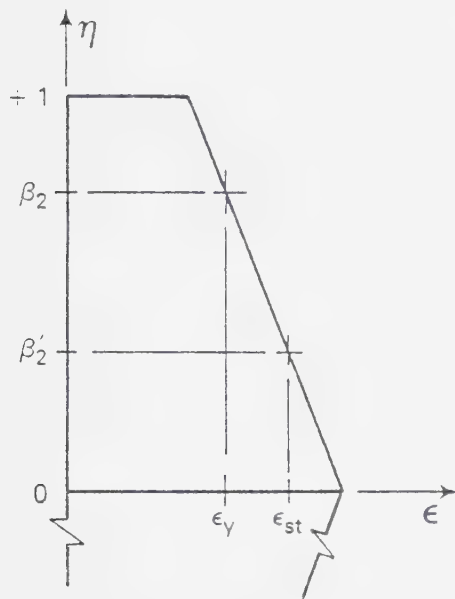
Yielding (y)



Strain Hardened (s)

Figure 4.7 Strain and Stress Distributions for Compression Zone of Web —  
(Case II - Center of Web Yielded)





(a) Strain Distribution

$$0 \leq \eta \leq 1$$

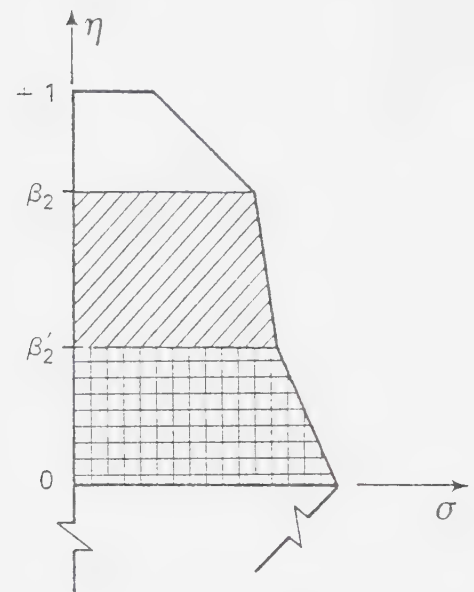
$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3 + (\epsilon_b - \epsilon_c - \epsilon_4 - \epsilon_3)\eta$$

$$\eta = 0$$

$$\epsilon = \epsilon_a + \epsilon_c + \epsilon_3$$

$$\eta = 1$$

$$\epsilon = \epsilon_b + \epsilon_a - \epsilon_4$$



(b) Stress Distribution



Figure 4.8 Strain and Stress Distributions for Compression Zone of Web — (Case III - Center of Web Strain-hardened)



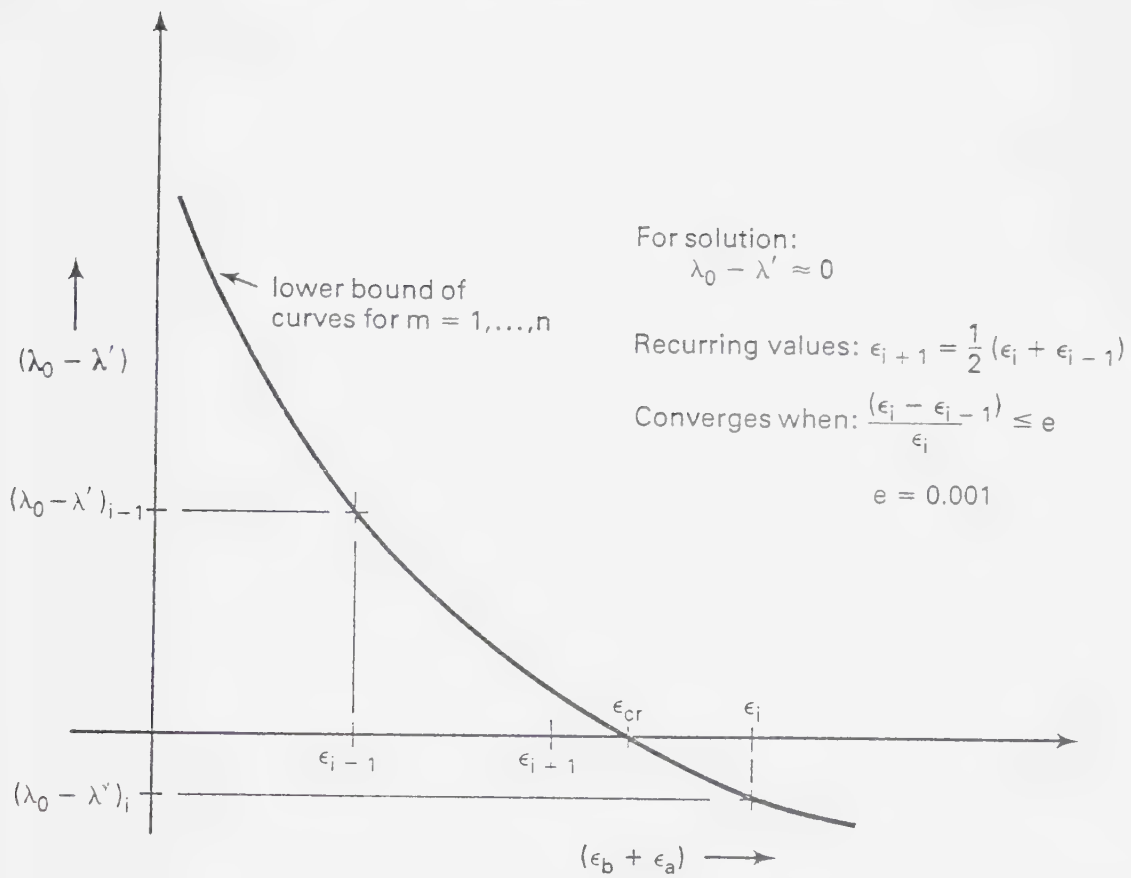


Figure 4.9 Iterative Technique for Critical Buckling Strain.



## Chapter 5

### COMPARISON OF THEORETICAL PREDICTIONS WITH TEST RESULTS

#### 5.1 Introduction

An analytical procedure for the calculation of critical loads causing local buckling of plate components of W shapes has been presented in Chapters 3 and 4. This procedure uses matrix techniques to predict critical local buckling loads which may occur either in the elastic or inelastic load region. Because of the large number of iterative calculations required it was necessary to use a computer program which was written for this purpose. In this chapter, theoretical results are compared with the results of laboratory tests performed on 57 specimens. These tests include six column specimens and six beam specimens tested by Haaïjer and Thurlimann<sup>10</sup>. Of the remaining 45 specimens, 4 were column specimens, 26 were beam specimens, and 15 were beam-column specimens all tested at the University of Alberta<sup>11,12,13,36</sup>. In all cases, local buckling of plate components of W shapes was the principal point of interest during testing.

#### 5.2 Prediction of Buckling Loads

In the analysis of local plate buckling as presented herein, it is assumed that portions of a cross-section having strains higher than the yield strain, have effective material properties corresponding to those in the strain-hardening region. As mentioned previously, this assumption has also been used successfully by other investigators in





this area<sup>25,34,51,53,54</sup>. These material properties are presented in Appendix B and have values dependent on the elastic modulus,  $E$ , the strain-hardening modulus,  $E_{st}$ , and Poisson's ratio,  $\nu$ . Where these values are not reported for a given laboratory specimen, values of  $E = 29,600$  ksi.,  $E_{st} = 800$  ksi., and  $\nu = 0.3$  are assumed. In the case where a residual stress is not available, a value of  $0.3 \sigma_y$  is assumed<sup>3,5</sup> as a maximum value of compressive and tensile residual stresses and the distribution configuration shown in Figure 3.11 is used. For the specimens tested by Haaijer and Thurlimann, specific values of  $D_x$ ,  $D_y$ ,  $D_{xy}$  and  $G_t$  were reported for a value of  $E_{st} = 900$  ksi. Consequently these values are used in the prediction of local buckling capacities for the specimens tested by Haaijer and Thurlimann.

### 5.3 Column Local Buckling Tests

Results of six column tests were published in 1958 by Haaijer and Thurlimann<sup>7</sup>. The specimens were designed to study the behaviour of W shape columns susceptible to local buckling beyond the elastic range. Each specimen was placed flat-ended between fixed plates in a testing machine and subjected to axial compression. During the tests, observations were made at each load increment to determine axial strains, web and flange deflections, and lateral movement. Column lengths varied between 23 and 32 inches while  $b\sqrt{F_y}/2t$  varied between 40 and 56 for the flanges and  $h\sqrt{F_y}/w$  varied between 147 and 265 for the webs.

The resulting critical column loads at the point of local buckling for these tests are shown in Table 5.1(a). The predicted loads as determined from the analytical procedure presented herein as



well as the ratios of the predicted to the experimentally determined loads are also shown in this table. The letters, F and W in brackets, indicate the plate component (flange or web) which initiated local buckling in each case.

In 1979 at the University of Alberta, G.L. Kulak tested four W shape column specimens for local buckling capacities. The end edges of the flanges and web of each specimen were rounded and fitted into grooved platens before being placed in a testing machine and subjected to axial compression. During the tests, local strains and plate deflections were recorded at various load levels so that a continuous monitoring of local, lateral and axial deflections was possible. The webs of all four specimens were proportioned to have a value of  $h\sqrt{F_y}/w = 200$ . Specimen numbers 1 to 3 were 36 inches long and had a value of  $b\sqrt{F_y}/2t = 72$  for the flanges. Specimen number four was 24 inches long and the value of  $b\sqrt{F_y}/2t$  for the flanges was set at 100 by milling the flanges to the required thickness. The results of these tests are presented in Table 5.1(b) where the values predicted by the analysis presented herein as well as the ratios of predicted to experimental values are also shown.

### 5.3.1 Discussion of Column Test Results

The ratios of predicted to experimental values of buckling loads presented in Table 5.1 vary between 0.97 and 1.08. It appears that there is better correlation of predicted and experimental values for Kulak's specimens than for those tested by Haaijer and Thurlimann. This difference in correlation for the two sets of test specimens is attributed to the fact that the test values for the specimens tested by



Haaiker and Thurlimann were scaled from published graphs whereas the measured values for the specimens tested by Kulak were directly available. Although this is considered to be the main source of error, other sources of error that are considered to be applicable to all test results presented herein are discussed in Section 5.6.

#### 5.4 Beam Local Buckling Tests

Theoretical values of critical moments causing local buckling in W shapes are compared with corresponding test results for 32 beam specimens. Six beam specimen test results were obtained from the work of Haaiker and Thurlimann<sup>10</sup> and twelve results were obtained from tests carried out by Holtz and Kulak<sup>11,12</sup>. The remaining 14 test results were obtained from experiments carried out by Lukey and Adams<sup>36</sup>.

The six beam sections tested by Haaiker and Thurlimann were identical to those tested in the column test series mentioned above. All beams were simply supported at the ends and loaded symmetrically by two concentrated loads so that local buckling could be expected to occur within the uniform moment region. Although the specimens were laterally braced failure was initiated by flange local buckling followed by some lateral movement between bracing points. It is to be expected that this combined failure mode affects the results predicted by the method presented herein although the extent of this effect is difficult to estimate.

In table 5.2(a) bending moments at failure for each of the six specimens tested by Haaiker and Thurlimann are compared with those predicted by the analysis presented herein. As before, the letter, F, in brackets indicates flange local buckling and in this case also, the



letter, L indicates the presence of lateral buckling. The ratios of experimental to predicted moment values are also shown.

In 1973, Holtz and Kulak<sup>11</sup> reported test results for a series of ten compact beam specimens. During testing, all specimens were simply supported and loaded symmetrically with equal concentrated loads so that a uniform moment region existed between load points. All specimens were laterally braced at the load and reaction points so that the possibility of lateral buckling was precluded. Similar tests were performed on a series of two non-compact beams in 1975<sup>12</sup>.

Table 5.2(b) shows the critical buckling moments obtained for the bending tests performed by Holtz and Kulak. The corresponding moments as determined by the analysis presented herein are shown as well as the ratios of predicted to experimental values. The letters, F and W, indicate which element (either flange or web) precipitated the local failure.

The beam specimen tests performed by Lukey and Adams<sup>36</sup> were designed for the purpose of studying the relationship between flange slenderness ratios and rotation capacities of W shape beams subjected to a moment gradient. All specimens were simply supported and loaded with a concentrated load placed at mid-span with lateral bracing placed at reaction and load points. The analytical method presented herein was not developed to predict local buckling capacities of beams subjected to moment gradients. It was assumed, however, that a buckle would normally occur in a localized region at the location of maximum moment and that adjacent moment gradient regions would not significantly affect the critical buckling moment. For this reason it was decided to include the results of the beams tested under a moment gradient as





described above.

The critical buckling moments for the specimens tested by Lukey and Adams are presented in Table 5.2(c). The critical element initiating local failure and the ratio of experimental to predicted buckling moment values are also indicated for each specimen.

#### 5.4.1 Discussion of Test Results

For the beam specimens tested by Haaijer and Thurlimann, the ratios of predicted to experimental values of the buckling moment vary between 0.97 to 1.13 as shown in Table 5.2(a). For five of the six specimens tested, the predicted values are within five per cent of the actual test values and for specimen number one the predicted value is slightly high. This slightly high value is not unacceptable, however, in view of the possible sources of error outlined in the next section.

The correlation between experimental and predicted buckling moments for the specimens tested by Holtz and Kulak is generally quite good as can be seen from Table 5.2(b). The ratios of experimental to predicted values vary between 0.89 and 1.12. For ten of the twelve specimens tested, the predicted values of buckling moment are within six per cent of the test values. For specimen numbers 5 and 9 the error is +12 per cent and -11 per cent respectively. Again, in view of the possible sources of error, as discussed in the next section for all test series, these values are considered to be acceptable.

For the specimens tested by Lukey and Adams, the ratios of predicted to experimental values of buckling moment vary between 0.87 and 1.08 for eleven of the fourteen specimens tested. For specimen numbers 2, 4, and 13, the ratios of predicted to experimental values



vary between 1.18 and 1.46 which must be considered as unacceptable correlation between test and theory for these three specimens. In attempting to explain this discrepancy between predicted and test values, it must be pointed out that all of the beam specimens tested eventually failed by a combination of local and lateral buckling. The laterally buckled shape, in plan view, was an S-shaped buckle symmetrically formed about the mid-span lateral brace. As a result, the final buckling mode was that of a combined local and lateral buckle. It is possible, therefore, that for specimen numbers 2, 4, and 13, additional lateral bracing placed at the quarter-points may have prevented lateral buckling thus permitting a failure by pure local buckling. Under such circumstances a higher test load would be obtained and a better correlation between test and theory would result. For the eleven remaining specimens, for which good correlation was obtained, it is assumed that this effect was not as significant presumably because the moment required to cause pure local buckling failure was either less than or equal to that required to cause a pure lateral buckling failure.

### 5.5 Beam-Column Local Buckling Tests

Local buckling tests on nine beam-columns using compact<sup>4</sup> sections were carried out by Perlynn and Kulak<sup>12</sup>. An additional series of tests consisting of six non-compact<sup>4</sup> beam-column specimens was carried out by Nash and Kulak<sup>13</sup>. Each specimen for both test series was aligned in a universal testing machine which was used to apply the principal concentric load through steel rockers at the top and bottom of a specimen. A moment was superimposed by using a center-hole jack acting between loading arms rigidly connected to the ends of a specimen.



As the eccentric load was increased to provide an increment of moment the principal load was decreased so that the total axial load remained constant and equal to a prescribed value. At each increment of moment, web and flange deflections were monitored locally at various points along the specimen, and overall rotations and deflections were also recorded. Each specimen was laterally braced at mid-span and adequate torsional restraint was provided at the ends by means of the rigidly connected loading arms.

The critical buckling moments for the compact beam-columns tested by Perlynn and Kulak are presented in Table 5.3(a). The ratio of the applied axial load to the yield load is also shown for each specimen. As mentioned previously, the critical element (either flange or web) which precipitates a local failure is indicated by the letter F or W. The predicted value of the local buckling moment as well as the ratio of predicted to experimental moment is also shown for each specimen. In a similar manner the results of the non-compact beam-columns tested by Nash and Kulak are presented in Table 5.3(b).

#### 5.5.1 Discussion of Beam-Column Test Results

For the specimens tested by Perlynn and Kulak the predicted values of buckling moment are within 7 per cent of the test moment for seven of the nine specimens as shown in Table 5.3(a). For the remaining two specimens, the errors are +12 per cent and +13 per cent. In view of the possible sources of error as discussed in the next section, these values are considered to be acceptable.

As shown in Table 5.3(b), for the specimens tested by Nash and Kulak, the predicted values of buckling moment are in error with



respect to the test values by less than 5 per cent for three specimens and by +11 and -13 per cent for two of the remaining three specimens. These errors are considered to be acceptable in view of the possible sources of error as discussed in the next section. For specimen number six the predicted load was only 64 per cent of the maximum load recorded during the test. The validity of this test result is in doubt when compared to that obtained for specimen number five. This specimen was identical except for a 25 per cent increase in the  $h\sqrt{F_y}/w$  term which is unlikely to result in a 40 per cent decrease in moment as is apparent from Table 5.3(b). Additional doubt is cast upon the validity of this test result since difficulty of specimen alignment at high axial loads ( $P/P_y = 0.7$ ) was apparently evident during the testing procedure<sup>60</sup>.

## 5.6 Sources of Error

In addition to the sources of error discussed in the previous sections for specific test series, the following sources of error are applicable as noted:

### (a) Material Properties

This area is probably the most significant source of error as well as the least determinable. No investigator has as yet come up with an unquestionable evaluation of plate buckling properties that can be applied over the entire inelastic plate buckling range. Apparently, the most reliable guidance presently available for these values in the strain-hardening range is based on the works of Handelman and Prager<sup>22</sup>, Haaijer and Thurlimann<sup>7</sup>, and Lay<sup>33</sup>. In all cases these values are apparently closely related to the strain-hardening modulus,  $E_{st}$ , of a





simple tension coupon test and any error involved in determining  $E_{st}$  would doubtlessly be reflected in the theoretical values predicted. No estimate in the error involved in determining  $E_{st}$  values is available.

Because the specimens tested by Haaïjer and Thurlimann and Lukey and Adams were proportioned so that failure in the plastic range was expected, an evaluation of  $E_{st}$  was made for each specimen. The specimens tested by Kulak et al on the other hand, were not proportioned as plastic design sections and therefore the effect of  $E_{st}$  was not expected to be as significant. In these cases the value of  $E_{st}$  was not available for each specimen. However, a value of  $E_{st} = 800$  ksi. has been estimated from the available stress - strain curves.

Assuming that various material properties have been accurately determined from a simple tension test, it is generally accepted that these properties also apply to larger specimens of different shape subjected to compression. It is further assumed that the material is uniform throughout the test specimens. Local material discontinuities due to welding and forming specimens may contribute to error in this respect. No estimate of the error arising from the use of tension coupon material properties is available.

#### (b) Residual Stresses

The actual magnitudes and distributions of residual stresses were not available for the specimens tested by Haaïjer and Thurlimann, and of the tests carried out by Kulak et al, residual stresses were available only for the column specimens. In these cases the residual stress distribution shown in Figure 3.11 with a maximum value of



residual stress of  $0.3\sigma_y$  was assumed. Residual stresses vary in magnitude and distribution from specimen to specimen<sup>32</sup> and affect the local buckling capacity. The effect of changing residual stress values is studied parametrically in Chapter 6.

#### (c) End Conditions

The end conditions existing at the longitudinal extremities of a local buckle depend largely upon the method of testing and usually they will lie between the pinned and fixed end conditions. In the analysis presented herein, after careful consideration of the methods of testing used in each test series, it was decided that pinned end conditions best represent the end support conditions of the column specimens. Because of the elastic moment gradient region adjacent to either end of a local buckle developing in the beam specimens tested, fixed-end support conditions were assumed. The very rigid attachment of the loading arms to the beam-columns tested resulted also in the assumption of fixed ends for these specimens. The effects of end conditions vary with the length of a specimen<sup>2</sup> and therefore any resulting errors will be more significant for shorter members. With the reduction in plate bending stiffnesses due to inelastic action, it is expected that the affects of end conditions would be lessened. For these reasons it is felt that the effects of estimating degrees of end fixity are not likely to contribute significantly to error.

#### (d) Iterative Technique

All specimens buckled beyond the elastic range and therefore a considerable amount of iteration involving matrices was required to arrive at a solution for each specimen. Although tolerances were set at 0.1 per cent in the computer program, round-off error is expected to



contribute to the total error.

(e) Analytical Technique

The buckled shapes of the webs and flanges of specimens are approximated using polynomials which result in the least energy shapes for a given specimen. It has been shown<sup>6,18</sup> that this technique results in over-estimation of buckling loads when theoretical shapes do not exactly fit the true buckled shapes. The present technique and associated computer program were checked for this source of error by comparing elastic critical values of plates with values presented by Timoshenko<sup>18</sup> and Bleich<sup>19</sup>. It was found for rectangular plates with various boundary conditions and loadings that this error varied from zero per cent for simply supported plates to about 3.0 per cent for fixed boundary conditions. Since neither web plates nor flange plates of these specimens are fully fixed, it is expected this source of error would be below 2.0 per cent in the majority of cases.

(f) Scaling from graphs

The values reported by Haaiker and Thurlimann<sup>7</sup> and by Lukey and Adams<sup>36</sup> were presented in graphical form and the critical loads were scaled from these graphs. It is estimated that the error involved in this procedure would be about  $\pm 1.0$  per cent.

(g) Test Measurements

No test difficulties, other than those mentioned above, were reported by the investigators. There are no other significant errors attributable to this source, although, as in all tests, some error, either human-related or machine-related, or both, is probable.

(h) Mode of Failure

It is apparent that in several tests, local buckling was



closely followed by, or in fact coupled with lateral buckling. Since the analytical method presented herein specifically excludes the occurrence of lateral buckling, such effects have not been evaluated. In any case, the degree to which lateral buckling was involved in the actual failure mechanism is not clearly known, and no estimate of the error involved is available.

### 5.7 Summary of Test Results

As explained previously, four of the 57 test results included herein, are not considered to be valid for the purpose of verifying a theoretical method which does not include the effects of lateral buckling. For the 53 specimens remaining, the predicted results were within 5 per cent of the test results for 60 per cent of the specimens, and within 10 per cent of the test values for 85 per cent of the specimens. The error was between  $\pm 10$  percent and an overall maximum of  $\pm 13$  per cent for only 15 per cent of the specimens. Overall, the ratio of theoretical to test values (of either critical load or critical moment) varies from -0.87 to +1.13 with a mean value of 1.00 and a standard deviation of 0.065. In all but four cases, the prediction of the critical component (either flange or web) which initiated the local buckling failure agreed with buckling observations made during the tests.

### 5.8 Summary

As outlined above, a comprehensive survey of available test data was used to substantiate the validity of the theoretical analysis presented herein as well as the associated computer program.





The comparison between predicted and experimental results has indicated satisfactory agreement between test and theory. In Chapter 6 this same technique is used to evaluate the effects of various parameters which are considered to be of some significance in affecting theoretical predictions.



Specimen Number	Load at Buckling		
	Experimental $L_E$ -(kips)	Predicted $L_T$ -(kips)	Ratio ( $L_T/L_E$ )
1	330 (F)	357 (F)	1.08
2	232 (F,W)	253 (F)	1.09
3	442 (F)	429 (F)	0.97
4	514 (F,W)	548 (F,W)	1.07
5	380 (F)	374 (F)	0.98
6	207 (W)	221 (W)	1.07

1 kip = 4.448 kN

(a) Results of G. Haaiker and B. Thurlimann

Specimen Number	Load at Buckling		
	Experimental $L_E$ -(kips)	Predicted $L_T$ -(kips)	Ratio ( $L_T/L_E$ )
1	1010 (F)	1015 (F)	1.01
2	1010 (F)	1018 (F)	1.01
3	1000 (F)	1019 (F)	1.02
4	680 (F)	693 (F)	1.02

1 kip = 4.448 kN

(b) Results of G.L.Kulak

## 5.1 Comparison of Experimental and Predicted Values for Columns.



Specimen Number	Moment at Buckling		Ratio ( $M_T/M_E$ )
	Experimental $M_E$ -(in.-kips)	Predicted $M_T$ -(in.-kips)	
1	1325 (F)	1491 (F)	1.13
2	812 (F,L)	842 (F)	1.04
3	1825 (F,L)	1797 (F)	0.98
4	2951 (L)	2848 (F)	0.97
5	1280 (F,L)	1295 (F)	1.01
6	819 (F,L)	847 (F)	1.03

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1 in.-kip = 112.98 N·M

(a) Results of Haaijer and Thurlimann

Specimen Number	Moment at Buckling		Ratio ( $M_T/M_E$ )
	Experimental $M_E$ -(in.-kips)	Predicted $M_T$ -(in.-kips)	
1	3840 (F)	3851 (F)	1.00
2	4910 (F)	4893 (F)	1.00
3	5740 (W)	6091 (F,W)	1.06
4	6820 (W)	6796 (W)	1.00
5	3940 (W)	4400 (W)	1.12
6	3380 (F)	3571 (F,W)	1.06
7	3770 (W)	4005 (W)	1.06
8	3910 (F)	3790 (F)	0.97
9	4580 (F)	4092 (F)	0.89
10	4780 (F)	4588 (F)	0.96
11	5367 (W)	5452 (W)	1.02
12	5696 (W)	5939 (W)	1.04

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1 in.-kip = 112.98 N·M

(b) Results of Holtz and Kulak

## 5.2 Comparison of Experimental and Predicted Values for Beams.

....cont'd



Specimen Number	Moment at Buckling		
	Experimental $M_E$ -(in.-kips)	Predicted $M_T$ -(in.-kips)	Ratio ( $M_T/M_E$ )
1	2251 (F)	2324 (F)	1.03
2	1845 (F)	2698 (F)	1.46
3	549 (F)	476 (F)	0.87
4	463 (F)	548 (F)	1.18
5	487 (F)	491 (F)	1.01
6	488 (F)	463 (F)	0.95
7	390 (F)	423 (F)	1.08
8	701 (F)	628 (F)	0.90
9	656 (F)	643 (F)	0.98
10	659 (F)	645 (F)	0.98
11	698 (F)	639 (F)	0.92
12	679 (F)	647 (F)	0.95
13	391 (F)	525 (F)	1.34
14	440 (F)	403 (F)	0.92

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1 in.-kip = 112.98 N·M

(c) Results of Lukey and Adams

## 5.2 Comparison of Experimental and Predicted Values for Beams.





Specimen Number	$P/P_y$	Moment at Buckling		
		Experimental $M_E$ -(in.-kips)	Predicted $M_T$ -(in.kips)	Ratio ( $M_T/M_E$ )
1	0.2	2370 (F)	2376 (F)	1.00
2	0.2	2732 (F)	2534 (F)	0.93
3	0.2	2887 (W)	2796 (W)	0.97
4	0.4	1606 (F)	1808 (F)	1.13
5	0.4	1829 (F)	1781 (F)	0.97
6	0.4	2303 (F,W)	2157 (F,W)	0.94
7	0.8	738 (F)	825 (F)	1.12
8	0.8	694 (W)	660 (W)	0.95
9	0.8	582 (W)	588 (W)	1.01

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1 in.-kip = 112.98 N·M

(a) Results of Perlynn and Kulak for Compact Sections

Specimen Number	$P/P_y$	Moment at Buckling		
		Experimental $M_E$ -(in.-kips)	Predicted $M_T$ -(in.kips)	Ratio ( $M_T/M_E$ )
1	0.15	3704 (F)	3698 (F)	1.00
2	0.15	2622 (F)	2923 (F)	1.11
3	0.30	2827 (W,F)	2712 (W,F)	0.96
4	0.30	2488 (F)	2171 (F)	0.87
5	0.70	668 (W,F)	682 (W,F)	1.02
6	0.70	1095 (F)	705 (F)	0.64

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1 in.-kip = 112.98 N·M

(b) Results of Nash and Kulak for Non-Compact Sections

### 5.3 Comparison of Experimental and Predicted Values for Beam-Columns.



## Chapter 6

### THEORETICAL STUDY AND EVALUATION OF PARAMETERS

#### 6.1 Introduction

As presented in Chapter 5, test results are available for a limited number of local plate buckling specimens subjected to axial, flexural, and combined axial and flexural loadings. The number and variation of dimensions of these specimens are not sufficient in themselves to be able to establish general design parameters. Furthermore, the critical buckling loads may be significantly influenced by certain parameters which have not been specifically studied in the tests. In this chapter, analytical results in the form of critical plate buckling curves are presented and discussed for a wide variety of columns, beams, and beam-columns. Additionally, the effects of important parameters on local plate buckling capacities are evaluated and discussed. Unless specifically varied the following basic values are used in the parametric study:  $E = 29,600 \text{ ksi.}$ ,  $\sigma_y = 44 \text{ ksi.}$ ,  $\epsilon_y = \sigma_y/E$ ,  $\epsilon_{rc} = 0.3\epsilon_y$ ,  $E_{st} = 800 \text{ ksi.}$ , flange aspect ratio = 4, and web aspect ratio = 2.

#### 6.2 Columns

Figure 6.1 shows the variation of the critical load ratio ( $P_{cr}/P_y$ ) with respect to the web width-to-thickness term ( $h\sqrt{F_y}/w$ ) for various values of the flange width-to-thickness term ( $b\sqrt{F_y}/2t$ ). The knee portion of each curve results from the presence of unavoidable



residual stresses. For the sake of comparison a curve corresponding to a theoretical value of zero residual stress is also shown. By this comparison it can be seen that the effects of residual stresses are most pronounced in the elastic and partially elastic regions. Assuming a residual compressive stress  $\sigma_{rc} = 0.3\sigma_y$ , yielding begins at  $P_{cr}/P_y = 0.7$  and  $h\sqrt{F_y}/w = 475$ . As  $h\sqrt{F_y}/w$  decreases, flange buckling becomes predominant as the yield load is approached, and at lower values strain-hardening occurs. The dashed curve represents the elastic buckling solution.

The CSA S16.1-1975 Standard<sup>4</sup> specifies a flange slenderness value for column flanges of 100. As illustrated in Figure 6.1, the analysis presented herein predicts that this value is slightly unconservative. At the currently specified limitations<sup>4</sup> of  $b\sqrt{F_y}/2t = 100$  and  $h\sqrt{F_y}/w = 255$  for columns, the analysis presented herein predicts a value of  $P_{cr}/P_y = 0.9$ . Also, the figure shows that  $P_{cr}/P_y$  does not reach a value of 1.0 until  $b\sqrt{F_y}/2t$  has been reduced to 72 and values of  $h\sqrt{F_y}/w$  are less than 300.

This theoretical result for columns has been substantiated by the results of four column specimens tested by Kulak at the University of Alberta. As discussed in Section 5.3, three of the specimens had flange slenderness values of 72, and the fourth specimen had a flange slenderness value of 100. The value of web slenderness was 200 for all four specimens. The flanges of the three specimens having  $b\sqrt{F_y}/2t = 72$  buckled at or slightly above  $P_{cr}/P_y = 1.0$ , while the flanges of the specimen having  $b\sqrt{F_y}/2t = 100$  buckled at  $P_{cr}/P_y = 0.9$ . These test results as well as the theoretical analysis presented herein indicate that a value of  $b\sqrt{F_y}/2t = 72$  (as opposed to the current



limitation<sup>4</sup> of  $b\sqrt{F_y}/2t = 100$ ) should be used for columns. For this reason, subsequent parametric studies presented herein are based on a value of flange slenderness of 72 for columns.

In order to be able to later tie in column local buckling behaviour as one limiting case of beam-columns, the local buckling strength curves for axially loaded Class 1, Class 2, and Class 3 sections are shown in Figure 6.2. The rounded portion of each curve between values of  $h\sqrt{F_y}/w$  of about 300 and 475 is due to gradual yielding in the presence of residual stresses. A curve corresponding to zero residual stresses as mentioned previously is also shown in Figure 6.2. For web slenderness values greater than 475, elastic buckling of the web occurs and for values between 475 and 300, web buckling occurs in the inelastic range. For values of web slenderness less than about 300, flange buckling in the yielded and strain-hardening ranges occurs at values of  $P_{cr}/P_y$  equal to or greater than 1.0.

A column local buckling curve is shown in Figure 6.2 for a value of  $b\sqrt{F_y}/2t = 64$  corresponding to the present CSA-S16.1 specification<sup>4</sup> for a Class 2 section. For a Class 1 section with  $b\sqrt{F_y}/2t = 54$ , a similar column curve is shown in Figure 6.2. Although these two sections are not explicitly designated for use as columns, the corresponding column curves are presented here so that the effect of the  $b\sqrt{F_y}/2t$  term for columns may be evaluated. Also, these sections subjected to column action, represent a limit of the corresponding beam-columns when bending moments are negligible.

The effect of the flange slenderness values can be seen by comparing the curves in Figure 6.2. For values of  $h\sqrt{F_y}/w$  greater than





300, where web buckling is predominant, varying  $b\sqrt{F_y}/2t$  from 54 to 72 has virtually no effect on the web buckling capacity. The effect of this term is most significant for values of  $h\sqrt{F_y}/w$  less than 300 since that is the region wherein flange buckling is predominant. For  $b\sqrt{F_y}/2t = 72$  strain-hardening of the section begins at a value of  $h\sqrt{F_y}/w = 220$ . As  $b\sqrt{F_y}/2t$  is decreased, strain-hardening begins at progressively higher values of  $h\sqrt{F_y}/w$ , the upper limit being  $h\sqrt{F_y}/w = 325$  for a Class 1 section with a flange slenderness value of 54.

### 6.2.1 Effects of Residual Stresses

The effects of residual stresses on the local buckling loads of columns is shown in Figure 6.1 for  $b\sqrt{F_y}/2t$  ratios of 72, 80, 90, and 100, and in Figure 6.2 for  $b\sqrt{F_y}/2t$  ratios of 54, 64, and 72. In these figures, column local buckling curves for a theoretical value of  $\sigma_{rc} = 0$  are compared with those corresponding to a more practical value of  $\sigma_{rc} = 0.3\sigma_y$  and a residual stress distribution as shown in Figure 3.11. The value of  $\sigma_{rc} = 0.3\sigma_y$  is representative of values used by other investigators<sup>3,5,61,62</sup> for the purpose of including the effects of residual stresses when exact values are unknown.

By comparing the curves corresponding to  $\sigma_{rc} = 0$  with those corresponding to  $\sigma_{rc} = 0.3\sigma_y$ , it can be seen that, in the range where elastic web buckling occurs (at values of  $h\sqrt{F_y}/w$  greater than about 475), a residual stress of  $0.3\sigma_y$  reduces the critical buckling load ratio by about 25 per cent. The influence of residual stresses diminishes as the value of  $P_{cr}/P_y$  approaches 1.0, and completely disappears as strain-hardening becomes imminent at lower values of  $h\sqrt{F_y}/w$ .

According to the theoretical method presented herein, the



values of  $b\sqrt{F_y}/2t = 100$  as presently specified for W shape columns<sup>4</sup> appears to be too liberal when a residual stress of  $\sigma_{rc} = 0.3\sigma_y$  is assumed. In Figure 6.1, the curve corresponding to  $b\sqrt{F_y}/2t = 100$  and  $\sigma_{rc} = 0.3\sigma_y$  reaches a value of  $P_{cr}/P_y = 0.9$  at the presently specified<sup>4</sup> web slenderness ratio of 255. The curve corresponding to  $b\sqrt{F_y}/2t = 100$  and the theoretical value of  $\sigma_{rc} = 0$  on the other hand, reaches a value of  $P_{cr}/P_y = 1.0$  for values of web slenderness as high as 475. For this reason therefore, the analytical method presented herein suggests that the presently specified value of  $b\sqrt{F_y}/2t = 100$  is too liberal because of the effects of residual stresses. The analytical method presented herein predicts that when a W shape column is proportioned so that  $b\sqrt{F_y}/2t \leq 72$  and  $h\sqrt{F_y}/w \leq 300$  a value of  $P_{cr}/P_y \geq 1.0$  can be reached for an assumed residual stress of  $\sigma_{rc} = 0.3\sigma_y$ .

### 6.2.2 Effects of Strain-hardening Modulus

Figure 6.3 shows the effects of strain-hardening modulus values of 700, 800, and 900 ksi. on the local buckling capacity of columns. These values of strain-hardening moduli were chosen because they are representative of values that have been determined by several investigators in this area<sup>5, 7, 36, 54, 59, 61, 62</sup>. For columns proportioned so that  $b\sqrt{F_y}/2t = 72$ , variations in the values of the strain-hardening modulus are influential for values of  $h\sqrt{F_y}/w$  less than about 200. In this region the curve separates into three branches corresponding to the three different values of the strain-hardening modulus investigated. As would be expected in this region, for a given value of web slenderness, the value of  $P_{cr}/P_y$  increases as the strain-hardening modulus increases. As can be seen from Figure 6.3, in the practical ranges of plate



proportions for W shape columns ( $b\sqrt{F_y}/2t = 72$  and  $h\sqrt{F_y}/w = 300$ ), the value of the strain-hardening modulus has negligible effect on column local buckling strength.

### 6.3 Beams

Figure 6.4 shows values of the ratio of critical local buckling moment to the yield moment plotted against the web width-to-thickness term ( $h\sqrt{F_y}/w$ ), for various values of the flange width-to-thickness term ( $b\sqrt{F_y}/2t$ ). Again, the effects of residual stresses are included as is evident from the rounded knee portions of these curves. For values of  $h\sqrt{F_y}/w$  greater than about 1100, local buckling of the slender web occurs in the form of warping due to the presence of residual stresses.

#### 6.3.1 Class 3 Beams

According to the CSA S16.1-1975 Standard<sup>4</sup>, the flange width-to-thickness term for a Class 3 beam is set at 100 and is the same as that for a column. The currently specified web width-to-thickness ratio is 690. For these values, Figure 6.4 shows that  $M_{cr}/M_y = 0.9$ .

Comparing similar curves for various values of  $b\sqrt{F_y}/2t$  in Figure 6.4, it is seen that  $M_{cr}/M_y$  attains a value of 1.04 at  $b\sqrt{F_y}/2t = 72$  over a large range of web slenderness (up to about  $h\sqrt{F_y}/w = 800$ ). Although  $M_{cr}$  slightly exceeds  $M_y$  (by only 4 per cent) for  $b\sqrt{F_y}/2t = 72$ , this value seems to be appropriate for a Class 3 beam in that it corresponds with the value suggested for columns. In this way, the same value may be used throughout for a Class 3 beam-column which has as its limits, pure axial load at one end of the loading spectrum, and pure flexure at the other. For  $b\sqrt{F_y}/2t = 72$ , Figure 6.4 shows that for curves



of web slenderness greater than 975 the web buckles elastically. For values of web slenderness between 800 and 975 the web buckles in the inelastic range as a consequence of the influence of residual stresses. The mode of failure changes from web buckling at a value of 800 to flange buckling for values less than 800. At a web slenderness value of 260 the curve begins to increase rapidly as strain-hardening becomes influential.

### 6.3.2 Class 2 Beams

The Standard<sup>4</sup> specifies that Class 2 beams have a flange slenderness value not exceeding 64 and a web slenderness value not exceeding 520. In Figure 6.5 values of the ratio of critical moment,  $M_{cr}$ , to the plastic moment,  $M_p$ , are plotted against the web width-to-thickness term,  $h\sqrt{F_y}/w$ , for a flange width-to-thickness value of 64. For values of  $h\sqrt{F_y}/w$  greater than 1100, local buckling occurs in the form of warping of the slender web subjected to residual stresses. Referring to the curve corresponding to a value of  $b\sqrt{F_y}/2t = 64$ , local buckling of the web occurs in the inelastic range between values of  $h\sqrt{F_y}/w$  of about 800 and 975. For values of  $h\sqrt{F_y}/w$  less than 800, flange buckling occurs at a value of  $M_{cr}/M_y = 1.0$ , and at  $h\sqrt{F_y}/w = 300$  strain-hardening of the section begins and strength increases rapidly as  $h\sqrt{F_y}/w$  decreases further.

### 6.3.3 Class 1 Beams

For Class 1 beams the Standard<sup>4</sup> specifies a flange slenderness of 54 and a web slenderness limit of 420. Values of  $M_{cr}/M_p$  are plotted against  $h\sqrt{F_y}/w$  for a value of  $b\sqrt{F_y}/2t = 54$  as shown in Figure 6.5. For  $h\sqrt{F_y}/w$  greater than 800, the behaviour of these beams is similar to





that described previously for Class 2 sections. Because of the increased sturdiness of the flanges however, web buckling continues to be the mode of failure for values of  $h\sqrt{F_y}/w$  as low as 600. Below this value, flange buckling occurs. For values less than 800,  $M_{cr}/M_p$  is greater than 1.0 and this implies that the strain in the flanges can reach the strain-hardening range before local buckling occurs. Therefore, these sections can undergo sufficient rotation to permit redistribution of stresses while sustaining the plastic moment value, as required of Class 1 sections<sup>2,3,4</sup>.

#### 6.3.4 Effects of Residual Stresses

The critical buckling moment ratio is plotted against values of web slenderness for values of flange slenderness of 72, 80, 90, and 100 in Figure 6.4 and for values of flange slenderness of 54 and 64 in Figure 6.5. Again, the assumed residual stress value is  $\sigma_{rc} = 0.3\sigma_y$  and the assumed residual stress distribution is similar to that shown in Figure 3.11. For the purpose of comparison, these curves are also plotted for the theoretical value of  $\sigma_{rc} = 0.0$ . In the range where web buckling is critical ( $P_{cr}/P_y < 1.0$  and  $h\sqrt{F_y}/w > 800$ ) residual stresses have their greatest effect in reducing the critical buckling moment capacity. For values of  $h\sqrt{F_y}/w$  greater than about 1100, the residual stress causes web buckling in the form of warping of the very slender webs. As values of  $h\sqrt{F_y}/w$  decrease below 800, the significance of residual stresses diminishes and as strain-hardening is approached their effects become negligible.

As discussed for columns, it appears here also that the presently specified flange slenderness<sup>4</sup> of 100 is too liberal for



Class 3 beams when a value of  $\sigma_{rc} = 0.3\sigma_y$  is assumed. Referring to Figure 6.4, for a value of flange slenderness of 100 and  $\sigma_{rc} = 0.3\sigma_y$ , the critical local buckling moment is 90 per cent of the yield moment at the presently specified web slenderness<sup>4</sup> of 690. The theoretical curve for which  $b\sqrt{F_y}/2t = 100$  and  $\sigma_{rc} = 0$ , on the other hand, shows that the critical local buckling moment reaches the yield moment for values of web slenderness as high as 800. This suggests, therefore, that the value of  $b\sqrt{F_y}/2t = 100$  for Class 3 beams is too liberal because of the effects of residual stresses.

### 6.3.5 Effects of Strain-hardening Modulus

The effect of varying the strain-hardening modulus upon the local buckling behaviour of beams is shown in Figure 6.6 for a value of  $b\sqrt{F_y}/2t = 72$  and in Figure 6.7 for a value of  $b\sqrt{F_y}/2t = 54$ . As for columns, the same three values of strain-hardening modulus are used here for beams. Figure 6.6 shows that for sections with  $b\sqrt{F_y}/2t = 72$ , the effect of strain-hardening is evident only for values of  $h\sqrt{F_y}/w$  less than about 280. Although not explicitly shown herein, it has been determined that these same observations apply to compact sections<sup>4</sup> with a value of  $b\sqrt{F_y}/2t = 64$ . For plastic design sections<sup>4</sup>, Figure 6.7 shows that the effect of strain-hardening is influential for values of  $h\sqrt{F_y}/w$  less than 800. This is because the very sturdy flanges ( $b\sqrt{F_y}/2t = 54$ ) enable these sections to reach strain-hardening strains at relatively high web slenderness values.

For the values of  $E_{st}$  examined, there are no significant differences in local buckling strength for beams having flange slenderness values of 72 and 64 when practical values of web slenderness



are used. The local buckling strength of plastic design sections, on the other hand, are significantly affected by changes in the strain-hardening modulus for all values of web slenderness less than about 800. As would be expected, this effect increases as web slenderness values decrease.

#### 6.4 Beam-Columns

The previous section has indicated that suitable flange slenderness values for flexural members are  $b\sqrt{F_y}/2t = 54, 64, \text{ and } 72$  for Class 1, 2, and 3 respectively. Further, a suitable value for axially loaded members is  $b\sqrt{F_y}/2t = 72$ . These will therefore be the values principally investigated for beam-columns. The analysis will be based on the assumption that an axial strain less than the critical one will be applied first. A flexural strain is then superimposed and gradually increased until local buckling occurs.

At each increment of flexural strain, equilibrium of a cross-section is satisfied and the position of the neutral axis is updated to account for yielding of a section. For a given flange width-to-thickness term, values of the ratio of applied load to yield load can then be plotted against values of the ratio of critical moment to yield moment or plastic moment for various web width-to-thickness terms. The web width-to-thickness term is varied from 300 to 800, corresponding to the limits determined previously for a pure column (zero flexural strain) and a pure beam (zero axial strain). In this way, for a given  $b\sqrt{F_y}/2t$  value, a set of interaction curves is generated for various  $h\sqrt{F_y}/w$  values.



#### 6.4.1 Class 3 Beam-Columns

As discussed in Section 6.3.1, the present code value of  $b\sqrt{F_y}/2t = 100$  for a Class 3 section appears to be too high according to the theory presented herein. Furthermore, a value of  $b\sqrt{F_y}/2t = 72$  was found to be adequate for local buckling requirements of columns and for beams required to reach the yield moment,  $M_y$ , before local buckling occurs. In this section, therefore, interaction curves are generated for beam-columns corresponding to a  $b\sqrt{F_y}/2t$  value of 72 and various values of  $h\sqrt{F_y}/w$ .

Ratios of applied load to yield load are plotted against the ratios of critical moment to yield moment for various  $h\sqrt{F_y}/w$  values in Figure 6.8. Values along the vertical axis represent pure column behaviour and those along the horizontal axis represent pure beam behaviour. For each curve corresponding to a fixed  $h\sqrt{F_y}/w$  value, as  $M_{cr}/M_y$  increases above zero, a point of tangency is approached on the line joining the points  $P/P_y = 1.0$  and  $M_{cr}/M_y = 1.04$ . At the point of tangency the mode of failure changes from web local buckling to flange buckling and beyond this point each curve remains tangent to the line. The sloping dashed line,  $M_{yc} = M_y (1 - P/P_y)$ , represents the strength interaction equation for a Class 3 beam-column.

##### 6.4.1.1 Current Specifications

For Class 3 beam-columns the web limitations are currently specified as follows:

$$h\sqrt{F_y}/w = 690 (1 - 2.60 P/P_y) \quad 0 \leq P/P_y \leq 0.15 \quad (6.1)$$





$$h\sqrt{F_y}/w = 450 (1-0.43 P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.2)$$

These web limitations are based on test results obtained by Kulak and Nash<sup>13</sup>. Since the time of publication of these limitations, additional work in this area by Kulak and Nash<sup>13</sup> has indicated the following increases:

$$h\sqrt{F_y}/w = 690 (1-1.69 P/P_y) \quad 0 \leq P/P_y \leq 0.15 \quad (6.3)$$

$$h\sqrt{F_y}/w = 535 (1-0.28 P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.4)$$

Although the theory presented herein indicates that a  $b\sqrt{F_y}/2t$  value of 72 should be used for Class 3 sections, the specimens tested by Kulak and Nash were designed on the basis of the existing code value of  $b\sqrt{F_y}/2t = 100$ . Furthermore, the specimens were able to reach, and in some cases exceed the value of  $M_y$  reduced in the presence of axial load. The theory presented herein also predicts values in good agreement with these test results when the effects of the rigid plate boundary constraints (necessitated by the attachment of the loading arms to the test specimens) are included in the analysis. In practice, however, the effects of these rigid supports for longer members are negligible. Omitting the effects of these supports, the present method predicts that a value of  $b\sqrt{F_y}/2t = 72$  is indicated for Class 3 sections. For this value, the web limitations as predicted by the theory presented herein are discussed in the following section.

#### 6.4.1.2 Theoretical Limitations as Determined Herein

For a Class 3 section the beam-column interaction curves are



plotted in Figure 6.8 for various values of  $h\sqrt{F_y}/w$ . At various values of  $P/P_y$  each curve intersects the line described by:

$$M = M_y (1 - P/P_y). \quad (6.5)$$

In Figure 6.9 these values of  $P/P_y$  are plotted against the corresponding values of  $h\sqrt{F_y}/w$ . For pure beam action at  $P/P_y = 0$ , the predicted web limitation is  $h\sqrt{F_y}/w = 850$ . As  $P/P_y$  increases, the slope of the curve decreases rapidly and becomes constant in the region of  $P/P_y$  between 0.42 and 0.75. As  $P/P_y$  further increases the slope decreases slightly as pure axial strains are approached at  $P/P_y = 1.0$  and a minimum value of  $h\sqrt{F_y}/w = 300$  is reached.

A linear approximation to the curve described above is also shown in Figure 6.9 and it is given by the following relationship:

$$h\sqrt{F_y}/w = 725 (1.0 - 0.59(P/P_y)) \quad 0 \leq P/P_y \leq 1.0 \quad (6.6)$$

This approximation deviates markedly on the conservative side from the theoretical curve for low values of  $P/P_y$ . In this region however, a conservative limitation is desirable in order to account for small residual axial loads occurring in service and as well to avoid the sensitivity of the steep theoretical gradient in this region. The relationship of Equation 6.6 as well as the limitations described by Equations 6.1 and 6.2 are also shown in Figure 6.9.

Referring to Figure 6.9, the theoretical method presented herein indicates a web limitation of  $h\sqrt{F_y}/w = 725$  at  $P/P_y = 0.0$  for beams. As  $P/P_y$  increases, a linear decrease in  $h\sqrt{F_y}/w$  as a result of decreasing web flexural tension, is indicated. For pure column action, at  $P/P_y = 1.0$ , a minimum value of  $h\sqrt{F_y}/w$  of 300 is reached. This theoretical



limitation differs from that presently specified by Equations 6.1 and 6.4 in the region of pure beam action ( $P/P_y = 0.0$ ) by about +5 per cent. As  $P/P_y$  increases this difference reaches a maximum of about +60 per cent at  $P/P_y = 0.15$  and from there it decreases to +18 per cent at  $P/P_y = 1.0$ . This comparison should be considered in light of the fact that the theoretical web limitations are based on the assumption of a flange slenderness ratio of  $b\sqrt{F_y}/2t = 72$  ( as previously explained), whereas the present limitations are based on the results of test specimens whose flange slenderness ratios were  $b\sqrt{F_y}/2t = 100$ .

#### 6.4.2 Class 2 Beam-Columns

As already indicated, it will be assumed that a Class 2 beam-column is proportioned so that the flange width-to-thickness term is  $b\sqrt{F_y}/2t = 64$ . According to the theory presented herein, this value was found to be adequate for compact beam sections as illustrated in Figure 6.5. At the other extreme of beam-column action, namely pure axial load, these sections can reach a  $P_{cr}/P_y$  value of 1.0 for a value of  $h\sqrt{F_y}/w = 300$ , as shown in Figure 6.2. Accordingly, these beam-column sections were investigated for a range of  $P_{cr}/P_y$  ratios between zero and one, with the  $h\sqrt{F_y}/w$  values varying from 300 to 800. The results of this investigation are presented in Figure 6.10.

In this figure, values of the ratio of applied load to yield load are plotted against ratios of critical moment to plastic moment for various values of  $h\sqrt{F_y}/w$ . The flange width-to-thickness term for all curves has a constant value of 64. For these curves, the same discussion as presented for Class 3 beam-columns in Section 6.4.1, is valid here for Class 2 beam-columns.



#### 6.4.2.1 Current Specifications

The present limitations specified for web width-to-thickness terms for Class 2 beam-columns are as follows<sup>4</sup>:

$$h\sqrt{F_y}/w = 520 (1-1.28 P/P_y) \quad 0 \leq P/P_y \leq 0.15 \quad (6.7)$$

$$h\sqrt{F_y}/w = 450 (1-0.43 P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.8)$$

These web limitations are based on test results from a series of nine compact beam-columns tested by Perlynn and Kulak<sup>12</sup>. As presented in Chapter 5, the analytical method presented herein substantiates these results when the effects of rigid supports necessary for testing purposes are included in the analysis. Conservatively omitting the effects of these rigid supports however, the analysis presented herein shows that the conventional value of plastic moment reduced for the presence of axial load<sup>63,64</sup>,

$$M_{pc} = M_p \quad 0 \leq P/P_y \leq 0.15 \quad (6.9(a))$$

and,

$$M_{pc} = 1.18 M_p (1-P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.9(b))$$

cannot be attained by compact sections when  $h\sqrt{F_y}/w$  is greater than 250. Such a limitation however would severely restrict the choice of web sizes available for use in compact sections. As shown in Figure 6.10, the analytical method presented herein predicts a maximum local buckling strength for compact sections of:

$$M_{pc} = M_p (1-P/P_y), \quad (6.10)$$





when the web slenderness ratio varies between 300 and 800. Therefore, in accordance with the analytical results presented in Figure 6.10, Equation 6.10 is suggested for use with compact sections and, in this way, all values of  $h\sqrt{F_y}/w$  up to 800 can be utilized for Class 2 sections.

#### 6.4.2.2 Theoretical Limitations as Determined Herein

As shown in Figure 6.10 the curves corresponding to various values of  $h\sqrt{F_y}/w$  become tangent to the line described by Equation 6.10 at various values of  $P/P_y$ . In Figure 6.11 these values of  $h\sqrt{F_y}/w$  are plotted against the corresponding values of  $P/P_y$ . The resulting curve has a value of  $h\sqrt{F_y}/w = 800$  for pure beam action ( $P/P_y = 0.0$ ). As  $P/P_y$  increases, the slope of the curve rapidly increases to a constant value in the region of  $P/P_y$  between 0.42 and 0.75. As  $P/P_y$  further increases, the slope decreases slightly as pure axial strains are approached at  $P/P_y = 1.0$ . At this value of  $P/P_y$  a minimum value of  $h\sqrt{F_y}/w = 300$  is reached.

The curve described above may be approximated by a linear relationship given by:

$$h\sqrt{F_y}/w = 660 (1 - 0.55 (P/P_y)) \quad 0 \leq P/P_y \leq 1.0 \quad (6.11)$$

This relationship as well as the relationships defined by Equations 6.7 and 6.8 as established by Kulak and Perlynn<sup>12</sup> are shown in Figure 6.11.

In establishing the above linear approximation a line was drawn from point A at the end of the curve at  $P/P_y = 1.0$  to the point of tangency at B and extended to intersect the vertical axis at



$h\sqrt{F_y}/w = 660$ . This approximation results in conservative limitations of  $h\sqrt{F_y}/w$  for low values of  $P/P_y$ . This is desirable however in order to avoid the sensitivity of the steep theoretical gradient in this region as well as to recognize the possibility of small unavoidable axial loads that may occur in service.

As can be seen from Figure 6.11, the theoretical method presented herein predicts more liberal web limitations than those presently required for Class 2 sections. However, implicit in these limitations is the use of  $M_{pc}$  as defined by Equation 6.10 based on local buckling considerations rather than the use of  $M_{pc}$  as defined on a strength basis by Equations 6.9. Referring to Figure 6.11, the web limitation for pure beam action at  $P/P_y = 0.0$  is  $h\sqrt{F_y}/w = 660$ . As  $P/P_y$  increases, the web is subjected to decreasing amounts of flexural tension. As a result, the  $h\sqrt{F_y}/w$  limitation decreases linearly to a minimum value of 300 for pure column action at  $P/P_y = 1.0$ . In the region of pure bending ( $P/P_y = 0.0$ ), this theoretical limitation differs by about +27 per cent from that presently specified by Equations 6.7 and 6.8. As  $P/P_y$  increases this difference increases to a maximum of about 45 per cent at  $P/P_y = 0.15$  and decreases to a minimum of +18 per cent for pure column action at  $P/P_y = 1.0$ . As explained previously, the theoretical limitations are based on the assumption that the maximum moment in the presence of axial load is that given by Equation 6.10. The presently specified limitations, on the other hand, are based on the results of test specimens whose moment capacities equalled or exceeded that given by Equations 6.9. In addition to the above theoretical limitations, the analytical method presented herein also gives results in good agreement with these test



results when the effects of the rigid plate boundary constraints necessary for testing are included. The marked difference between the theoretical limitations and the presently specified limitations is therefore largely attributable to the effects of necessary constraints used during testing.

#### 6.4.3 Class 1 Beam-Columns

A Class 1 beam-column is proportioned so that the flange width-to-thickness term  $b\sqrt{F_y}/2t = 54$ . As discussed in Sections 6.2 and 6.3.3, this value was found to be adequate at the two extremes of beam-column action, namely pure axial strain and pure flexural strain. The behaviour of Class 1 beam-columns was investigated for a range of values of  $P/P_y$  between zero and one, and for values of  $h\sqrt{F_y}/w$  between 325 and 800. The results of this investigation are presented in the form of interaction curves in Figure 6.12, where values of  $P/P_y$  are plotted against values of  $M_{cr}/M_p$  for various values of  $h\sqrt{F_y}/w$ .

Unlike those for Class 2 and Class 3 sections, the interaction curves for Class 1 sections do not become tangential to a line joining the points at  $P/P_y = 1.0$  and  $M_{cr}/M_p = 1.0$ . For Class 2 and Class 3 sections it was found that at a given value of  $P/P_y$ , the maximum critical moment value is determined by flange local buckling capacity. However, in the present case, local buckling is controlled by the web for values of  $M_{cr}/M_p$  less than about 1.05. Since web buckling controls, for a given value of  $P/P_y$  an increase in the sturdiness of the web (corresponding to a reduced  $h\sqrt{F_y}/w$  value) will result in an increase in the value of  $M_{cr}/M_p$ . Thus the curves in Figure 6.12 do not become tangential but rather, they are separated to



the right of one another as  $h\sqrt{F_y}/w$  decreases.

In the region of pure flexural strains, an upward sweep of the curves is evident for values of  $h\sqrt{F_y}/w$  less than or equal to 600. This is apparently due to two factors. Firstly, as the region of pure flexural strain is approached in the vicinity of  $M_{cr}/M_p = 1.05$ , the mode of failure changes from web buckling to flange buckling. The second factor which contributes to this upward sweep is the effect of strain-hardening on the flange. (It is shown in Figure 6.15 that a reduction in the value of  $E_{st}$  reduces this upward sweep).

Referring again to Figure 6.12, the equation of the line joining the points,  $P/P_y = 1.0$  and  $M_{cr}/M_p = 1.0$  is given by Equation 6.10 and is restated here as follows:

$$M_{cr}/M_p = (1 - P/P_y) \quad (6.10)$$

For all values of  $h\sqrt{F_y}/w$  corresponding to the curves shown in the figure, the moment ratio given by Equation 6.10 is either equalled or exceeded for certain values of  $P/P_y$ . This implies that the strain-hardening strain may be reached or exceeded for these values and therefore the hinge rotation necessary for plastic design sections is attainable. For values of  $h\sqrt{F_y}/w$  equal to 325 and 350, the effect of strain-hardening is more pronounced as indicated by the increased slopes of the corresponding curves.

The cross-sectional strength interaction equation commonly used for plastic design sections<sup>63,64</sup> is also shown in Figure 6.12. This is described by Equations 6.9 as stated previously and repeated here for convenience as follows:





$$M_{pc} = M_p \quad 0 \leq P/P_y \leq 0.15 \quad (6.9(a))$$

$$M_{pc} = 1.18 M_p (1 - P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.9(b))$$

These values are applicable as long as local and overall instability do not occur. The validity of Equations 6.9 has been verified experimentally using W12 x 36, W8 x 31, W4 x 13, and W14 x 78 sections<sup>5,63,64,66</sup> with values of  $b\sqrt{F_y}/2t$  of 38, 60, 39 and 53, and values of  $h\sqrt{F_y}/w$  of 206, 153, 84 and 186, respectively. For these plate width-to-thickness terms the theory presented herein verifies that local buckling indeed is not critical in these sections and therefore does not interfere with the strength limitation as given by Equations 6.9. However, it should be noted that the specimens used for the experimental verification of the strength limitation are considerably sturdier than those presently designated as Class 1 sections.

#### 6.4.3.1 Current Specifications

As mentioned previously, Class 1 sections are those for which the flange width-to-thickness term is presently limited to 54. Under pure axial loading the web width-to-thickness term for all W shapes is presently limited to 255 and for pure beam action the limit<sup>4</sup> is set at 420. In the intermediate range where beam-column action is required, a bi-linear curve is applicable. This bi-linear relationship is presently expressed as follows<sup>4</sup>:

$$h\sqrt{F_y}/w = 255 \quad 0.28 \leq P/P_y \leq 1.0 \quad (6.12)$$



$$h\sqrt{F_y}/w = 420 (1 - 1.4 P/P_y) \quad 0 \leq P/P_y \leq 0.28 \quad (6.13)$$

The current code width-to-thickness limitation for column webs is set at  $h\sqrt{F_y}/w = 255$ . This value is based upon the results of specimen D6, one of six tested by Haaijer and Thurlimann<sup>7</sup>. The current web limitations for Class 1 beam-columns are based on semi-empirical values obtained by Haaijer and Thurlimann. Although no beam-columns were tested, they used the test results of specimens D2, D4, and D6 in which the webs were uniformly compressed. It was then assumed that values obtained at the level of critical strain for these specimens could be applied at the level of mean strain in the compression zone of a beam-column. Although the analysis did not directly incorporate the effects of residual stresses, in the region of critical stress between the proportional limit and complete yield, a transition curve was fitted on the basis of geometric considerations.

According to the present theory, the current value of  $b\sqrt{F_y}/2t = 54$  is an adequate limitation for Class 1 sections and no reduction or increase is indicated. It is apparent, however, that the present web width-to-thickness limitations for these sections are conservative. The present theory predicts that the value of  $h\sqrt{F_y}/w$  can be increased for all values of  $P/P_y$ .

#### 6.4.3.2 Theoretical Limitations as Determined Herein

Class 1 beam-columns are required to reach and sustain the reduced plastic moment capacity through a hinge rotation sufficient for the redistribution of stresses within a structure prior to collapse<sup>4,33,63</sup>. Because of the stricter requirements, these sections



should be expected to satisfy the strength limitations of Equations 6.9. In fact, the theoretical method presented herein predicts that this is so and that the present web limitations are somewhat conservative for Class 1 sections.

Values of  $P/P_y$  versus  $M_{cr}/M_p$  are plotted for various values of  $h\sqrt{F_y}/w$  for Class 1 sections as shown in Figure 6.12. The strength relationship given by Equations 6.9 is also plotted in this figure. As shown in Figure 6.12 the local buckling capacity equals or exceeds the strength capacity of Equations 6.9 for some values of  $h\sqrt{F_y}/w$ . Values of  $P/P_y$  may be determined at the points of intersection of the local buckling curves with the strength curve of Equations 6.9. The corresponding values of  $h\sqrt{F_y}/w$  are plotted against  $P/P_y$  values in Figure 6.13.

The resulting curve has a value of  $h\sqrt{F_y}/w = 800$  for pure beam action ( $P/P_y = 0.0$ ). For a relatively small increase of  $P/P_y$  from zero to about 0.08 the  $h\sqrt{F_y}/w$  values decrease rapidly to a value of 430. As  $P/P_y$  increases, a sharp knee portion of the curve occurs and at  $P/P_y = 0.23$  and  $h\sqrt{F_y}/w = 370$  the curve becomes linear with a slight drop in  $h\sqrt{F_y}/w$  to about 360 as  $P/P_y$  increases to 0.75. As  $P/P_y$  increases further, a slightly rounded portion of the curve occurs with a gradual decrease in slope. The  $h\sqrt{F_y}/w$  term reaches a value of 325 for pure column action ( $P/P_y = 1.0$ ). (This value of  $h\sqrt{F_y}/w$  is in contrast to a value of 300 for Class 2 and 3 sections. The slightly higher value for Class 1 sections is due to the effect of the sturdier flanges).

The curve described above may be approximated by a bi-linear relationship given by:



$$h\sqrt{F_y}/w = 430 (1-0.93(P/P_y)) \quad 0 \leq P/P_y \leq 0.15 \quad (6.14)$$

and,

$$h\sqrt{F_y}/w = 382 (1-0.22(P/P_y)) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.15)$$

This approximation as well as the present web limitations given by Equations 6.12 and 6.13 are also shown in Figure 6.13.

In arriving at the above approximation, point A is located at  $P/P_y = 1.0$  and  $h\sqrt{F_y}/w = 300$ . This value of  $h\sqrt{F_y}/w$  is used here so that for pure column action, the web limitations for Class 1, 2, and 3 beam-columns would coincide at a value of  $h\sqrt{F_y}/w = 300$ . (As explained previously, the value of  $h\sqrt{F_y}/w = 325$  for Class 1 sections is due to the sturdier flanges). Point C on Figure 6.13 is located on the vertical axis opposite point D where the initial portion of the curve first deviates from the tangent to the curve at  $P/P_y = 0$ . Through points A and C, tangents to the curve are drawn and extended to intersect at point B ( $P/P_y = 0.15$ ).

The reason for constructing line BC in this manner is twofold. In the region of low axial loads (corresponding to line CD) the web width-to-thickness term is very sensitive to small changes in the axial load. This is witnessed by the very steep gradient of the initial portion of the curve. Since a value of low axial load would be very difficult to pin-point with any great degree of accuracy (in a practical design case) line BC is constructed to eliminate the effect of this high sensitivity for practical applications. Furthermore, in practical cases, it is very likely that residual axial loads of low magnitudes will be present to some degree even in members





designed for pure bending. This effect is also accounted for by the above construction.

As seen from Figure 6.13, the theoretical method presented herein predicts more liberal web limitations than those presently required for Class 1 sections. For pure beam action at  $P/P_y = 0.0$ , the present theory indicates a web limitation of  $h\sqrt{F_y}/w = 430$  which is about 2 per cent greater than the presently specified value of 420 (Equation 6.13). As  $P/P_y$  increases to 0.15,  $h\sqrt{F_y}/w$  decreases to 370 and, as  $P/P_y$  further increases, the maximum difference between the presently specified value of  $h\sqrt{F_y}/w$  and that predicted by the analysis presented herein occurs at  $P/P_y = 0.28$ . At this point, the presently specified value of  $h\sqrt{F_y}/w$  is 255 whereas the theory presented herein predicts a value about 40 per cent higher. As  $P/P_y$  increases, this difference decreases and at  $P/P_y = 1.0$  the theory presented herein indicates a value about 18 per cent higher than the presently specified value of  $h\sqrt{F_y}/w = 255$ .

#### 6.4.4 Effects of Residual Stresses

The effects of residual stresses on the local buckling capacities of beam-column sections were investigated for values of  $b\sqrt{F_y}/2t = 54, 64, \text{ and } 72$ . Since the effects are similar in all three cases, only one such case will be presented here. For a value of flange slenderness of 54 (corresponding to a Class 1 section) and various values of web slenderness, beam-column interaction curves are shown in Figure 6.12 for  $\sigma_{rc} = 0.3\sigma_y$  and in Figure 6.14 for a theoretical value of  $\sigma_{rc} = 0$ .

The rounded knee portions of the curves in Figure 6.12 are



due to residual stresses which result in a gradual yielding within a cross-section. As shown in Figure 6.14, these rounded knee portions change to an abrupt transition in the absence of residual stresses. As can be seen, a Class 1 beam-column can perform adequately over a range of web slendernesses between 440 and 1000 where the residual stress has a theoretical value of zero. In the presence of a practical value of residual stress of  $\sigma_{rc} = 0.3\sigma_y$ , on the other hand, Figure 6.12 shows that these web slenderness values are reduced to 325 for pure column action and 800 for pure beam action.

#### 6.4.5 Effects of Strain-hardening Modulus

The effect of varying the strain-hardening modulus on the local buckling capacities of beam-columns was studied for flange slenderness values of 54, 64, and 72. The same three values of strain-hardening modulus used in the study of column and beam local buckling were also used here. For  $b\sqrt{F_y}/2t$  values of 72 and 64 and practical values of  $h\sqrt{F_y}/w$  between 300 and 800 as used in the interaction diagrams and shown in Figures 6.8 and 6.10, the values of  $E_{st}$  investigated had no effect on local buckling capacities. For Class 1 sections ( $b\sqrt{F_y}/2t = 54$ ) and practical values of web slenderness between 325 and 800 (as used in Figure 6.12) the effect of varying the strain-hardening modulus is significant for certain values of  $h\sqrt{F_y}/w$ .

For the values of strain-hardening modulus considered, Figure 6.15 shows the interaction curves corresponding to  $b\sqrt{F_y}/2t = 54$  and values of  $h\sqrt{F_y}/w$  of 325, 500 and 800. These three values were chosen as being representative of the range of web slendernesses considered practical for Class 1 sections. When  $h\sqrt{F_y}/w = 325$ , the



effect of strain-hardening is present over the entire length of the curve. For pure beam action ( $P/P_y = 0$ ), the effect of the strain-hardening modulus is quite significant. As  $P/P_y$  increases this effect diminishes and is least significant when pure column action ( $P_{cr}/P_y = 1.0$ ) is approached. As  $h\sqrt{F_y}/w$  is increased the significance of strain-hardening (for the values investigated) becomes less noticeable. At  $h\sqrt{F_y}/w = 500$  the effect of varying  $E_{st}$  is noticeable at the point of pure beam action ( $P/P_y = 0$ ). As  $P/P_y$  increases, this effect decreases and is no longer evident for values of  $P/P_y$  between 0.5 and the critical load ratio. As  $h\sqrt{F_y}/w$  is further increased, the effect of varying the strain-hardening modulus over the range considered also decreases. At  $h\sqrt{F_y}/w = 800$  no effect of changing the value of the strain-hardening modulus is evident. As can be seen from Figure 6.15, for the range of values of strain-hardening modulus considered, a Class 1 beam-column section has adequate performance with regard to local buckling capacity.

#### 6.4.6 Effects of Specimen Length

It has been thoroughly demonstrated by several investigators<sup>2, 3, 6, 16, 26</sup> that critical plate buckling stresses approach relatively high values with decreasing values of aspect ratio (the ratio of the length of a plate in the direction of uniaxial stress to its transverse dimension). At a theoretical aspect ratio equal to zero, the critical buckling stress is infinite and as the aspect ratio increases for a given specimen, the predicted critical buckling stress rapidly decreases. At some point a value of the aspect ratio will be reached above which the predicted critical stress becomes stable. This



behaviour is illustrated in Figure 6.16 for a column having a flange slenderness value of 72 and a web slenderness value of 300. As the length,  $L$ , increases from zero, the critical load ratio drops rapidly and reaches a stable value for length values greater than 12. This corresponds to an aspect ratio of 2.4 for the flanges and 1.2 for the web for this particular specimen.

Investigations similar to the above have been carried out for additional columns, beams, and beam-columns of various dimensions. From this analysis it was determined that stable values of critical local buckling stress would be reached for all specimens having minimum aspect ratios of 4 for the flanges and 2 for the webs.

Figure 6.17 summarises the findings of this investigation by showing the length effects on the interaction diagrams of Class 1 beam-columns having web slenderness values of 325, 500, and 800. A Class 1 beam-column was chosen so that the length effect could be evaluated for a range of values of load ratio and moment ratio as well as a range of material conditions from elastic to fully strain-hardened. These include those values that occur: (1) at an elastic or partially yielded stress level ( $h\sqrt{F_y}/w = 800$ ), (2) at an elastic, partially yielded, or a strain-hardened stress level ( $h\sqrt{F_y}/w = 500$ ), and (3) at a strain-hardened stress level ( $h\sqrt{F_y}/w = 325$ ).

Each curve in Figure 6.17 was plotted for values of the length,  $L$ , of 15, 20, 25, and 30. These values correspond to flange aspect ratios of 3, 4, 5, and 6, and to web aspect ratios of 1.5, 2.0, 2.5, and 3.0. For all specimens the length effect is negligible in the region for which  $M_{cr} > M_p(1-P/P_y)$ . This corresponds to the region where complete yielding has occurred and strain-hardening is imminent.





Presumably a negligible effect of the variation of aspect ratios investigated occurs in this region because of the reduced stiffness of the material in the strain-hardening region. In the elastic region (the initially flat portion of the curves corresponding to  $h\sqrt{F_y}/w = 500$  and  $800$ ) and the partially yielded region (the remaining portion of these curves up to the line  $M_{cr} = M_p(1-P/P_y)$ ), the length effects are noticeable. For all cases considered however, the length effects are negligible for values of length of 20 or greater. A length value of 20 corresponds to a flange aspect ratio of 4 and a web aspect ratio of 2. As a result of this investigation, these aspect ratios were the minimum values used for all specimens in the parametric studies presented herein.



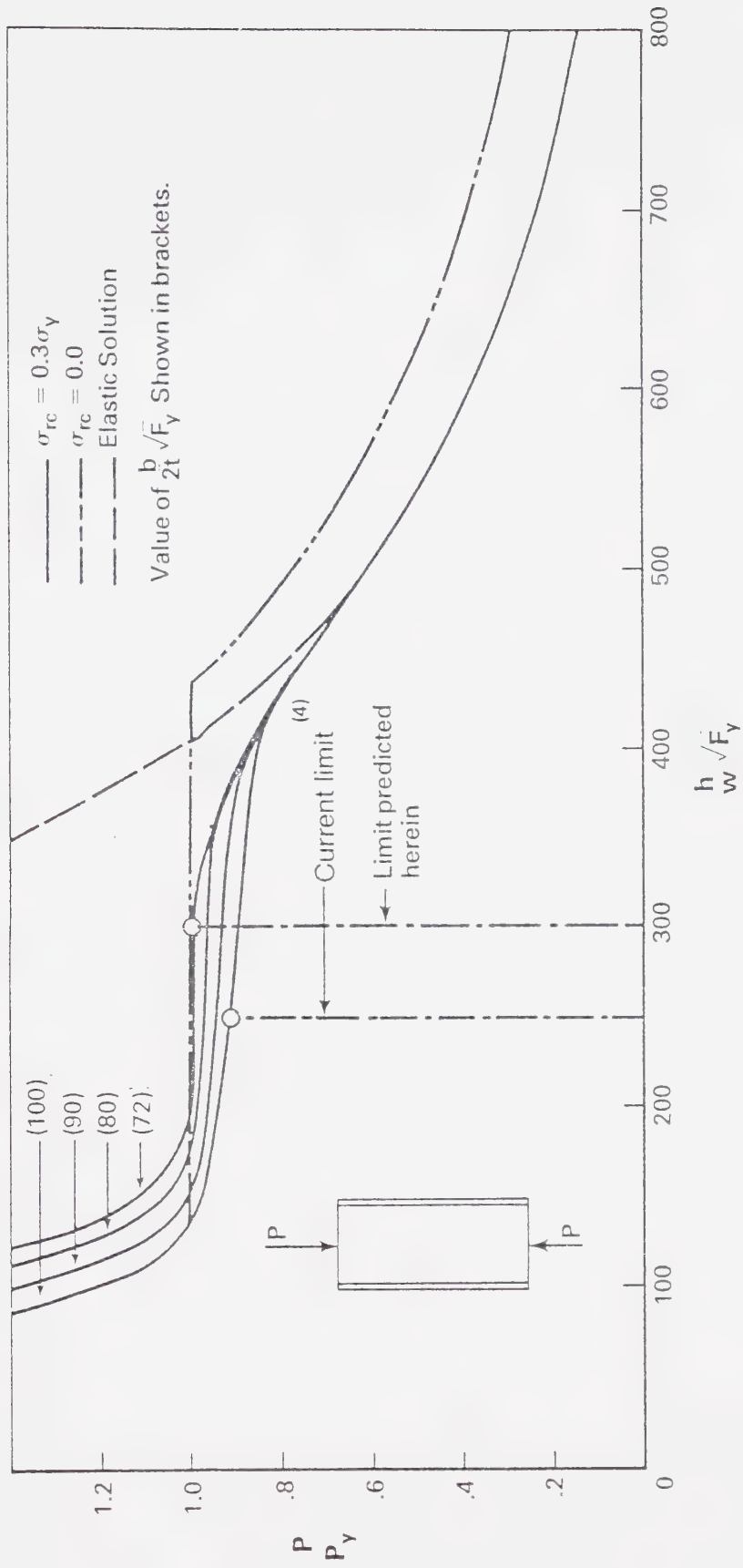


Figure 6.1 Effect of  $\frac{h}{w} \sqrt{F_y}$  on  $\frac{P}{P_y}$  for Various Values of  $\frac{b}{2t} \sqrt{F_y}$



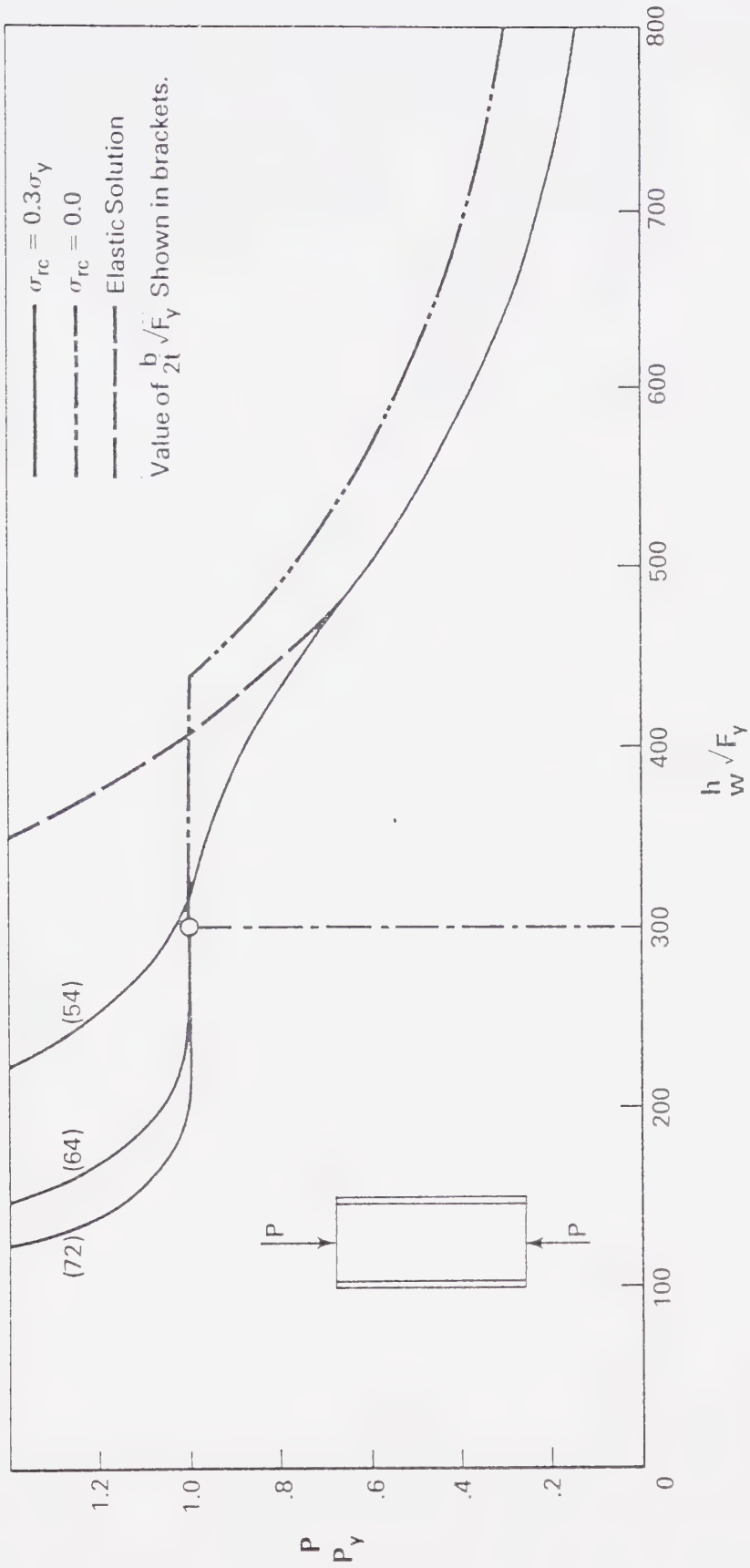


Figure 6.2 Effect of  $h/w\sqrt{F_y}$  on  $P/P_y$  for  $b/2t\sqrt{F_y}$  Values of 54, 64, and 72.



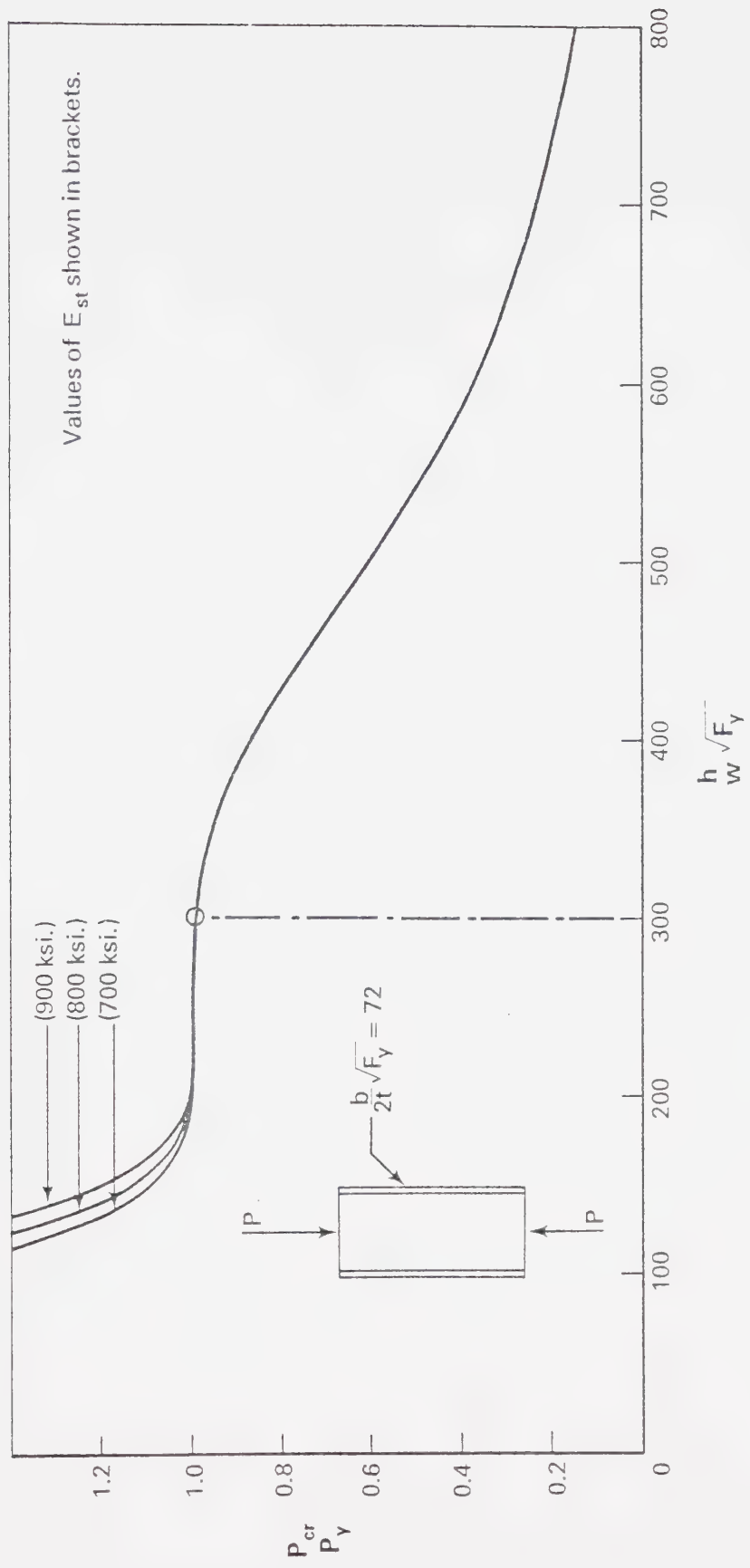


Figure 6.3  $\frac{P_{cr}}{P_y}$  vs.  $\frac{h}{w} \sqrt{F_y}$  for Values of  $E_{st} = 700, 800, \text{ and } 900 \text{ ksi.}$





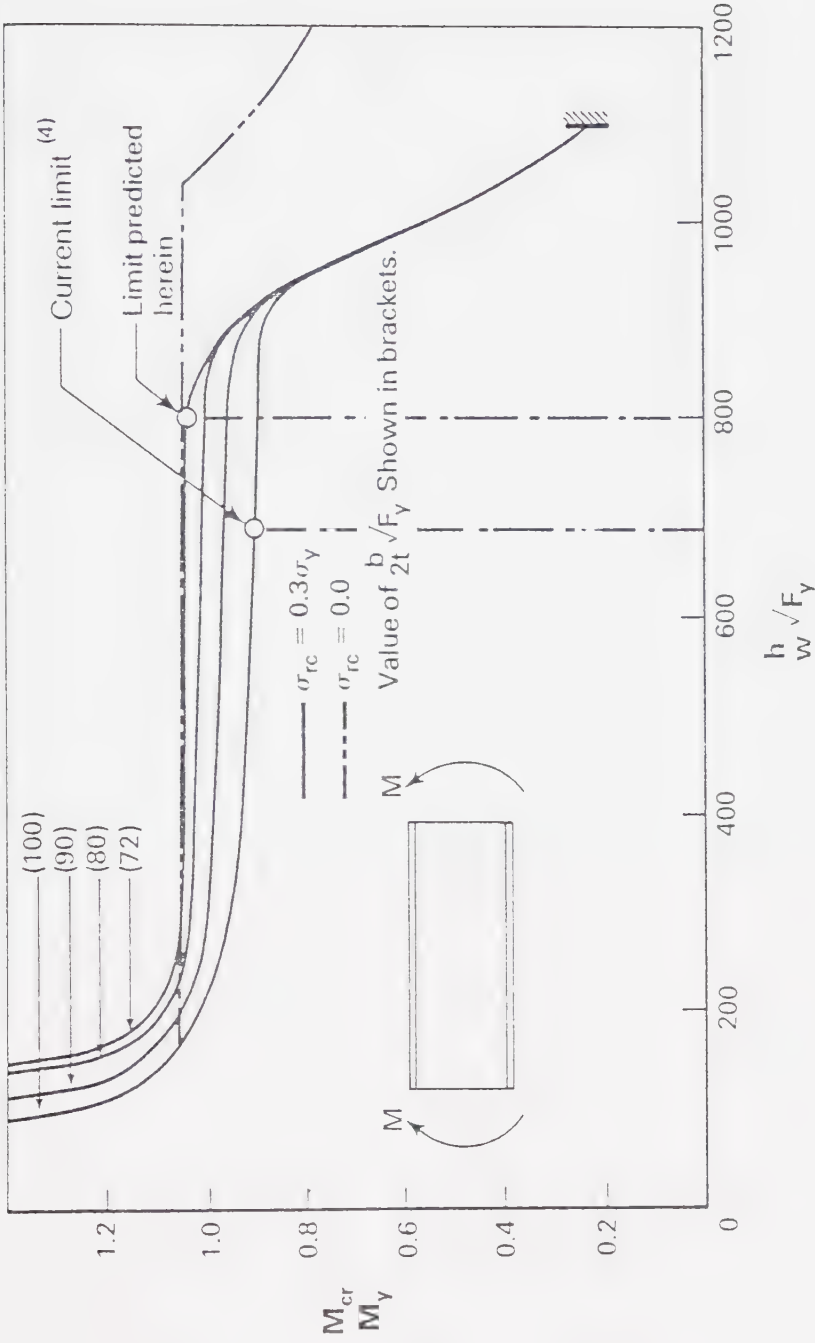


Figure 6.4 Effect of  $\frac{h}{w} \sqrt{F_y}$  on  $\frac{M_{cr}}{M_y}$  for Various Values of  $\frac{b}{2t} \sqrt{F_y}$



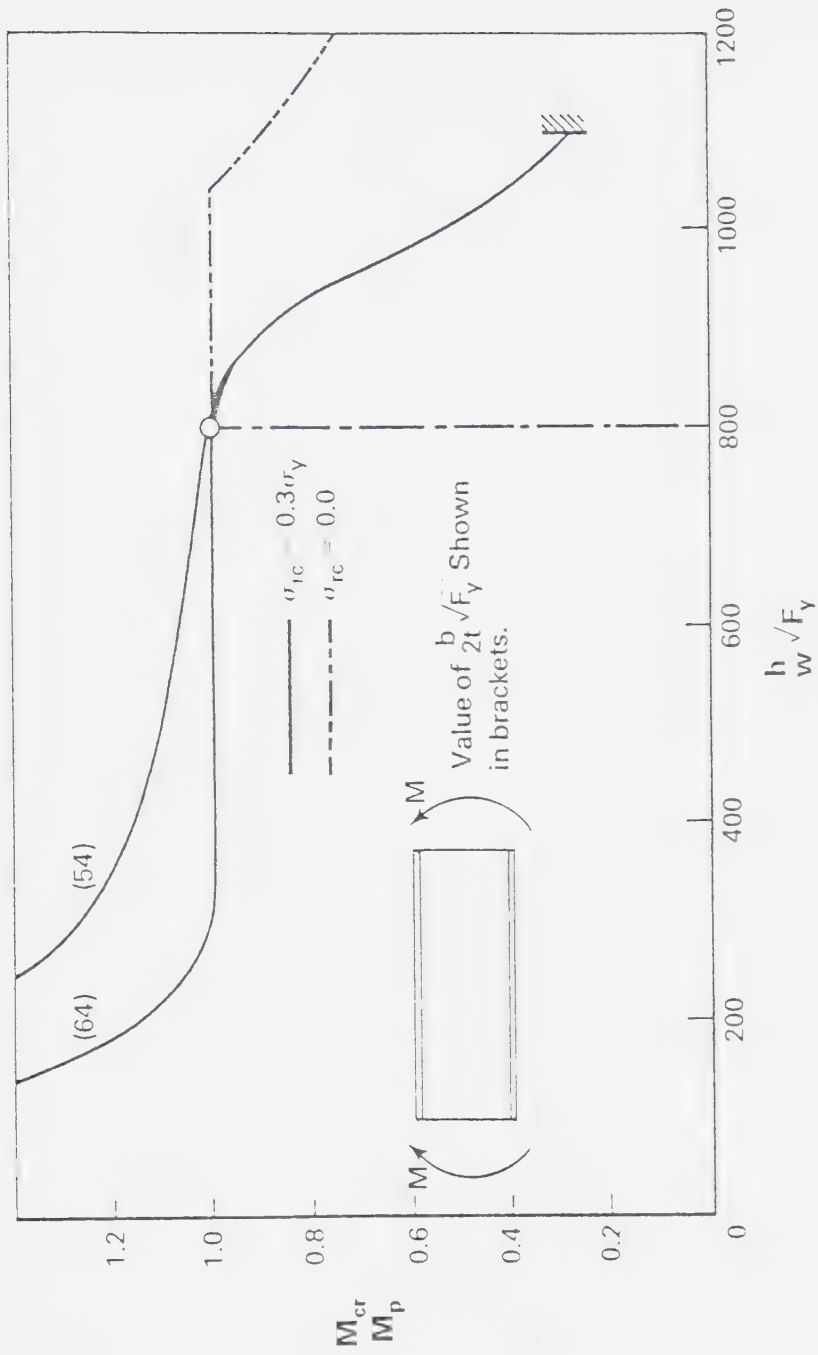


Figure 6.5 Effect of  $\frac{h}{w}\sqrt{F_y}$  on  $\frac{M_{cr}}{M_p}$  for Various Values of  $\frac{b}{2t}\sqrt{F_y}$



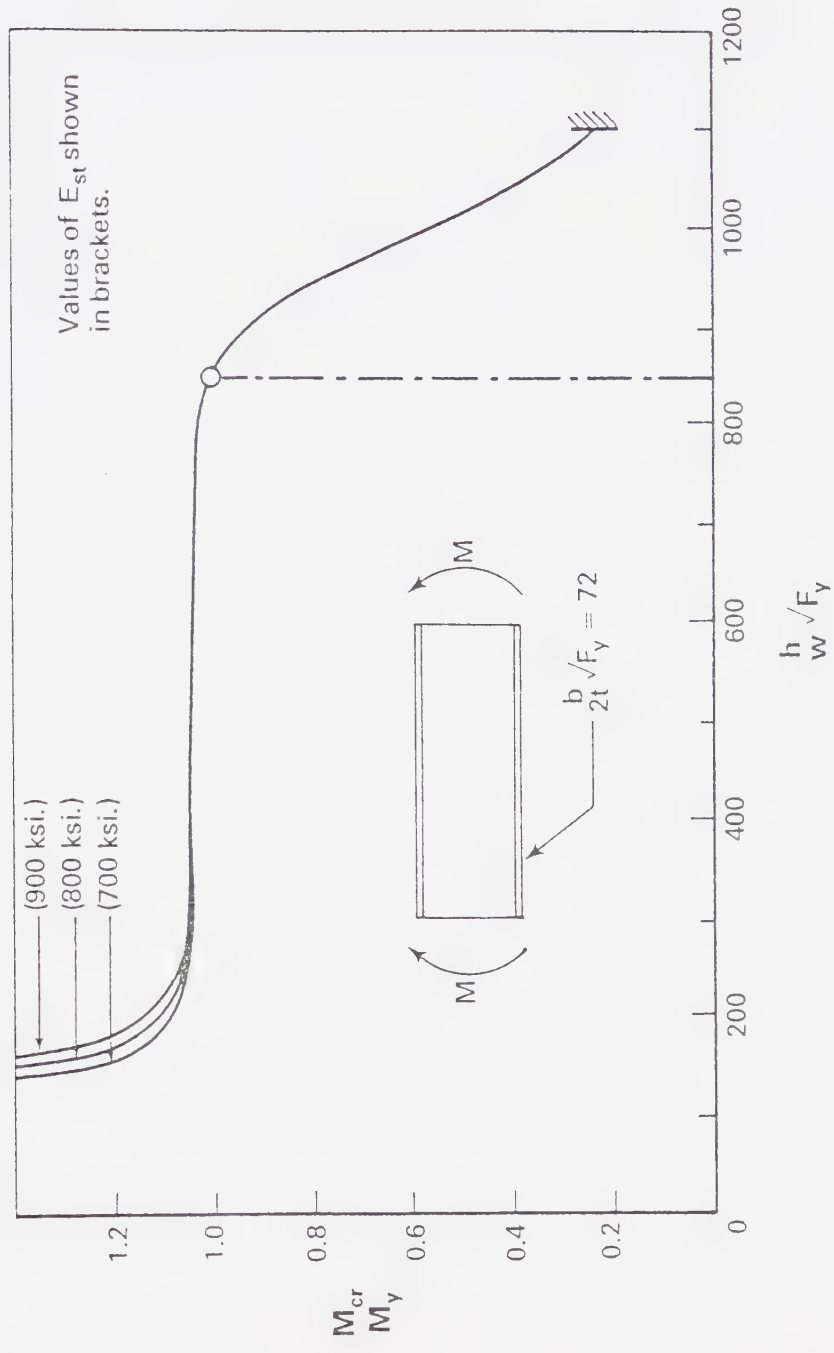


Figure 6.6  $\frac{M_{cr}}{M_y}$  vs.  $\frac{h}{w} \sqrt{F_y}$  for Values of  $E_{st} = 700, 800$ , and 900 ksi.



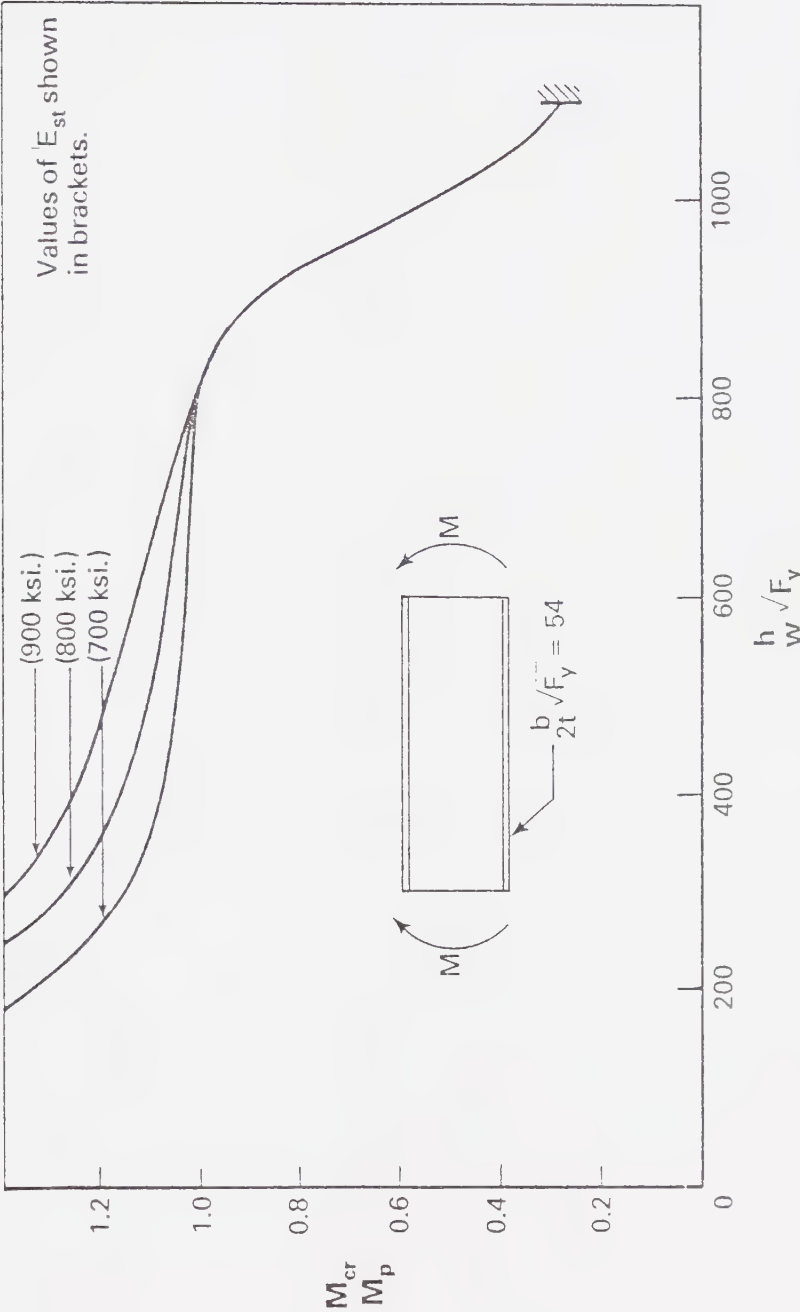


Figure 6.7  $M_{cr}$  vs.  $\frac{h}{w} \sqrt{F_y}$  for Values of  $E_{st} = 700, 800$ , and 900 ksi.





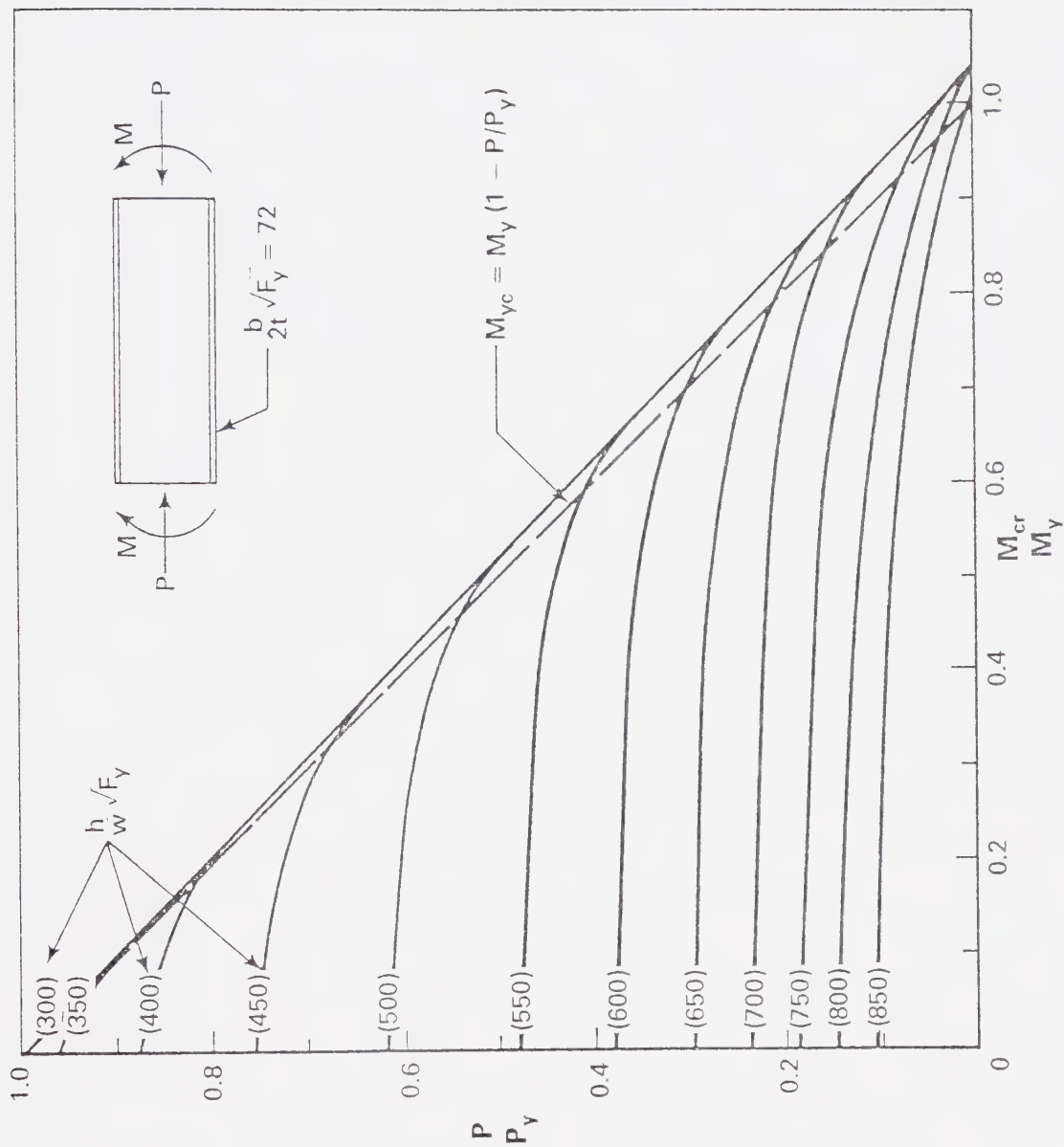


Figure 6.8 Effect of  $\frac{P}{P_y}$  on  $\frac{M_{cr}}{M_y}$  for Various Values of  $\frac{h_w}{\sqrt{F_y}}$



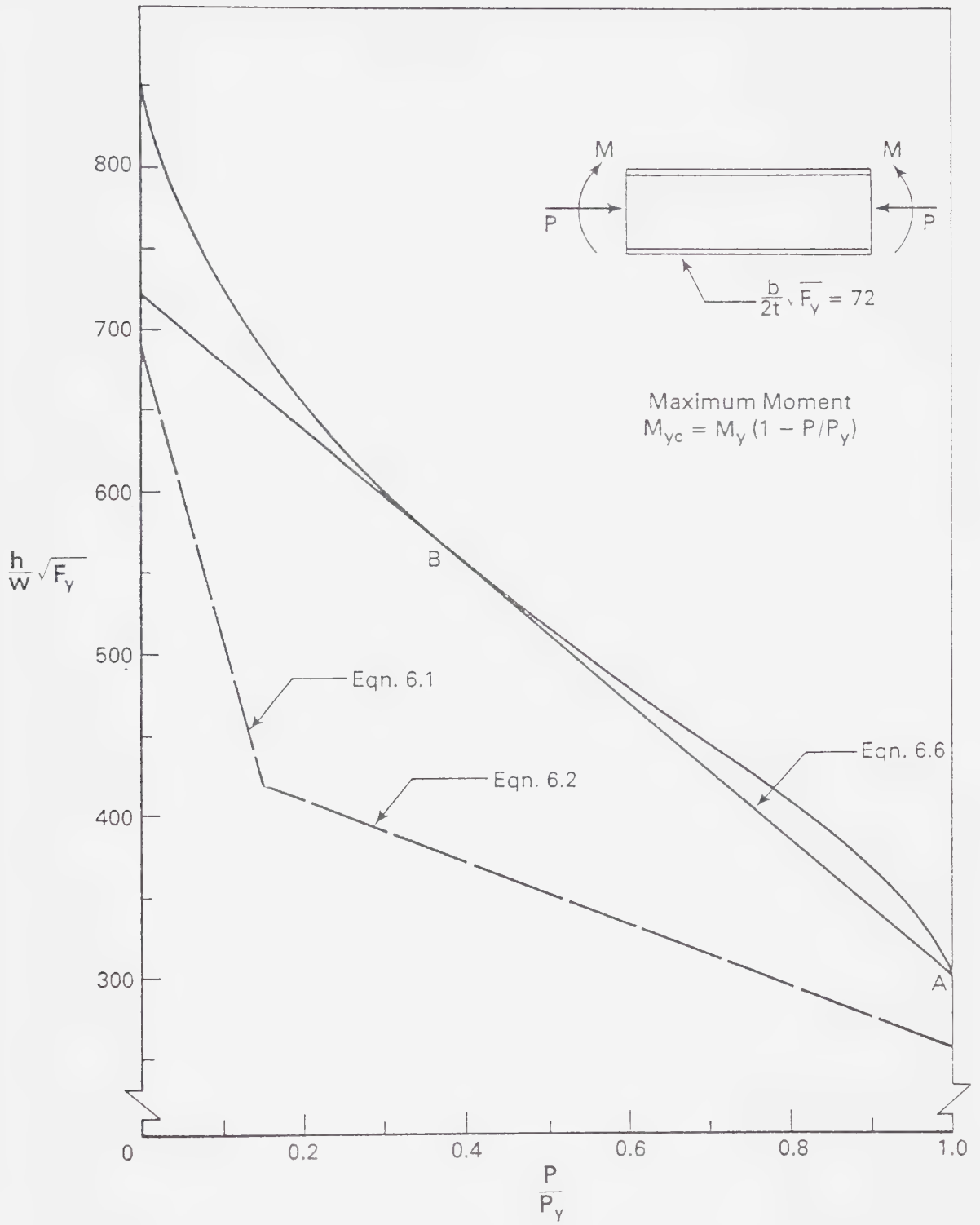


Figure 6.9  $\frac{h}{w} \sqrt{F_y}$  vs.  $\frac{P}{P_y}$  for a Class 3 Section



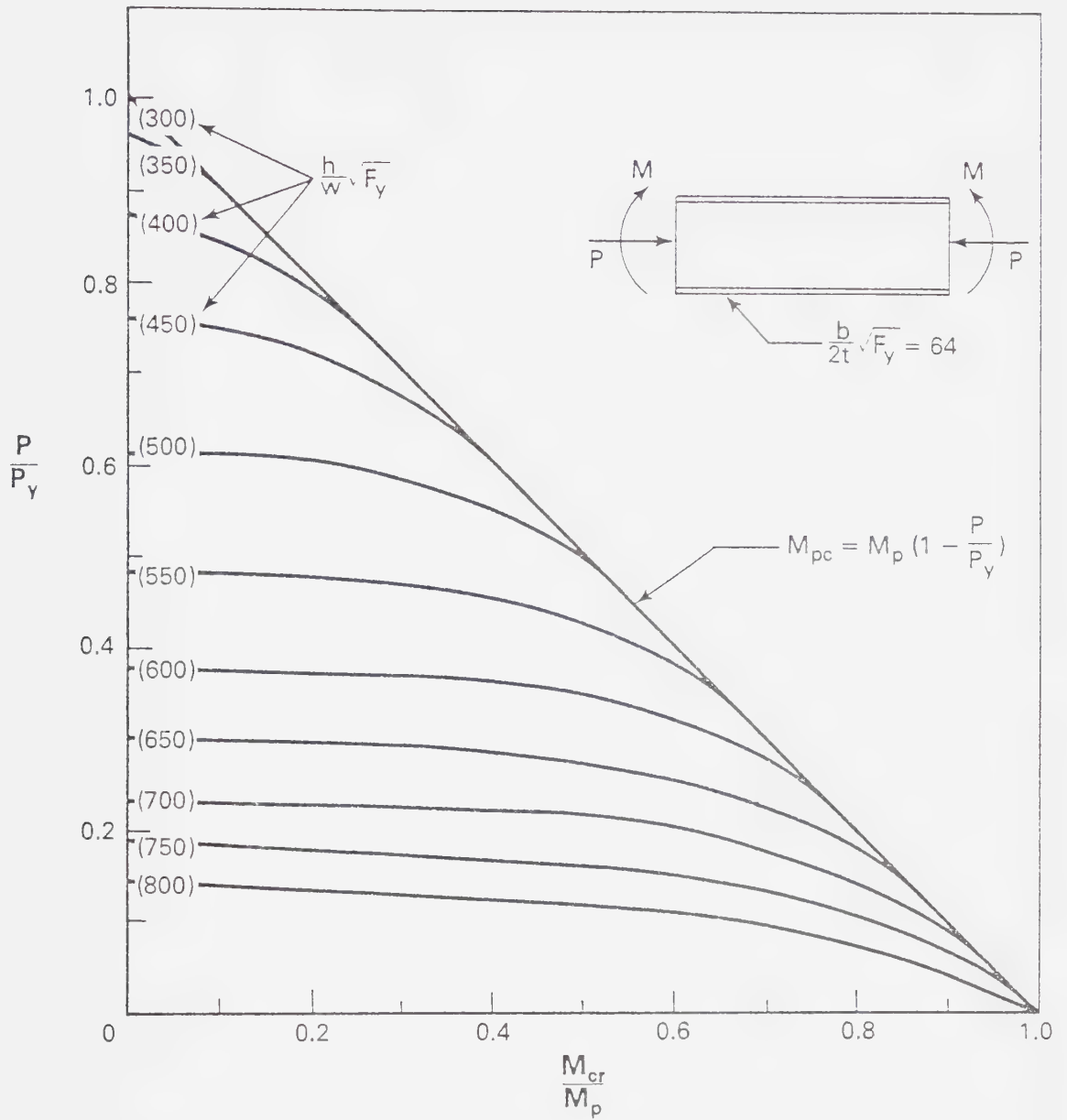


Figure 6.10 Effect of  $\frac{P}{P_Y}$  on  $\frac{M_{cr}}{M_p}$  for Various Values of  $\frac{h}{w} \sqrt{F_Y}$



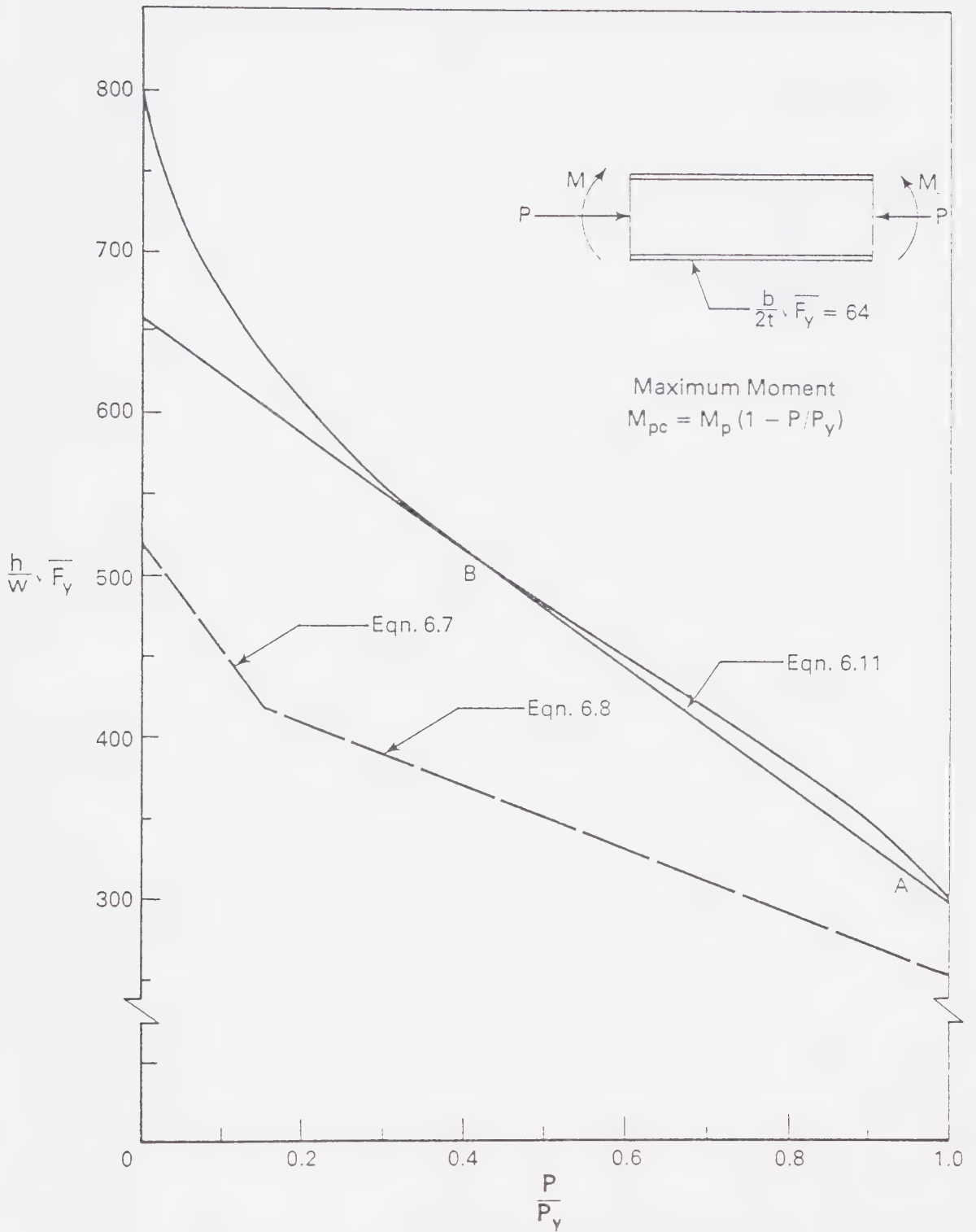


Figure 6.11  $\frac{h}{w} \sqrt{F_y}$  vs.  $P/P_y$  for a Class 2 Section





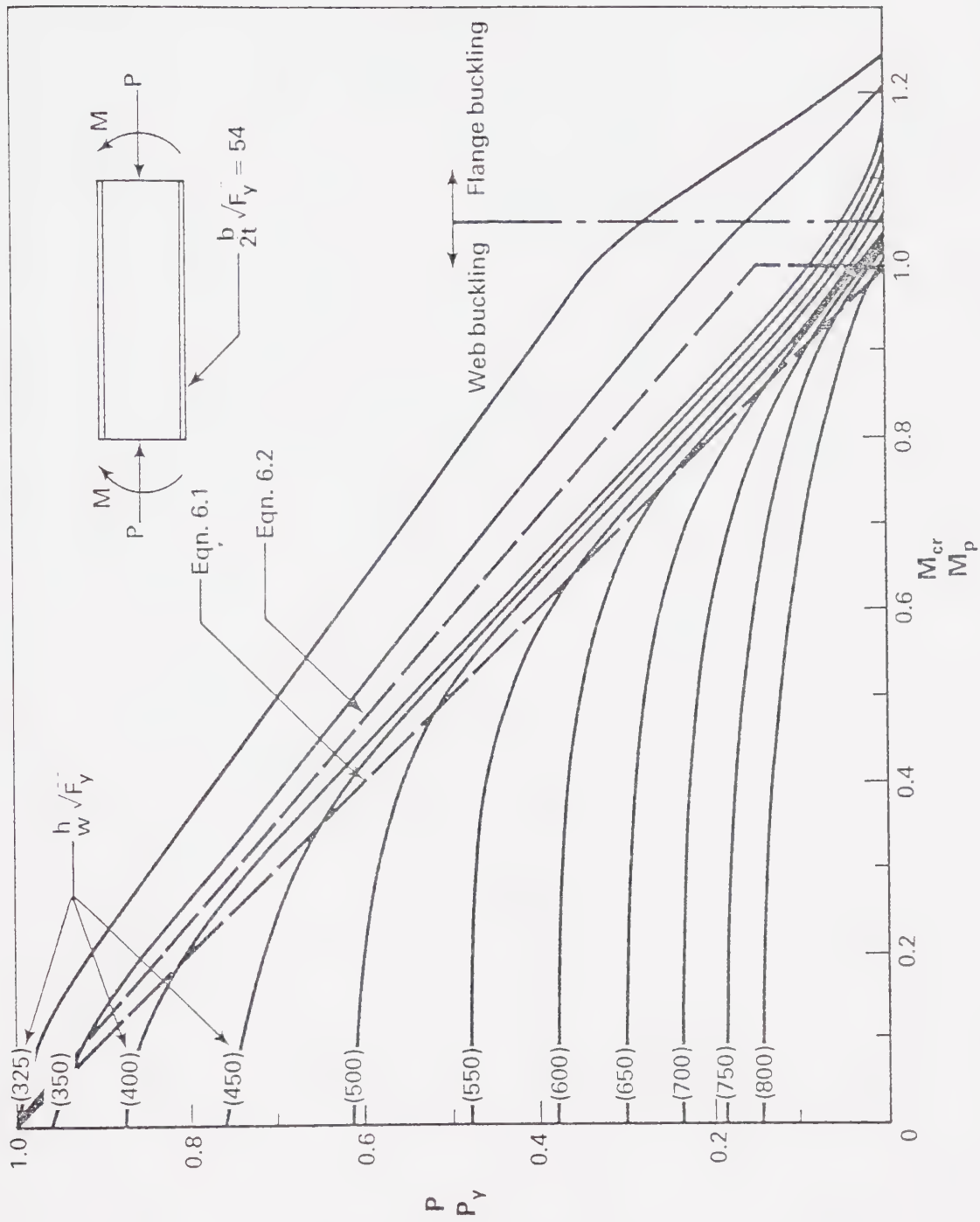


Figure 6.12 Effect of  $\frac{P}{P_y}$  on  $\frac{M_{cr}}{M_p}$  for Various Values of  $\frac{h}{w} \sqrt{F_y}$



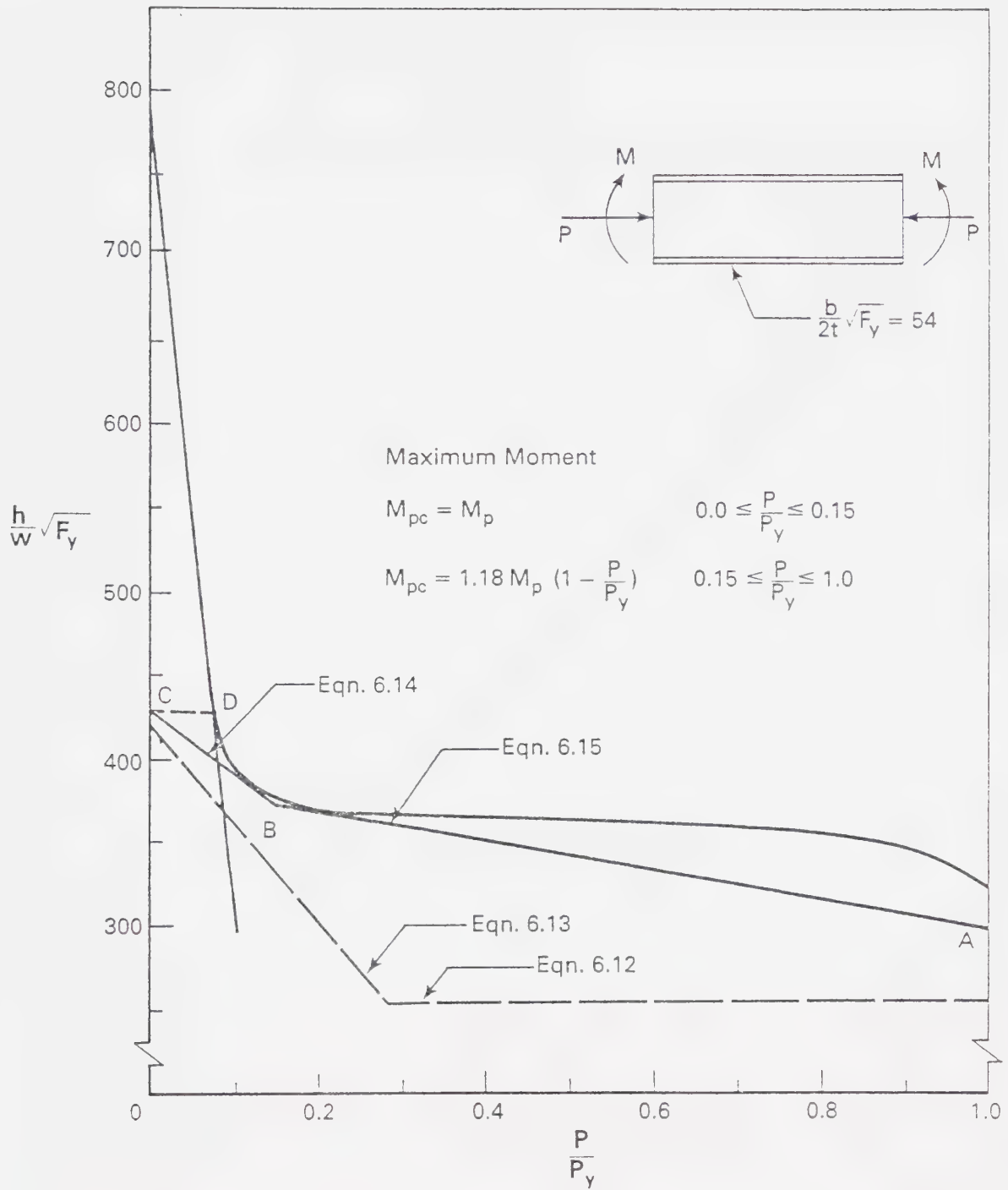


Figure 6.13  $\frac{h}{w} \sqrt{F_y}$  vs.  $\frac{P}{P_y}$  for a Class 1 Section



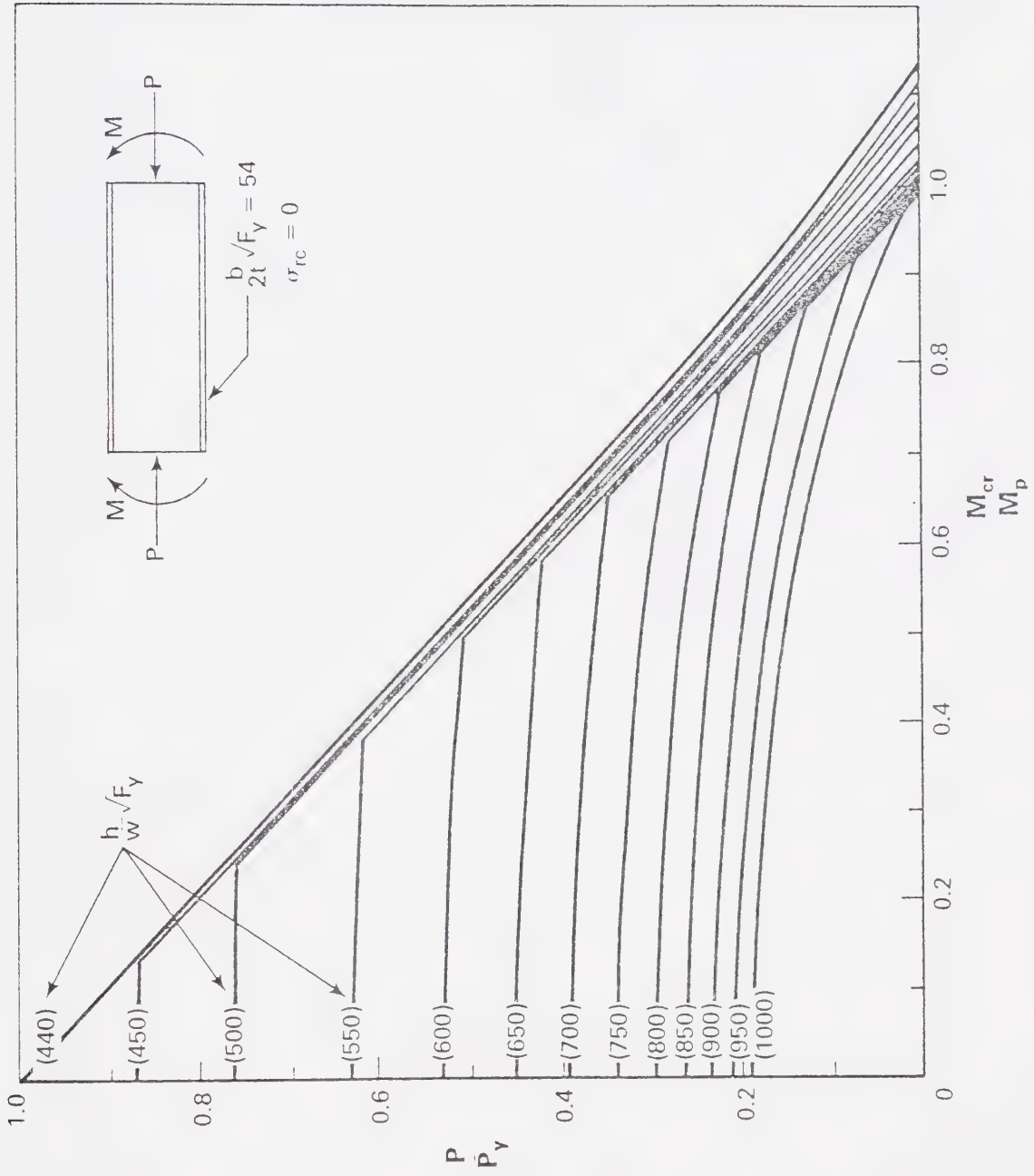


Figure 6.14  $\frac{P}{P_y}$  vs.  $\frac{M_{cr}}{M_p}$  for Various Values of  $\frac{h}{w} \sqrt{F_y}$  and  $\sigma_{rc} = 0$



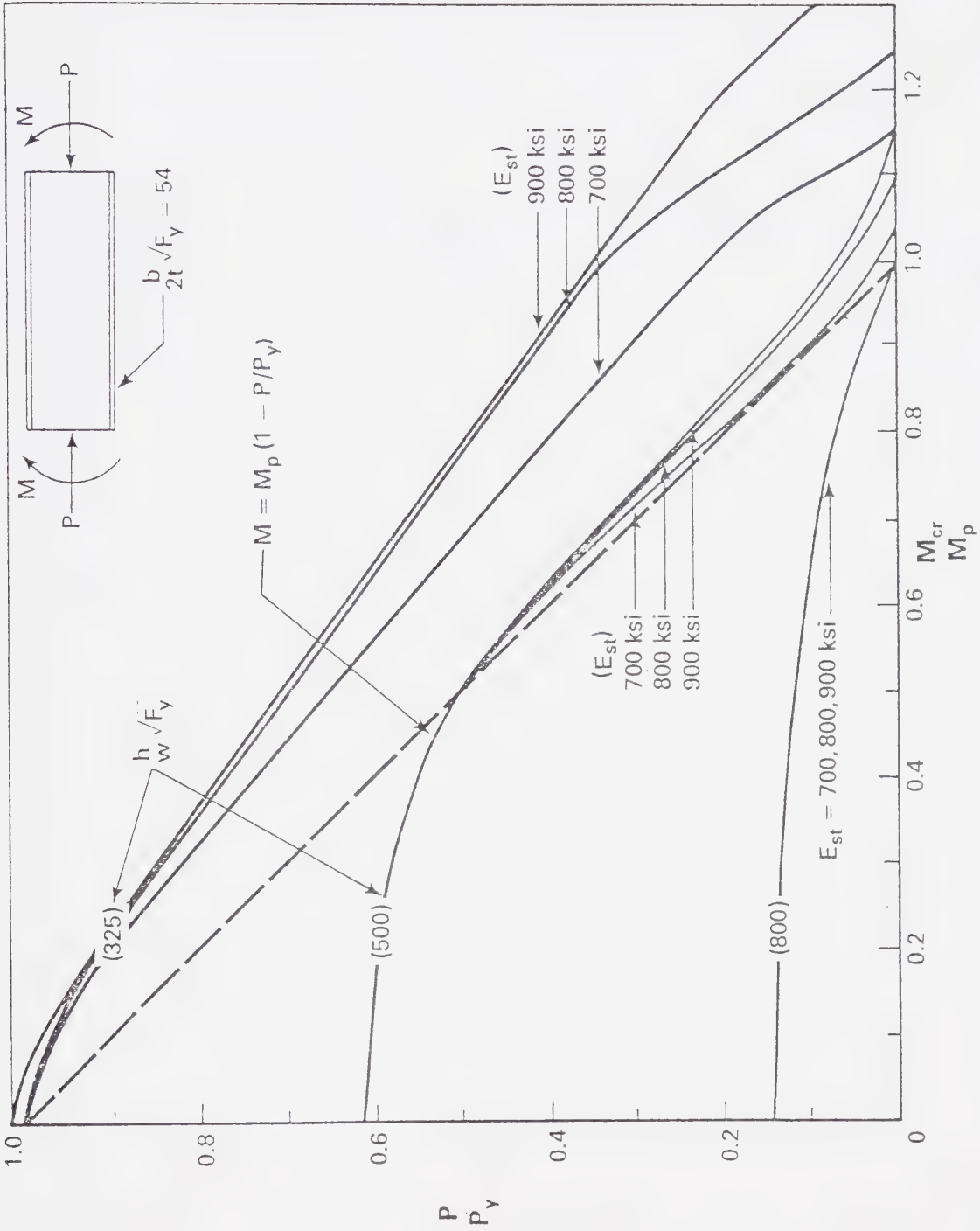


Figure 6.15 Effect of  $E_{st}$  on the Interaction of  $P/P_y$  and  $M/M_p$





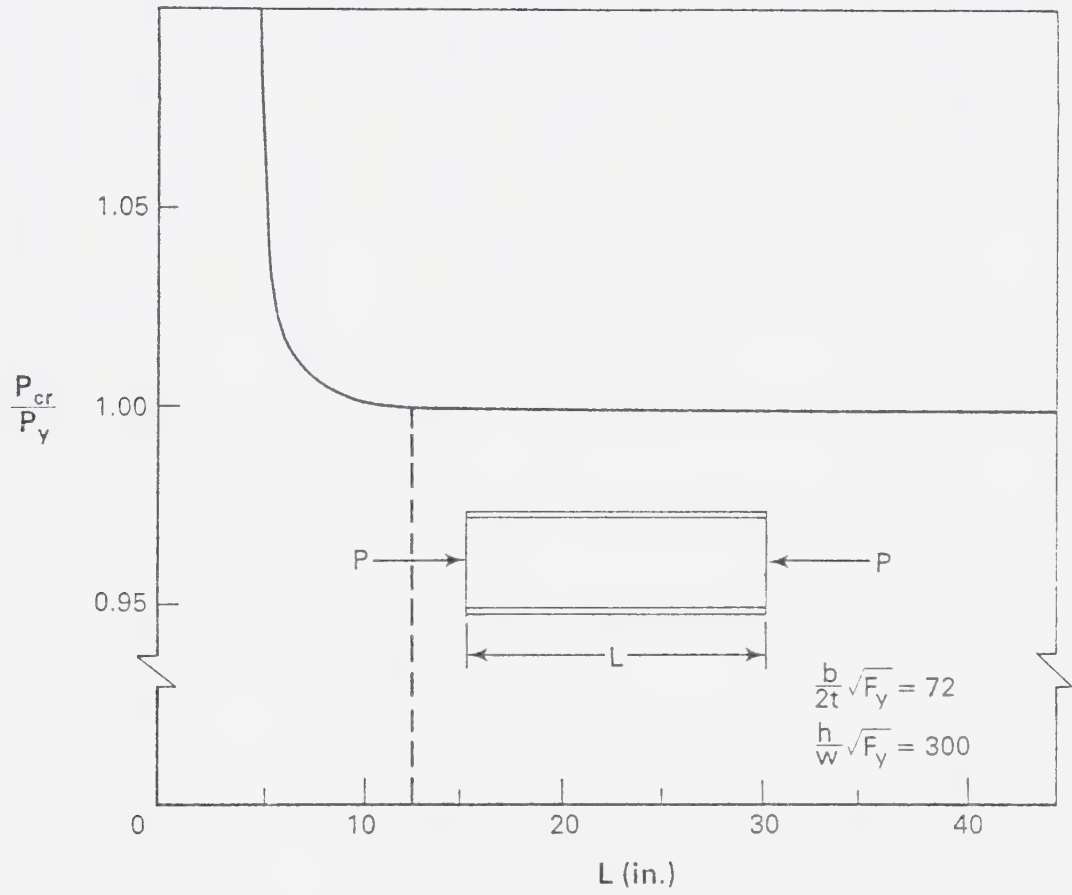


Figure 6.16 Effect of Length on Critical Load Prediction



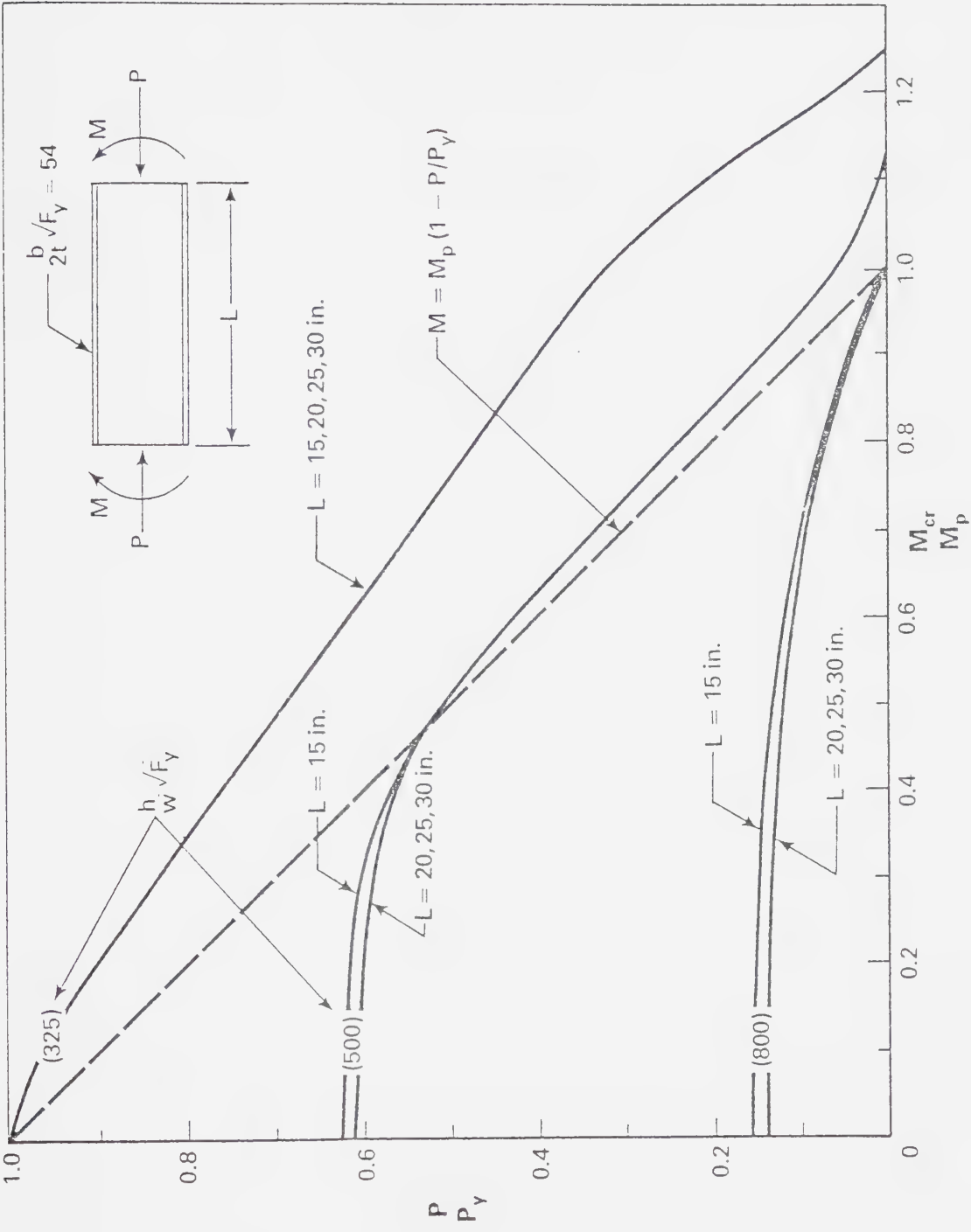


Figure 6.17 Effect of Length on the Interaction of  $\frac{P}{P_y}$  and  $\frac{M_{cr}}{M_p}$



## Chapter 7

### SUMMARY AND CONCLUSIONS

#### 7.1 Introduction

The problem of local buckling of flange- and web-plate components of W-shapes subjected to axial compression, flexural compression, and axial and flexural compression combined has been extensively investigated. A review of the available literature in this area has revealed that a few investigators have previously attempted to deal with various aspects of this problem which is complex with respect to the practical difficulties of laboratory testing as well as with respect to theoretical modelling. At the present time, among the more notable contributions in this area are those of Haaijer and Thurlimann<sup>7</sup>, Bleich<sup>6</sup>, Timoshenko<sup>2,26</sup>, and Kulak<sup>10,11,12,13</sup>. In all cases, however, the theoretical investigations have been either totally or partially empirical or semi-empirical in nature. Furthermore, only the latter investigator has attempted to include all aspects of the problem in an experimental program for Class 1, 2, and 3 sections subjected to the three types of loading mentioned above. The present investigation is an extension of this work and is primarily an attempt to formulate a sound theoretical basis for predicting both elastic and inelastic local buckling capacities of W shapes. In the following sections the scope of the theory is summarised, the findings are discussed with respect to existing design limitations and suggested modifications are indicated.



## 7.2 Summary of the Theoretical Method

The theoretical method presented herein has been formulated for the analysis of local buckling capacities of uniform members of W shape cross-sections. These members may be end-loaded in axial compression, flexural compression or combined axial and flexural compression. The flange - web restraint interaction is included directly in the formulation and the ends of a member may be rigidly supported or pinned with respect to plate buckling. The presence of residual stresses is accounted for by including their effects directly in the formulation of the local buckling geometric stiffness matrices of the plate components. Using an eigenvalue matrix iteration technique the elastic local buckling capacity is determined. If this value exceeds the proportional limit, an applied linear strain is gradually incremented above this limit and, at each level, gradual yielding of the section is evaluated and the position of the neutral axis is updated so that equilibrium conditions on the cross-section are satisfied. The member is then analysed for a critical eigenvalue strain. Essentially, when the critical eigenvalue strain is zero, the applied strain corresponds to the critical buckling strain. In this manner, the increase in strains and gradual yielding of a cross-section simulate actual conditions. Because of the flexibility of the shape functions used in the formulation, the method is capable of predicting separate flange or web buckling or a combination of both.

## 7.3 Summary of Findings

The present theoretical method has been verified by comparison of predicted results with the results of 53 test specimens of various dimensions and subjected to various load combinations.





Additionally, in the elastic range, the method gives results which agree with those predicted using widely acceptable classical methods<sup>2,6</sup>. A large number of hypothetical beams, columns, and beam-columns for each class of section has also been investigated. As a result of this study the analytical method presented herein indicates that the present code limitation for flanges of Class 3 sections is inadequate. This limitation is based on a purely theoretical torsional analysis of the flange<sup>1</sup>. For Class 1 and 2 sections, on the other hand, the analytical method presented herein substantiates the flange slenderness limitations of 54 and 64 which have been well established by experimental investigations. It is clear therefore, that the same theory which substantiates flange slenderness values of 54 and 64 based on experimental results for Class 1 and 2 sections, casts doubt upon the flange slenderness value of 100 based on an approximate torsional analysis of a Class 3 section flange. Assuming the above flange modification as indicated, presently specified web plate width-to-thickness limitations may be increased for columns, beams, and beam-columns.

#### 7.4 Recommendations for Design

As a result of the theoretical investigation presented herein certain modifications of the present code values<sup>4</sup> of plate width-to-thickness limitations are indicated. These modifications apply to W shapes subjected to axial, flexural, and combined axial and flexural loadings that are uniform along the member lengths. Additionally, certain flange width-to-thickness limitations presently set by the code have been substantiated. These results, as summarised in Figure 7.1, are discussed in the following sections.



#### 7.4.1 Class 1 Sections

The present theory substantiates the use of  $b\sqrt{F_y}/2t = 54$  for the flanges of Class 1 sections. It is indicated that the present limitation of  $h\sqrt{F_y}/w = 255$  for pure axial loading can be safely increased to 300. For pure flexural loading an increase of  $h\sqrt{F_y}/w$  from the present code value of 420 to a value of 430 is indicated. In the intermediate range, where combined axial and flexural loadings occur, the following increases in  $h\sqrt{F_y}/w$  are indicated:

$$h\sqrt{F_y}/w = 430 (1 - 0.93 (P/P_y)) \quad 0 \leq P/P_y \leq 0.15 \quad (6.14)$$

$$h\sqrt{F_y}/w = 382 (1 - 0.22 (P/P_y)) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.15)$$

For these values the present theoretical method indicates that the following maximum moments reduced for axial load, can be sustained for adequate plastic design behaviour:

$$M_{pc} = M_p \quad 0 \leq P/P_y \leq 0.15 \quad (6.9(a))$$

$$M_{pc} = 1.18 M_p (1 - P/P_y) \quad 0.15 \leq P/P_y \leq 1.0 \quad (6.9(b))$$

#### 7.4.2 Class 2 Sections

According to the present theoretical method, the value of  $b\sqrt{F_y}/2t = 64$ , as presently set by the code for Class 2 sections, is adequate with respect to local buckling provided that the maximum reduced moment value is set at:



$$M_{pc} = M_p (1 - P/P_y) \quad (6.10)$$

For pure axial loading it is indicated that the present web limitation of  $h\sqrt{F_y}/w = 255$  can be increased to 300. For pure flexural loading the present value of  $h\sqrt{F_y}/w = 520$  can be increased to 660. In the intermediate range of combined axial and flexural loading the following increases are indicated:

$$h\sqrt{F_y}/w = 660 (1 - 0.55 (P/P_y)) \quad 0 \leq P/P_y \leq 1.0 \quad (6.11)$$

#### 7.4.3 Class 3 Sections

The present theoretical method indicates that the value of  $b\sqrt{F_y}/2t = 100$  for flanges of Class 3 sections is too high. A reduction to a value of 72 is indicated. In the presence of pure axial loads the present value of  $h\sqrt{F_y}/w = 255$  can be increased to 300 and, for beams, the present code value of  $h\sqrt{F_y}/w = 690$  can be increased to 725. In the intermediate range where combined axial and flexural loadings occur the following increases are indicated:

$$h\sqrt{F_y}/w = 725 (1 - 0.59 (P/P_y)) \quad 0 \leq P/P_y \leq 1.0 \quad (6.6)$$

For these values a maximum moment value of

$$M_{yc} = M_y (1 - P/P_y) \quad (6.5)$$

can be reached in the presence of axial load. The above indicated modifications for Class 1, 2, and 3 sections are summarised in



Figure 7.1.

## 7.5 Further Recommendations

As a result of the tests performed by Kulak et al<sup>10,11,12,13</sup>, increases in web limits for Class 2 and 3 sections have recently been implemented by the present code<sup>4</sup>. The theory presented herein indicates that these increases are also justified on a purely theoretical basis. Furthermore, according to this theoretical method, additional increases in web slendernesses are indicated with the reservations that Class 3 flange limits be set at  $b\sqrt{F_y}/2t = 72$  and that  $M_{pc}$  for Class 2 beam-columns be limited to the value given by Equation 6.10. Increases in web limits for Class 1 sections are also indicated by the theoretical results presented herein.

Before these additional increases are implemented, however, it is suggested that further testing of laboratory specimens be carried out. These tests should be performed in the light of certain implications arising from the present theoretical analysis. The items of particular relevance to test specimens and testing procedure are listed below:

### 1. End Conditions

As nearly as possible, the end supports of test specimens should approximate simply-supported plate edges with respect to local buckling. For axially loaded members this is not a difficult problem. However, for members that must be subjected to axial and flexural loads combined, it would be necessary to use end-moment connections requiring very rigid support of the plate edges. A solution to this problem





would be to use members sufficiently long so that the effects of rigid edge restraints are reduced. An aspect ratio of at least 4 for webs and flanges is suggested.

## 2. Lateral Support

To ensure a local buckling mode of failure, adequate lateral support must be provided in order to preclude overall lateral-torsional instability. Lateral supports should be designed so as not to interfere with local buckling. It is suggested that knife-edge lateral supports be placed at web-to-flange junctions if possible.

## 3. Residual Stresses

If local buckling is expected to occur in the elastic or partially yielded regions, an exact determination of residual stress magnitude and distribution is especially important. In the present correlation of theoretical and test results this information was not available and therefore, typical values had to be assumed. As a result, partially due to this lack of information, discrepancies between theoretical predictions and test results are evident. Future experimental investigations of local buckling should incorporate the exact determination of residual stresses as part of the test program.

## 4. Material Properties

At the present time there are conflicting opinions<sup>5,22,24,27,33</sup> as to what values of material properties are applicable in the range of strain between the yield strain and the strain at the onset of strain-hardening. Further investigations in this area



would be desirable. Values of the strain at the onset of strain-hardening as well as values of the strain-hardening modulus were not available for several of the test specimens referred to in the present investigation. Therefore, typical values had to be assumed and this partially contributed to discrepancies between theoretical predictions and test results. In future testing, it is recommended that attempts be made to determine strain-hardening strains and moduli during standard tension coupon tests.

## 7.6 Conclusions

A sound theoretical analysis similar to a finite strip technique has been developed and verified for the purpose of determining critical local buckling loads for W shape structural members. These members may be end-loaded by axial loads, flexural loads, or axial and flexural loads combined. The method predicts local buckling of flanges and webs in the elastic and inelastic ranges and the interaction of the flanges and web is accounted for in the formulation of the problem. Also, the effects of residual stresses are included directly into the theoretical formulation. The calculations were performed by computer and the theoretical results were verified by comparison with available test results as well as with the predictions of classical analysis for elastic plate buckling.

A wide range of Class 1, 2, and 3 sections of varying dimensions were analysed for axial loadings, flexural loadings, and axial and flexural loadings combined. As a result, interaction diagrams were generated for each class of section and these diagrams were used to determine maximum web limitations for a range of axial loads varying



from zero to the yield load. It was generally indicated that present web limitations are too restrictive and appropriate increases have been suggested. Additionally, it has been suggested that more testing of laboratory specimens be carried out and appropriate recommendations have been made regarding the design of such tests.



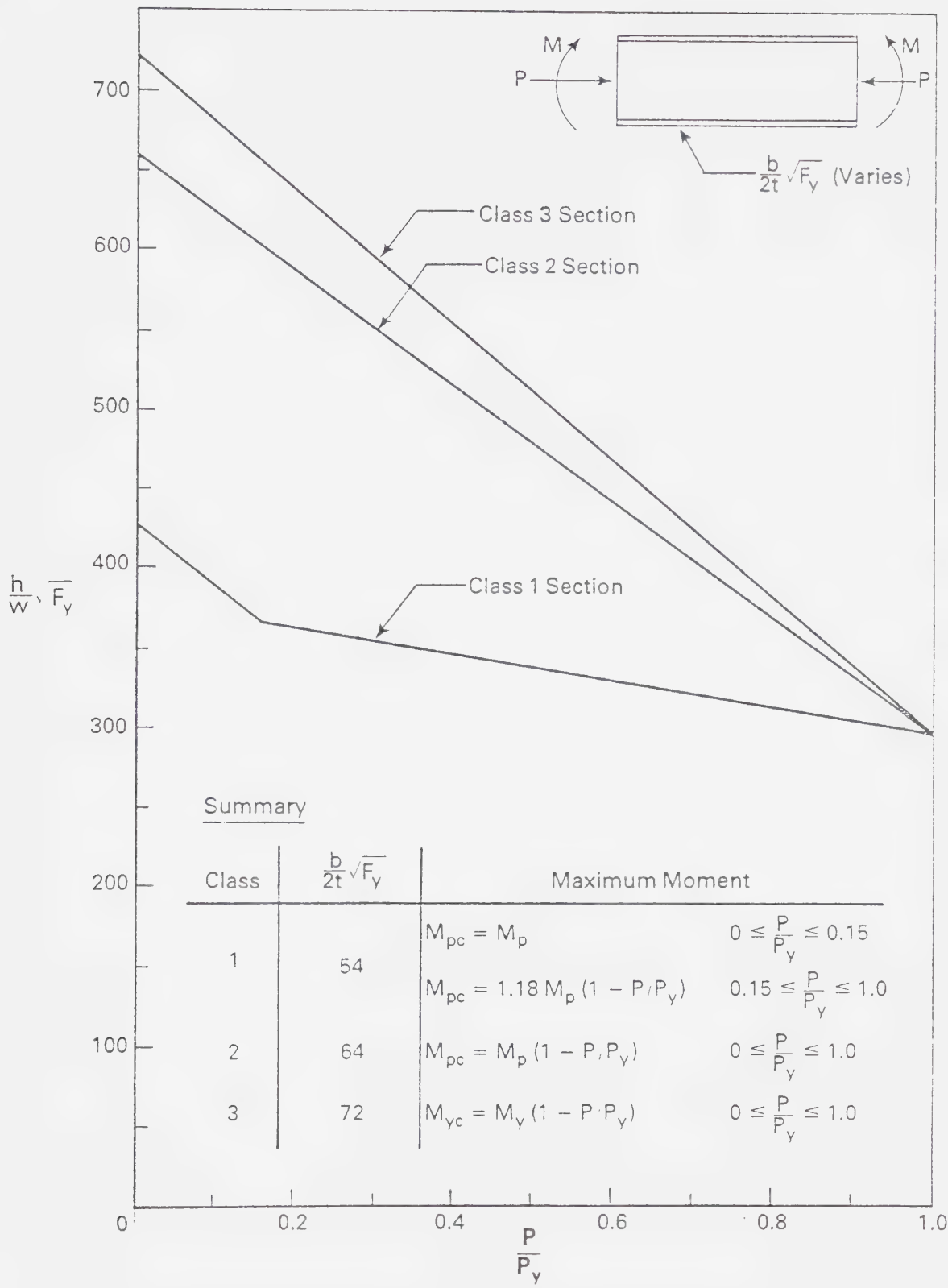


Figure 7.1 Summary of Indicated Modifications





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## APPENDIX A

### DERIVATION OF A PLATE BUCKLING CONDITION

#### A-1 Introduction

In this section a plate buckling condition is established using the principle of virtual work<sup>2,8,55</sup>. For a body in equilibrium, the principle states that the work done by an internal equilibrium stress field,  $\sigma_{ij}$ , is equal to the work done by the surface tractions,  $T_i$ , when the body is subjected to a virtual displacement field,  $\delta u_i$ . The principle may be expressed as follows:

$$\int_V \sigma_{ij} \delta e_{ij} dV = \int_S T_i \delta u_i dS \quad (A-1)$$

where  $\delta e_{ij}$  is the virtual strain field resulting from a virtual displacement field,  $\delta u_i$ ,  $V$  refers to the volume, and  $S$  refers to the surface area over which surface tractions,  $T_i$ , are specified. In this expression, the effects of body forces,  $F_i$ , have been neglected.

#### A-2 Assumptions

In the following derivation it is assumed that:

1. second order strain terms resulting from in-plane displacement components due to buckling are small and may be neglected<sup>8</sup>.
2. at the point of bifurcation of a plate, an in-plane or a buckled equilibrium configuration is possible while



- the system of external forces remains constant,
3. a state of plane stress<sup>46</sup> exists within the plate,
  4. the plate thickness is small relative to the surface dimensions,
  5. deflections are small relative to the plate thickness,
  6. straight lines perpendicular to the middle surface of an undeformed plate remain straight and perpendicular to the middle surface after buckling,
  7. longitudinal strips of a plate may be elastic or inelastic,
  8. stresses are constant or vary linearly across the thickness of a plate,
  9. stretching of the middle plane of the plate during buckling is small and may be neglected.

### A-3 Strain - Displacement Relationships

The strain - displacement relationships in tensor form may be written as follows<sup>8</sup>:

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_{k,i} u_{k,j}) \quad (A-2)$$

where  $i, j$ , and  $k$  cyclically represent the subscripts  $x, y$ , and  $z$  corresponding to Cartesian coordinates. In this and subsequent expressions, the comma-notation is used to represent differentiation,





are the in-plane components of strain<sup>2</sup>. Here also,  $e_{xx}$ ,  $e_{yy}$ , and  $e_{xy}$  represent normal and shear components, respectively,

In the present case of plane stress, Equation A-1 may be equated to the following components of strain:

$$e_{xx} = \epsilon_{xx} = \frac{1}{L} \left( u_{,x}^2 + v_{,x}^2 + w_{,x}^2 \right) \quad (A-3)$$

$$e_{yy} = \epsilon_{yy} = \frac{1}{L} \left( u_{,y}^2 + v_{,y}^2 + w_{,y}^2 \right) \quad (A-4)$$

$$e_{xy} = \frac{1}{L} \left( u_{,y} + v_{,x} + w_{,x} v_{,y} + w_{,y} v_{,x} \right) \quad (A-5)$$

where  $u$ ,  $v$ , and  $w$  represent displacement components in the  $x$ -,  $y$ -, and  $z$ -coordinate directions, and subscripts  $x$ ,  $y$ , and  $z$  indicate differentiation with respect to that particular variable.

According to Equation (A-1), strain terms depending on the squares and products of  $u_{,xx}$ ,  $u_{,xy}$ ,  $u_{,yy}$ , and  $w_{,y}$  may be neglected. Second order terms such as  $w_{,xx}$  and  $w_{,xy}$  result from a double configuration and therefore they are retained. Thus, Equations A-3 to A-5 may be rewritten as follows:

$$e_{xx} = \epsilon_{xx} = \frac{1}{L} \left( u_{,x}^2 + v_{,x}^2 \right) \quad (A-6)$$

$$e_{yy} = \epsilon_{yy} = \frac{1}{L} \left( v_{,y}^2 \right) \quad (A-7)$$

$$e_{xy} = \frac{1}{L} \left( u_{,y} + v_{,x} + v_{,x} w_{,y} + w_{,y} v_{,x} \right) \quad (A-8)$$



#### A-4 Displacement Components

Figure A-1 shows a plate oriented in a 3-dimensional Cartesian coordinate system. The displacement components of a point in the middle plane of the plate during buckling are given as:

$$u = u' \quad (A-9)$$

$$v = v' \quad (A-10)$$

$$w = w \quad (A-11)$$

Figure A-2 shows a portion of a plate before and after buckling in the  $x - y$  plane. During buckling, a point  $P$  moves tangentially to  $P'$  by a distance  $u'$ . Due to out-of-plane deflection,  $P'$  moves to  $P''$  by a distance equal to  $w$ . As a result of rotation of a cross-section of the plate,  $P''$  moves to  $P'''$ . Making use of Assumption No. 5 concerning small deflections, and recognizing similar observations for buckling in the  $y - z$  plane, the following displacement components are obtained:

$$u = u' - zw_{,x} \quad (A-12)$$

$$v = v' - zw_{,y} \quad (A-13)$$

$$w = w \quad (A-14)$$

Using these results in Equations A-6 to A-8, the following strain-displacement relationships are obtained:

$$e_x = u'_{,x} - zw_{,xx} + \frac{1}{2} w_{,x}^2 \quad (A-15)$$



$$e_y = v'_{,y} - zw_{,yy} + \frac{1}{2} w_{,y}^2 \quad (\text{A-16})$$

$$e_{xy} = \frac{1}{2}(u'_{,y} + v'_{,x}) - zw_{,xy} + \frac{1}{2} w_{,x} w_{,y} \quad (\text{A-17})$$

#### A-5 Application of the Principle of Virtual Work

Using Equations A-15 to A-17, the left hand side of Equation A-1, for the case of plane stress, (Assumption No. 3), may be written as follows:

$$\begin{aligned} \int_V \sigma_{ij} \delta e_{ij} dV = & \int_x \int_y \left[ N_x \delta u'_{,x} + N_{xy} (\delta u'_{,y} + \delta v'_{,x}) + N_y \delta v'_{,y} \right] dx dy \\ & + \int_x \int_y \left[ N_x w_{,x} \delta w_{,x} + N_{xy} (w_{,x} \delta w_{,y} + w_{,y} \delta w_{,x}) \right. \\ & \left. + N_y w_{,y} \delta w_{,y} \right] dx dy + \int_x \int_y \left[ M_x \delta w_{,xx} \right. \\ & \left. + 2M_{xy} \delta w_{,xy} + M_y \delta w_{,yy} \right] dx dy \end{aligned} \quad (\text{A-18})$$

In developing this expression, the following relationships were used:

$$\sigma_{ij} \delta e_{ij} = \sigma_x \delta e_x + 2\sigma_{xy} \delta e_{xy} + \sigma_y \delta e_y \quad (\text{A-19})$$



$$\delta e_x = \delta u',_x - z\delta w,_{xx} + w,_{xx}\delta w,_{xx} \quad (A-20)$$

$$\delta e_y = \delta v',_y - z\delta w,_{yy} + w,_{yy}\delta w,_{yy} \quad (A-21)$$

$$\begin{aligned} \delta e_{xy} = & \frac{1}{2} (\delta u',_y + \delta v',_x) - z\delta w,_{xy} \\ & + \frac{1}{2} (w,_{xx}\delta w,_{yy} + w,_{yy}\delta w,_{xx}) \end{aligned} \quad (A-22)$$

from Equation A-15 to A-17, and,

$$N_x = \int_{-t/2}^{t/2} \sigma_x dy \quad (A-23a)$$

$$N_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} dz \quad (A-23b)$$

$$N_y = \int_{-t/2}^{t/2} \sigma_y dz \quad (A-23c)$$

$$M_x = - \int_{-t/2}^{t/2} \sigma_x z dz \quad (A-24a)$$

$$M_{xy} = - \int_{-t/2}^{t/2} \sigma_{xy} z dz \quad (A-24b)$$

$$M_y = - \int_{-t/2}^{t/2} \sigma_y z dz \quad (A-24c)$$

where  $N_x$ ,  $N_{xy}$ , and,  $N_y$  are forces and  $M_x$ ,  $M_{xy}$ , and  $M_y$  are moments per unit length of a plate having a thickness,  $t$ .

The first integral on the right hand side of expression A-18, may be integrated by parts as follows:





$$\begin{aligned}
& \int_y \left[ N_x \delta u' \Big|_0^a + N_{xy} \delta v' \Big|_0^a \right] dy + \int_x \left[ N_y \delta v' \Big|_0^b + N_{xy} \delta u' \Big|_0^b \right] dx \\
& - \int_x \int_y \delta u' (N_{x,x} + N_{xy,y}) dx dy - \int_x \int_y \delta v' (N_{y,y} + N_{xy,x}) dx dy
\end{aligned} \tag{A-25}$$

where a and b refer to the length and width of a rectangular plate as shown in Figure A-1. The first two integrals of this expression represent the virtual work done by the applied forces per unit length evaluated at the boundaries of the plate. If body forces are negligible, the conditions of equilibrium<sup>2,8,46</sup> require that the last two integrals of this expression equal zero. Therefore the first integral on the right hand side of expression A-18 represents the virtual work done by the applied external forces acting through mid-plane displacements.

It is assumed that the strain in the middle plane of a plate at buckling is negligible. Therefore, from Equations A-15 to A-17,

$$u'_{,x} = -\frac{1}{2} w'^2_{,x} \tag{A-26}$$

$$u'_{,y} = -\frac{1}{2} w'^2_{,y} \tag{A-27}$$

$$\frac{1}{2}(u'_{,y} + v'_{,x}) = -\frac{1}{2} w'_{,x} w'_{,y} \tag{A-28}$$

Substituting these values into the first two integrals of



expression A-25 gives:

$$\begin{aligned} & \int_{\mathbf{x}} \int_{\mathbf{y}} \left[ N_{\mathbf{x}} \delta u',_{\mathbf{x}} + N_{\mathbf{xy}} (\delta u',_{\mathbf{y}} + \delta v',_{\mathbf{x}}) + N_{\mathbf{y}} \delta v',_{\mathbf{y}} \right] dx dy = \\ & - \int_{\mathbf{x}} \int_{\mathbf{y}} \left[ N_{\mathbf{x}} w,_{\mathbf{x}} \delta w,_{\mathbf{x}} + N_{\mathbf{xy}} (w,_{\mathbf{x}} \delta w,_{\mathbf{y}} + w,_{\mathbf{y}} \delta w,_{\mathbf{x}}) + N_{\mathbf{y}} w,_{\mathbf{y}} \delta w,_{\mathbf{y}} \right] dx dy \end{aligned} \quad (\text{A-29})$$

Using this relationship in Equation A-18, the left hand side of Equation A-1 finally reduces to:

$$\int_{\mathbf{V}} \sigma_{ij} \delta e_{ij} dV = \int_{\mathbf{x}} \int_{\mathbf{y}} \left[ M_{\mathbf{x}} \delta w,_{\mathbf{xx}} + M_{\mathbf{xy}} \delta w,_{\mathbf{xy}} + M_{\mathbf{y}} \delta w,_{\mathbf{yy}} \right] dx dy \quad (\text{A-30})$$

The right hand side of Equation A-1 represents the work done by the surface tractions when the body is subjected to a virtual displacement field. This quantity has already been evaluated in expression A-29 above. Using expressions A-29 and A-30, Equation A-1 finally reduces to:

$$\begin{aligned} & \int_{\mathbf{x}} \int_{\mathbf{y}} \left[ M_{\mathbf{x}} \delta w,_{\mathbf{xx}} + 2M_{\mathbf{xy}} \delta w,_{\mathbf{xy}} + M_{\mathbf{y}} \delta w,_{\mathbf{yy}} \right] dx dy \\ & = - \int_{\mathbf{x}} \int_{\mathbf{y}} \left[ N_{\mathbf{x}} w,_{\mathbf{x}} \delta w,_{\mathbf{x}} + N_{\mathbf{xy}} (w,_{\mathbf{x}} \delta w,_{\mathbf{y}} + w,_{\mathbf{y}} \delta w,_{\mathbf{x}}) + N_{\mathbf{y}} w,_{\mathbf{y}} \delta w,_{\mathbf{y}} \right] dx dy \end{aligned} \quad (\text{A-31})$$



In this equation, the integral on the left hand side represents the strain energy of bending of an equilibrium stress field when a virtual displacement field is superimposed on a buckled configuration. The integral on the right hand side represents the virtual work done by the in-plane stresses acting at the boundaries. In this form, Equation A-31 defines the buckling condition of a plate subjected to in-plane stresses.

In the present analysis, the equilibrium stress field is derived from the general stress-strain relationships for an orthotropic plate. The moments per unit length can be expressed in terms of the plate deflections as follows<sup>7</sup>:

$$M_x = -D_x w_{,xx} - D_{xy} w_{,yy} \quad (A-32)$$

$$M_y = -D_y w_{,yy} - D_{yx} w_{,xx} \quad (A-33)$$

$$M_{xy} = -2G_t I w_{,xy} \quad (A-34)$$

where  $D_x$ ,  $D_y$ ,  $D_{xy}$ , and  $D_{yx}$  are plate bending rigidities,  $G_t$  is the tangent shear modulus, and  $I$  is the moment of inertia per unit length of the plate. These properties are further discussed in Appendix B.

Substituting Equations A-32 to A-34 into Equation A-31, the buckling condition for a uniaxially stressed plate may be expressed as:



$$\begin{aligned}
& \int_x \int_y (D_x w,_{xx} \delta w,_{xx} + D_y w,_{yy} \delta w,_{yy} + D_{xy} w,_{yy} \delta w,_{xx} + D_{yx} w,_{xx} \delta w,_{yy} \\
& + 4G_t I w,_{xy} \delta w,_{xy}) dx dy - \int_x \int_y N_x w,_{x} \delta w,_{x} dx dy = 0
\end{aligned}
\tag{A-35}$$

#### A-6 Development of a Matrix Buckling Condition

The first integral of Equation A-35 leads to the bending stiffness matrix of a plate and the second integral results in a geometric stiffness matrix. The subscript,  $x$ , refers to integration along the length of a plate, and the subscript,  $y$ , refers to integration along its width. In the present analysis, the integration along  $x$  is continuous since material properties along the length of a strip of plate under uniaxial stress will be constant. In the transverse direction of a plate however, the uniaxial stress may vary in intensity and therefore certain longitudinal strips may be yielded while others are still elastic or strain-hardened. To account for this, integration in the  $y$  direction is done in a piecewise manner and the appropriate material properties for a given strip are incorporated into the integration in a piecewise fashion.

As explained in Chapter 3, a Rayleigh-Ritz technique is applied using a displacement function of the form:

$$w = f\langle\phi\rangle\{\theta\} \tag{A-36}$$

In this expression,  $f$  is a function of  $x$  only and it describes the buckled shape in the longitudinal direction,  $\langle\phi\rangle$  is a row vector of





interpolating functions of  $y$  only and  $\{\theta\}$  is a column vector of nodal coordinate displacements. Together,  $\langle\phi\rangle$  and  $\{\theta\}$  define the transverse buckled shape of a plate.

The derivatives and corresponding variations for  $w$  are defined as follows:

$$w_{,xx} = f_{,xx} \langle\phi\rangle \{\theta\} \quad \delta w_{,xx} = f_{,xx} \langle\phi\rangle \{\delta\theta\} \quad (\text{A-37})$$

$$w_{,yy} = f_{,yy} \langle\phi,_{yy}\rangle \{\theta\} \quad \delta w_{,yy} = f_{,yy} \langle\phi,_{yy}\rangle \{\delta\theta\} \quad (\text{A-38})$$

$$w_{,xy} = f_{,x} \langle\phi,_{y}\rangle \{\theta\} \quad \delta w_{,xy} = f_{,x} \langle\phi,_{y}\rangle \{\delta\theta\} \quad (\text{A-39})$$

$$w_{,x} = f_{,x} \langle\phi\rangle \{\theta\} \quad \delta w_{,x} = f_{,x} \langle\phi\rangle \{\delta\theta\} \quad (\text{A-40})$$

Substituting these expressions into Equation A-35 gives:

$$\begin{aligned} \langle\delta\theta\rangle \left[ F_1[\Phi_1] + F_2[\Phi_2] + F_3[\Phi_3] + F_4[\Phi_4] \right] \{\theta\} \\ - \langle\delta\theta\rangle \left[ F_5[\Phi_5] \right] \{\theta\} = 0 \end{aligned} \quad (\text{A-41})$$

where the following relationships have been used:

$$F_1 = D_x \int_x f^2_{,xx} dx, \quad (\text{A-42})$$

$$F_2 = D_y \int_x f^2 dx, \quad (\text{A-43})$$



$$F_3 = 2D_{xy} \int_x f \cdot f_{,xx} dx, \quad (A-44)$$

$$F_4 = 4G_t I \int_x f_{,x}^2 dx, \quad (A-45)$$

$$F_5 = t \int_x f_{,x}^2 dx, \quad (A-46)$$

and,

$$[\Phi_1] = \int_y \{\phi\} \langle \phi \rangle dy \quad (A-47)$$

$$[\Phi_2] = \int_y \{\phi_{,yy}\} \langle \phi_{,yy} \rangle dy \quad (A-48)$$

$$[\Phi_3] = \int_y \left[ \{\phi_{,yy}\} \langle \phi \rangle + \{\phi\} \langle \phi_{,yy} \rangle \right] dy \quad (A-49)$$

$$[\Phi_4] = \int_y \{\phi_{,y}\} \langle \phi_{,y} \rangle dy \quad (A-50)$$

$$[\Phi_5] = \int_y \sigma_x \{\phi\} \langle \phi \rangle dy \quad (A-51)$$

where, for a plate of constant thickness,  $t$ ,  $N_x = \sigma_x t$  is used.

In Equation A-41, the virtual displacement coordinates are completely arbitrary and therefore the relationship must hold for all values  $\langle \delta \theta \rangle$ . Therefore the buckling condition defined by Equation A-41 may be written as follows:



$$[K]\{\theta\} - [K_G]\{\theta\} = \{0\} \quad (A-52)$$

where the bending stiffness matrix,  $[K]$  is given by:

$$[K] = F_1[\Phi_1] + F_2[\Phi_2] + F_3[\Phi_3] + F_4[\Phi_4] = F_i[\Phi_i] \quad (A-53)$$

where,  $i = 1, 2, 3, 4$ , and repeated subscripts indicate summation, and the geometric stiffness matrix,  $[K_G]$  is given by:

$$[K_G] = F_5[\Phi_5] \quad (A-54)$$

In Equations A-53 and A-54, the  $\Phi$  matrices are evaluated by piecewise integration across the width of a plate. The integration is performed for each strip of plate which may be in the elastic, the yielded, or the strain-hardening region. For each such strip, the appropriate bending rigidities, as defined in Appendix B, are used in Equation A-53.



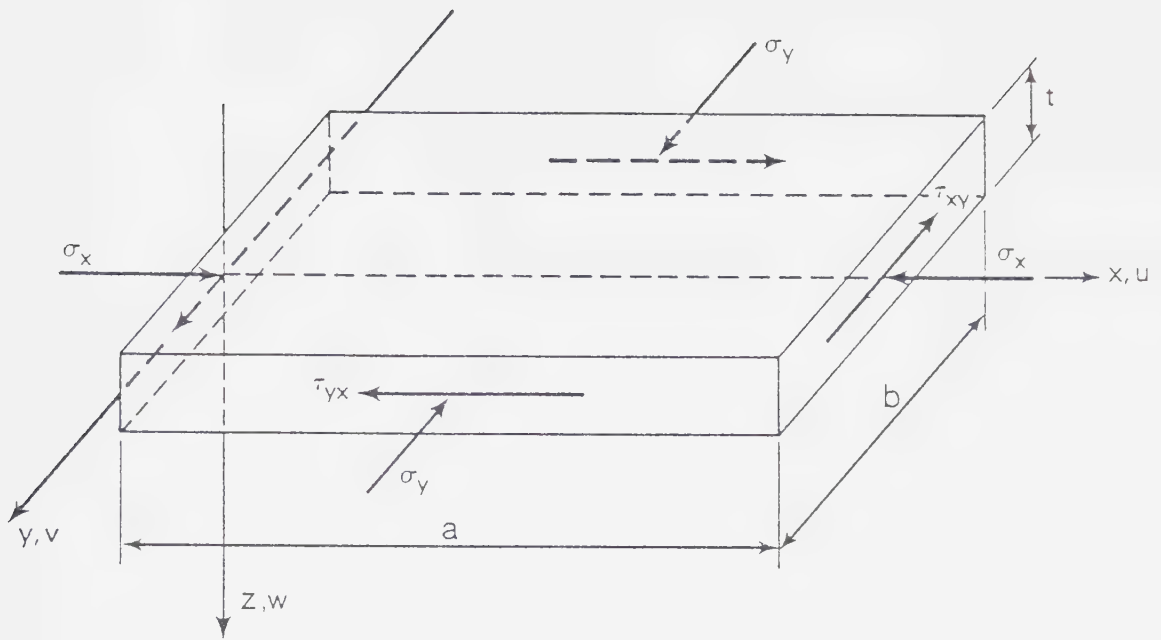


Figure A-1 Rectangular Plate Subjected to Plane Stress

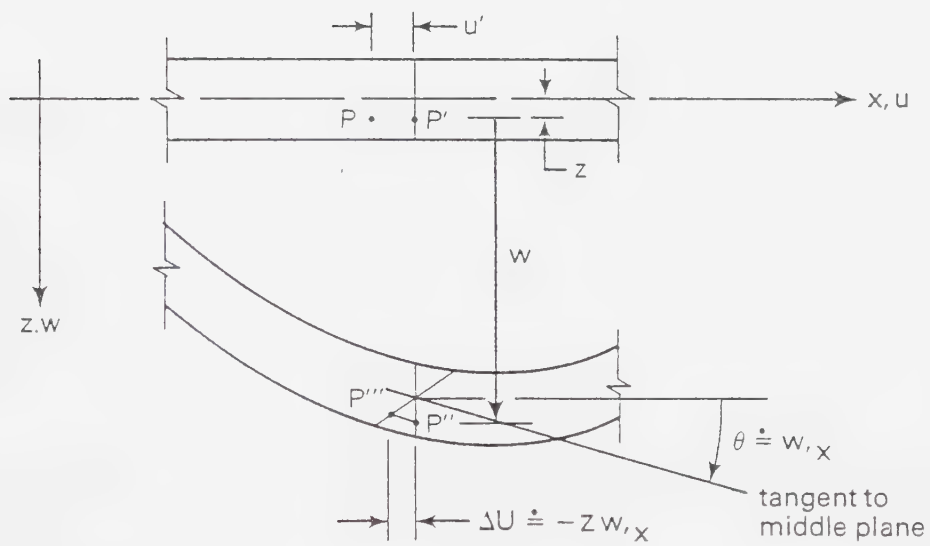


Figure A-2 Plate Buckling in the x-y Plane





## APPENDIX B

### MATERIAL PROPERTIES

#### B-1 Introduction

The determination of material properties used in the present study is based on a tangent modulus concept<sup>5</sup>, and therefore, the material properties that determine a critical buckling strain, are those that exist in a plate at the instant before buckling. This concept is considered to give very good correlation between predicted and test results in inelastic buckling of columns<sup>3,5,6</sup>, and the correlation is further improved with the inclusion of residual stress effects<sup>5</sup>, as in the present study.

#### B-2 Incremental Stress-Strain Relationships

Figure B-1(a) shows a stress-strain relationship for a strain-hardening material. An increment of total strain,  $\Delta\bar{\epsilon}$ , is assumed to be sufficiently small so that it may be separated into an elastic strain increment and a plastic strain increment<sup>71</sup>. Thus,

$$de_{ij} = de_{ij}^{(e)} + de_{ij}^{(p)} \quad (B-1)$$

Elastic strain increments are related to elastic stress increments as follows<sup>71</sup>:

$$de_{ij}^{(e)} = \left(\frac{1+\nu}{E}\right)d\sigma_{ij} - \frac{\nu}{E} d\sigma_{kk}\delta_{ij} \quad (B-2)$$



In these relationships, Cartesian tensor notation is used<sup>46</sup>, and

$$\delta_{ij} = 1 \text{ for } i = j, \text{ and}$$

$$\delta_{ij} = 0 \text{ for } i \neq j,$$

and  $\nu$  and  $E$  are Poisson's ratio and Young's modulus respectively<sup>46</sup>.

The relationship between plastic strain increments and corresponding stress increments may be defined for an assumed yield criterion, an associated flow rule, and a work hardening rule<sup>45,71</sup>. For a material which yields according to von Mises yield criterion<sup>8,71</sup>, the plastic strain increment tensor coincides with the plastic stress deviator tensor and the following relationship may be used<sup>45,71</sup>:

$$de_{ij}^{(p)} = s_{ij} d\lambda \quad (B-3)$$

where  $d\lambda$  is a positive constant of proportionality and the stress deviator tensor is defined as:

$$s_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij} \quad (B-4)$$

According to von Mises yield criterion, yielding occurs when the second invariant of the stress deviator tensor reaches a critical value<sup>71</sup>. This may be expressed as:

$$J_2 - k^2 = 0 \quad (B-5)$$

where,

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \quad (B-6)$$



and  $k$  is a critical constant.

When a work-hardening material is subjected to a uniaxial stress state,  $\sigma$ , Equation B-6 gives:

$$J_2 = \frac{1}{3} \sigma^2 \quad (B-7)$$

or,

$$\bar{\sigma} = \sqrt{3J_2} = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \quad (B-8)$$

where  $\bar{\sigma}$  is an effective stress invariant corresponding to an effective plastic strain,  $\bar{e}^{(p)}$ . In a similar manner, an effective plastic strain increment,  $d\bar{e}^{(p)}$  may be obtained. For a uniaxial stress state, Equation B-3 gives:

$$d\bar{e}^{(p)} = \frac{2}{3} \sigma d\lambda \quad (B-9)$$

Equation B-3 may be written as:

$$de_{ij}^{(p)} de_{ij}^{(p)} = s_{ij} s_{ij} d\lambda^2 = \frac{2}{3} \sigma^2 d\lambda^2 \quad (B-10)$$

Combining Equations B-9 and B-10 results in the following effective plastic strain increment:

$$d\bar{e}^{(p)} = \sqrt{\frac{2}{3} de_{ij}^{(p)} de_{ij}^{(p)}} \quad (B-11)$$

Figure B-1(b) shows a plot of the effective stress  $\bar{\sigma}$ , plotted against effective plastic strain,  $\bar{e}^{(p)}$ . The slope of this curve is defined as:



$$H' = \frac{d\bar{\sigma}}{d\bar{\epsilon}^{(p)}} \quad (B-12)$$

It is assumed that for all monotonic loading paths the same  $\bar{\sigma} - \bar{\epsilon}^{(p)}$  relationship is obtained.

Combining Equations B-8 and B-11 with Equation B-10 gives:

$$d\lambda = \frac{3}{2} \left( \frac{d\bar{\epsilon}^{(p)}}{d\bar{\sigma}} \right) \frac{d\bar{\sigma}}{\bar{\sigma}} = \frac{3}{2} \frac{d\bar{\sigma}}{H' \bar{\sigma}} \quad (B-13)$$

where the value  $H'$  is defined in Equation B-12. Substituting this value of  $d\lambda$  into Equation B-3 gives the following relationship between plastic strain and corresponding stress increments:

$$de_{ij}^{(p)} = \frac{3}{2} \frac{d\bar{\sigma}}{H' \bar{\sigma}} S_{ij} \quad (B-14)$$

Referring to Figure B-1(b), the slope of the effective stress vs. plastic strain curve is given by:

$$H' = \frac{d\bar{\sigma}}{d\bar{\epsilon}^{(p)}} = \lim_{\Delta\bar{\epsilon} \rightarrow 0} \frac{\Delta\bar{\sigma}}{\Delta\bar{\epsilon} - \Delta\bar{\sigma}/E} \quad (B-15)$$

where  $\Delta\bar{\epsilon}$  is the total effective strain increment,  $\Delta\bar{\sigma}$  is the effective stress increment, and  $E$  is Young's modulus. In this relationship,  $\Delta\bar{\sigma}/E$  is the elastic portion of the total strain increment. Multiplying the numerator and the denominator of Equation B-15 by  $E/\Delta\epsilon$  and taking the limit gives:

$$H' = \lim_{\Delta\bar{\epsilon} \rightarrow 0} \frac{E\Delta\bar{\sigma}/\Delta\bar{\epsilon}}{E - \Delta\bar{\sigma}/\Delta\bar{\epsilon}} = \frac{E \cdot E_t}{E - E_t} \quad (B-16)$$

where the tangent modulus is defined as:

$$E_t = \frac{d\bar{\sigma}}{d\bar{\epsilon}} \quad (B-17)$$





#### B-4 Stress-Strain Relationships for Plane Stress

In the case of plane stress<sup>46</sup>,

$$\sigma_z = \tau_{zx} = \tau_{yz} = 0 \quad (\text{B-18})$$

and the effective stress for this case may be obtained from Equation B-8 as follows:

$$\bar{\sigma} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}. \quad (\text{B-19})$$

The effective stress increment is obtained from Equation B-19 as:

$$d\bar{\sigma} = \frac{1}{2\bar{\sigma}} \left[ (2\sigma_x - \sigma_y)d\sigma_x + (2\sigma_y - \sigma_x)d\sigma_y + 6\tau_{xy}d\tau_{xy} \right] \quad (\text{B-20})$$

In the present analysis a plate element before buckling is in a uniaxial stress state defined by  $\sigma_x \neq 0$ ,  $\sigma_y = \tau_{xy} = 0$ , and therefore:

$$\bar{\sigma} = \sigma_x \quad (\text{B-21})$$

Immediately after buckling the same element is subjected to a state of plane stress. In applying a tangent modulus theory, only the resulting increment of effective stress is of interest, and it is given by Equation B-20 as:

$$d\bar{\sigma} = d\sigma_x - \frac{1}{2} d\sigma_y \quad (\text{B-22})$$



The corresponding increments in plastic strain are obtained by using these values of  $\bar{\sigma}$  and  $d\bar{\sigma}$  in Equation B-14 where  $S_{ij}$  is the stress deviator tensor for a state of uniaxial stress prior to buckling. The resulting plastic strain increments are:

$$de_x^{(p)} = \frac{d\sigma_x}{H'} - \frac{d\sigma_y}{2H'} \quad (B-23)$$

$$de_y^{(p)} = -\frac{d\sigma_x}{2H'} + \frac{d\sigma_y}{4H'} \quad (B-24)$$

$$de_{xy}^{(p)} = 0 \quad (B-25)$$

Combining Equations B-2, B-23, B-24, and B-25 with Equation B-1, the increments of total strain may be obtained for the strain-hardening region as:

$$de_x = \left(\frac{1}{E} + \frac{1}{H'}\right)d\sigma_x - \left(\frac{\nu}{E} + \frac{1}{2H'}\right)d\sigma_y \quad (B-26)$$

$$de_y = -\left(\frac{\nu}{E} + \frac{1}{2H'}\right)d\sigma_x + \left(\frac{1}{E} + \frac{1}{4H'}\right)d\sigma_y \quad (B-27)$$

$$de_{xy} = \frac{1+\nu}{E} d\tau_{xy} \quad (B-28)$$

#### B-5 Stress - Strain Relationships for an Orthotropic Plate

For the general case of a homogeneous elastic body, the generalized stress - strain relationship is given by<sup>46</sup>:



$$\sigma_{ij} = C_{ijkl} e_{kl} \quad (B-29)$$

where  $C_{ijkl}$  is a fourth-order tensor representing a total of 81 constants. This number is reduced to 21 independent constants when the symmetry of the stress and strain tensors, and the existence of a potential energy function are considered<sup>8,46,72</sup>.

If a material is orthotropic and the x, y, and z axes coincide with the principle directions of the material, the following stress-strain relationships apply<sup>47,72</sup>:

$$e_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx} \sigma_y}{E_y} - \frac{\nu_{zx} \sigma_z}{E_z} \quad (B-30)$$

$$e_y = -\frac{\nu_{xy} \sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy} \sigma_z}{E_z} \quad (B-31)$$

$$e_z = -\frac{\nu_{xz} \sigma_x}{E_x} - \frac{\nu_{yz} \sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (B-32)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}} \quad (B-33)$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G_{yz}} \quad (B-34)$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} \quad (B-35)$$

where each Poisson ratio,  $\nu_{ij}$  represents the strain in the j direction



per unit strain in the  $i$  direction for  $i, j = x, y, z$ . The above relationships contain 9 independent material constants and:

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x}, \quad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x}, \quad \frac{\nu_{zy}}{E_z} = \frac{\nu_{yz}}{E_y} \quad (\text{B-36})$$

Referring to Equations B-30 to B-35, the incremental stress - strain relationships for an orthotropic plate element subjected to a state of plane stress may be written as follows:

$$de_x = \frac{1}{E_x} d\sigma_x - \frac{\nu_y}{E_y} d\sigma_y \quad (\text{B-37})$$

$$de_y = -\frac{\nu_x}{E_x} d\sigma_x + \frac{1}{E_y} d\sigma_y \quad (\text{B-38})$$

$$de_{xy} = \frac{1}{2G} d\tau_{xy} \quad (\text{B-39})$$

#### B-6 Effective Moduli in the Strain Hardening Range

A steel plate subjected to a uniaxial stress yields by the formation of slip bands along preferential shear planes characterised by a dissipation of minimum plastic distortional energy<sup>67,68</sup>. For a thin steel plate these shear planes will be normal to the stress axis and inclined with respect to the middle plane of the plate. As a result, in the strain-hardening range, the steel plate will be orthotropic at the instant before buckling.

Based on a tangent modulus theory, the effective moduli in the strain-hardening range may be obtained by comparing Equations B-37





to B-39 with Equations B-26 to B-28<sup>69</sup>. Using the results of this comparison and the value of  $H'$  as given by Equation B-16 gives the following effective tangent moduli:

$$E_x = E_t \quad (B-40)$$

$$E_y = \frac{4E \cdot E_t}{3E_t + E} \quad (B-41)$$

$$G_t = G = \frac{E}{2(1+\nu)} \quad (B-42)$$

and the corresponding Poisson's ratios:

$$\nu_x = \frac{(2\nu-1)E_t + E}{2E} \quad (B-43)$$

$$\nu_y = \frac{4\nu E}{3E_t + E} \quad (B-44)$$

## B-7 Material Properties Used in the Present Analysis

In the elastic range, the tangent modulus is equal to Young's modulus,  $E$ , and Equations B-40 to B-44 reduce to:

$$E_x = E_y = E \quad (B-45)$$

$$G_t = G \quad (B-46)$$

$$\nu_x = \nu_y = \nu \quad (B-47)$$



In the yielding range, the tangent modulus  $E_t = 0$  and Equations B-40 to B-44 give:

$$E_x = E_y = 0 \quad (B-48)$$

$$G_t = G \quad (B-49)$$

$$\nu_x = 0.5 \quad (B-50)$$

$$\nu_y = 2.0 \quad (B-51)$$

The results given by Equations B-40 to B-44 were originally presented by Handelsmann and Prager<sup>22</sup>. However, since that time it has been well recognized that these values may result in significant error in the prediction of inelastic critical plate buckling stresses. It has further been observed that these discrepancies are mainly due to an over-estimation of the effective shear modulus,  $G_t$ , at stresses above the yield<sup>7,28,33,70</sup>.

For  $E = 29,600$  ksi. and  $\nu = 0.3$ , Equation B-42 gives a value of  $G_t = 11,385$  ksi. Using experimental results, Haaijer and Thurlimann determined that the value of  $G_t$  for steel above the yield should be between 2000 and 3000 ksi.<sup>7</sup>, with an actual value selected at 2400 ksi. This 79 per cent reduction in the theoretical value of  $G_t$ , which resulted in better correlation with test results, was attributed to the effects of initial imperfections. Independently, Lay<sup>33</sup> disregarded the effects of initial imperfections and, using slip field theory, predicted a value of  $G$  of about 3000 ksi. As a result of this work, Lay



presented the following expression for the tangent shear modulus for strain-hardening:

$$G_t = \frac{2G}{1 + \frac{E}{4E_t(1+\nu)}} \quad (B-52)$$

where  $G$  is the elastic shear modulus,  $E$  is Young's modulus,  $E_t$  is the tangent modulus, and  $\nu$  is Poisson's ratio. For  $E = 29,600$  ksi.,  $G = 11,385$  ksi.,  $\nu = 0.3$ , and  $E_t = 800$  ksi., Equation B-51 gives a value of  $G_t = 2806$  ksi. In the present study, Equations B-40, B-41, B-43, B-44, and B-51 are used to determine material properties in the inelastic range.

#### B-8 Plate Bending Rigidities

The plate bending rigidities  $D_x$ ,  $D_y$ ,  $D_{xy}$ , and  $D_{yx}$  were introduced in Appendix A. In terms of the material properties discussed previously the plate bending rigidities may be expressed as follows<sup>73</sup>.

$$D_x = \frac{E_x I}{1 - \nu_x \nu_y} \quad (B-53)$$

$$D_y = \frac{E_y I}{1 - \nu_x \nu_y} \quad (B-54)$$

$$D_{xy} = \nu_y D_x \quad (B-55)$$



$$D_{yx} = \nu_x D_{xy} \quad (B-56)$$

where  $I$  is the moment of inertia per unit length of a plate.





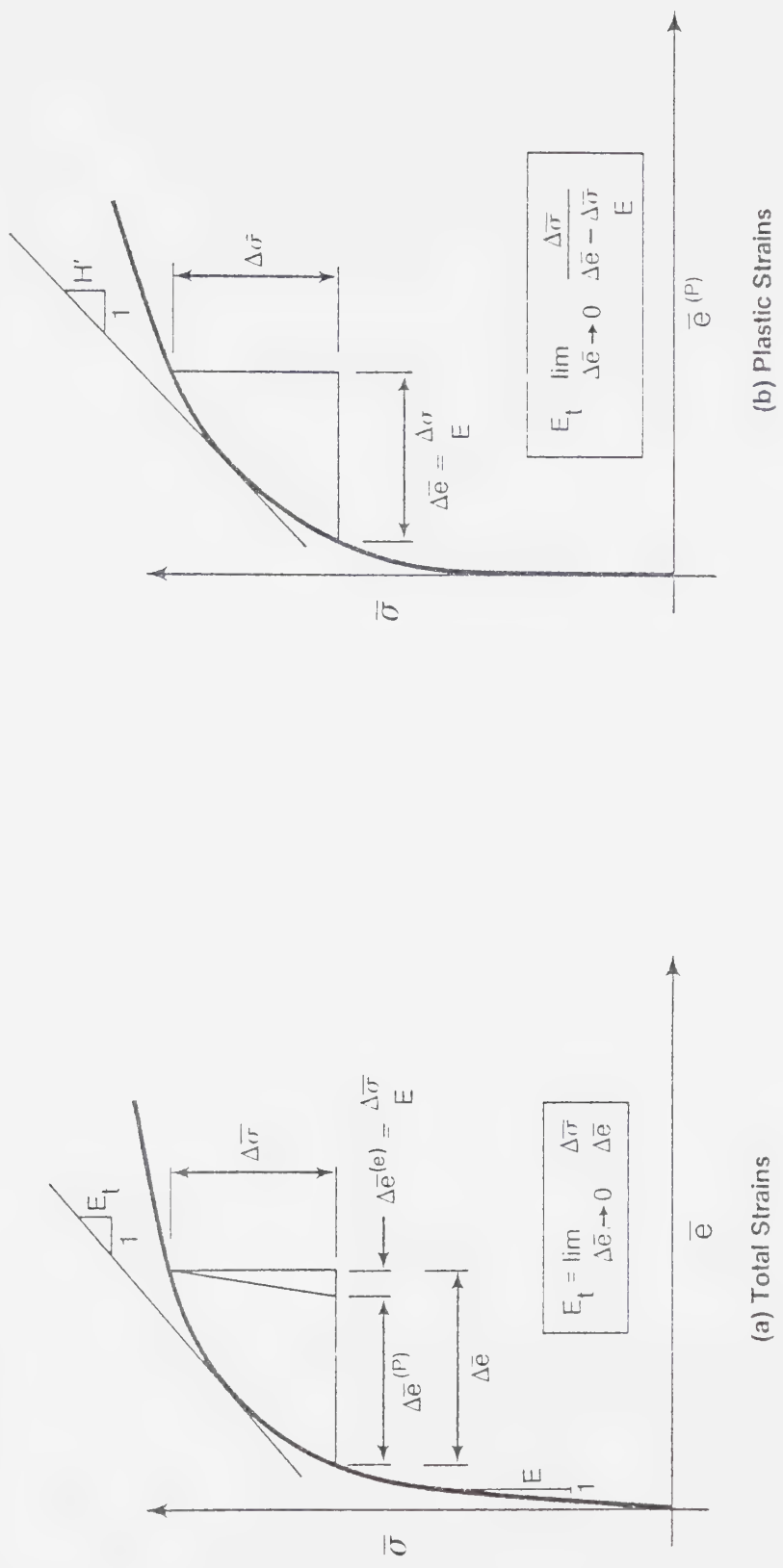


Figure B-1 Effective Stress - Effective Strain Relationships



## APPENDIX C

### COMPUTER PROGRAM

#### C.1 Introduction

This appendix contains a listing of the computer program which was developed and used to predict the test results presented in Chapter 5. It was also used in the parametric study presented in Chapter 6. The program consists of one main program and forty-one subroutines. The symbolic names used in the program are defined in the first subroutine in which they are principally used. The purposes of the main program as well as the subroutines are explained by extensive comments placed at the beginning of each. Numerous comments are also used throughout the program to segment and explain blocks of logic. A general outline of the program is given below.

#### C-2 General Outline

The number of specimens to be analysed, the modulus of elasticity, and Poisson's ratio are read in in the main program. Values indicating the type of cross-section (either a single plate or a W shape configuration) as well as the type of edge supports and the type of loading are also read in. These, as well as the other variables are explained in the comments at the beginning of the program.

The main routine calls one of PLATE1, PLATE2, PLATE3, PLATE4, or PLATE5 depending on the type of problem. PLATE1 and PLATE2 are used to analyse single plate configurations while PLATE3, PLATE4



and PLATE5 apply to W shape configurations used for columns, beams, and beam-columns, respectively. As stated previously, the functions of these subroutines are fully explained by comments inserted at the beginning of each one. Each of the PLATE-series of subroutines reads in additional information necessary for the solution of a problem. Since many aspects of the PLATE-series of subroutines are similar, it will be assumed here (for the purpose of continuing with the general outline of the program) that PLATE5 has been called from the main routine.

PLATE5 analyses a W shape section loaded as a beam-column. At the beginning of this subroutine all subsequent data necessary for the analysis are read in and section properties and residual stress values are calculated. This subroutine, during iteration, must calculate several critical local buckling loads before the correct one is determined. The critical axial load (either elastic or inelastic) is first determined and compared with the applied axial load. If the applied axial load is less than the critical one, the analysis for the critical superimposed bending moment continues (using an iterative technique if inelastic action occurs).

As explained by the comments within the subroutine, each time a critical load is calculated, PLATE5 calls three additional subroutines which calculate tension and compression flange stiffness matrices as well as the web stiffness matrix for an assumed value of strain. A fourth subroutine, ESFORM, is then called. ESFORM formulates the flange and web plate stiffness matrices into a total global stiffness matrix for an assumed number of half wavelengths of  $m$ . A matrix iterative subroutine, EIGEN, is then called by ESFORM to



calculate the corresponding eigenpair. This process is continued for values of  $m$  incremented by unity until a minimum eigenvalue is obtained. This value is returned to PLATE5 and compared with the previous value and if convergence has not occurred the entire process (beginning with new flange and web stiffness matrices) is repeated.

When this process has converged, the resulting eigenvalue is used to calculate the corresponding critical bending moment in subroutine MCALC. The critical axial load and the critical superimposed bending moment as well as the applied load, deformed configuration during local buckling, and other pertinent data are printed out as illustrated in the sample problems presented in Appendix D.

In subroutines, PLATE1 and PLATE2, since only elastic local buckling stresses are considered, it is necessary to formulate flange and web stiffness matrices only once. Iteration on the number,  $m$ , of half wavelengths must still be performed, however. In subroutines, PLATE3 and PLATE4, for columns and beams, respectively, the entire iterative process must be carried out when inelastic local buckling stresses are required.





Listing of  
Computer Program











```

226      CW(5,5)=HW**2/64 ODO
227      CW(6,6)=-CW(2,6)
228      CW(6,7)=-HW**2/32 ODO
229      CW(6,8)=HW**2/64 ODO
230      CW(6,9)=HW**2/64 ODO
231      CW(7,4)=-HW**11 ODO/32 ODO
232      CW(7,5)=-CW(1,5)
233      CW(7,6)=HW*9 ODO/16 ODO
234      CW(7,7)=HW**11 ODO/16 ODO
235      CW(7,8)=-HW*7 ODO/32 ODO
236      CW(7,9)=-CW(1,9)
237      RETURN
238      END
239      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
240      C
241      C
242      C          SUBROUTINE 4
243      C
244      C
245      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
246      C
247      C          SUBROUTINE EIGEN
248      C
249      C          THIS ROUTINE PERFORMS INVERSE ITERATION
250      C          TO DETERMINE THE EIGENVALUE AND BUCKLING
251      C          SHAPE OF THE CROSS-SECTION FOR AN ASSUMED
252      C          NUMBER OF HALF SINE WAVES IN THE LONGITUDINAL
253      C          DIRECTION.
254      C          WHEN A NEGATIVE EIGENVALUE IS ENCOUNTERED A
255      C          TECHNIQUE IS USED TO REMOVE THE ASSOCIATED MODE
256      C          AND A POSITIVE EIGENVALUE IS FOUND. AN EIGENVALUE
257      C          SHIFT IS USED TO INCREASE THE CONVERGENCE RATE.
258      C
259      C          SG = GEOMETRIC STIFFNESS MATRIX
260      C          ABG = PRODUCT OF INVERSE STIFFNESS AND GEOMETRIC
261      C          STIFFNESS MATRICES
262      C          EVAL = CURRENT EIGENVALUE
263      C          SVEC = CURRENT STARTING VECTOR
264      C
265      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
266      C          SUBROUTINE EIGEN(ABG,SG,EVAL,IG,SVEC,*)
267      C          IMPLICIT REAL*8(A-H,O-Z)
268      C          DIMENSION ABG(15,15),SVEC(15),EVEC(15),AVEC(15,2),SG(15,15)
269      C          DIMENSION ABGP(15,15)
270      C          COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
271      C          COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRGB,SRTT,SRCT
272      C
273      C
274      C          INITIALISE ISH, IG, AND ICH
275      C          ICH=SWEEP COUNTER
276      C          IG=NUMBER OF HALF WAVELENGTHS
277      C          ISH=SHIFT COUNTER
278      C
279      C          ISH=0
280      C          IG=IG+1
281      C          ICH=0
282      C          IF (IG.GT.1) GO TO 20
283      C
284      C
285      C          INITIALISE A STARTING VECTOR SVEC(1)
286      C          ABG(1,J)=[INVERSE STIFF. MATRIX]X(GEOM. STIFF. MATRIX)
287      C          SVAL=STARTING EIGENVALUE TO EVALUATE INITIAL SHIFT
288      C
289      C          10 CALL START(ABG,SVEC,SVAL)
290      C          ISH=0
291      C          SH=0.1DO/DABS(SVAL)
292      C          SH=SH*(-1)**ISH
293      C          GO TO 30
294      C
295      C
296      C          INITIALISE ABGP(I,J)=ABG(I,J) AND PERFORM SHIFT
297      C
298      C          20 SH=0.1DO/DABS(EVAL)
299      C          SH=SH*(-1)**ISH
300      C          30 DO 40 I=1,N
301      C          DO 40 J=1,N
302      C          ABGP(I,J)=ABG(I,J)
303      C          DO 60 I=1,N
304      C          ABGP(I,I)=ABG(I,I)+SH*ISH
305      C          70 NIT=0
306      C          80 AMAX=0 ODO
307      C
308      C
309      C          BEGIN MATRIX ITERATION
310      C
311      C          DO 100 I=1,N
312      C          SUM=0 ODO
313      C          DO 90 J=1,N
314      C          90 SUM=SUM+ABGP(I,J)*SVEC(J)
315      C          EVEC(I)=SUM
316      C          IF (DABS(SUM) LE.AMAX) GO TO 100
317      C          K=1
318      C          AMAX=DABS(SUM)
319      C          100 CONTINUE
320      C
321      C
322      C          NORMALISE THE SHAPE VECTOR AND CHECK CONVERGENCE
323      C
324      C          AVAL=1 ODO/EVEC(K)
325      C          ADIF=0 ODO
326      C          DO 110 I=1,N
327      C          REP=SVEC(I)
328      C          SVEC(I)=EVEC(I)*AVAL
329      C          DREP=DABS(REP)
330      C          DSVEC=DABS(SVEC(I))
331      C          ADIF=ADIF+DABS(DREP-DSVEC)
332      C          110 CONTINUE
333      C
334      C
335      C          NIT=NUMBER OF ITERATIONS
336      C          IF NIT EXCEEDS 250 CHANGE VALUE OF SHIFT
337      C
338      C          NIT=NIT+1

```





```

339      IF (ADIF.LT.1.0D-10) GO TO 130
340      IF (INIT.GT.250) GO TO 120
341      GO TO 80
342      C
343      C
344      C      CONTINUE SHIFTING UP TO 4 TIMES IF CONVERGENCE
345      C      IS NOT REACHED.
346      C
347      120      ISH=ISH+1
348      IF (ISH.GT.4) GO TO 140
349      SH=0.100/DABS(AVAL)
350      SH=SH*(-1)**ISH
351      GO TO 50
352      C
353      C
354      C      WHEN SHAPE HAS CONVERGED DETERMINE THE EIGENVALUE.
355      C
356      130      CALL RAYLE(SVEC,ABGP,BVAL)
357      EVAL=1.000/[(1.000/BVAL)-SH*ISH)
358      C
359      C
360      C      IF THE EIGENVALUE IS NEGATIVE, SWEEP OUT THE
361      C      ASSOCIATED MODE AND CONTINUE...OTHERWISE RETURN
362      C
363      IF (EVAL.GE.0.000 AND BVAL.GE.0.000) RETURN
364      ICH=ICH+1
365      IF (ICH.GT.5) GO TO 160
366      CALL SWEEP(ABG,SG,SVEC,ICH)
367      GO TO 10
368      140      NIT=200
369      C
370      C
371      C      WRITE ERROR MESSAGES.
372      C
373      WRITE (6,190) NIT,ISH,NS
374      150      WRITE (6,180)
375      RETURN 1
376      180      FORMAT (' ',5X,'EIGENVALUE REMAINS NEGATIVE IN 5 SWEEPS')
377      190      FORMAT ('-',5X,'SUBROUTINE EIGEN FAILS TO CONVERGE IN ',I3,' ITERA
378      &TIONS AND ',I3,' SHIFTS FOR SPECIMEN NO.',I3)
379      END
380      C
381      C
382      C      SUBROUTINE 5
383      C
384      C
385      C
386      C
387      C
388      C      SUBROUTINE EPHI
389      C
390      C      THIS ROUTINE INTEGRATES THE GEOMETRIC
391      C      SUBMATRICES RESULTING FROM THE STRAIN ENERGY
392      C      ASSOCIATED WITH A STRESS GRADIENT WITHIN THE
393      C      PLANE OF A PLATE.
394      C
395      C
396      C
397      C
398      C
399      C
400      C
401      C
402      C
403      C
404      C
405      C
406      C
407      C
408      C
409      C
410      C
411      C
412      C
413      C
414      C
415      C
416      C
417      C
418      C
419      C
420      C
421      C
422      C
423      C
424      C
425      C
426      C
427      C
428      C
429      C
430      C
431      C
432      C
433      C
434      C
435      C
436      C
437      C
438      C
439      C
440      C
441      C
442      C
443      C
444      C
445      C
446      C
447      C
448      C
449      C
450      C
451      C

```



```

452      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
453      IG=0
454      XMIN=1.0D0
455      C
456      C
457      C      BEGIN ITERATION ON THE NUMBER OF HALF WAVELENGTHS
458      C
459      DD 140 M=1,10
460      RM=DFLOAT(M)
461      C
462      C
463      C      CALCULATE LONGITUDINAL BUCKLING FACTORS, RM1, RM2, RM3,
464      C      RM4, RM5, RM6, RM7, AND RM8 FOR VARIOUS BOUNDARY
465      C      CONDITIONS
466      C
467      IF (ICLAMP.EQ.1) GO TO 10
468      RM1=RM**4
469      RM2=1.0D0
470      RM3=RM**2
471      RM4=RM3
472      GO TO 20
473      C
474      C
475      C      PERFORM GAUSSIAN INTEGRATION OF BUCKLING FACTORS
476      C      FOR OTHER THAN PINNED ENDS
477      C
478      10 CALL GOUAD(M,RM1,RM2,RM3,RM4)
479      RM5=RM1
480      RM6=RM2
481      RM7=RM3
482      RM8=RM4
483      IF (ICLAMP.NE.2) GO TO 30
484      CALL GOUAD(M,RM5,RM6,RM7,RM8)
485      30 IF (ITYPE.LE.2) GO TO 40
486      IF (ITYPE.LE.6) GO TO 60
487      IF (ITYPE.LE.7) GO TO 80
488      C
489      C
490      C      ASSEMBLE GLOBAL STIFFNESSES FOR FLANGE-TYPE PLATES
491      C
492      40 N=5
493      DO 50 I=1,N
494      DO 50 J=1,N
495      SB(I,J)=RM1*FB1(I,J)+RM2*FB2(I,J)+RM3*FB3(I,J)+RM4*FB4(I,J)
496      SB(J,I)=SB(I,J)
497      SG(I,J)=RM4*FB5(I,J)
498      SG(J,I)=SG(I,J)
499      50 IF (ITYPE.EQ.2) SB(5,5)=1.D50
500      GO TO 120
501      C
502      C
503      C      ASSEMBLE GLOBAL STIFFNESSES FOR WEB-TYPE PLATES
504      C
505      60 N=7
506      DO 70 I=1,N
507      DO 70 J=1,N
508      SB(I,J)=RM1*WB1(I,J)+RM2*WB2(I,J)+RM3*WB3(I,J)+RM4*WB4(I,J)
509      SB(J,I)=SB(I,J)
510      SG(I,J)=RM4*WB5(I,J)
511      SG(J,I)=SG(I,J)
512      IF (ITYPE.EQ.4) SB(1,1)=1.D50
513      IF (ITYPE.EQ.5) SB(7,7)=1.D50
514      IF (ITYPE.EQ.6) SB(1,1)=1.D50
515      IF (ITYPE.EQ.6) SB(7,7)=1.D50
516      GO TO 120
517      C
518      C
519      C      ASSEMBLE GLOBAL STIFFNESSES FOR W-SHAPES.
520      C
521      80 N=15
522      DO 90 I=1,N
523      DO 90 J=1,N
524      SB(I,J)=0.0D0
525      SG(J,I)=0.0D0
526      DO 100 I=1,5
527      DO 100 J=1,5
528      SB(I,J)=RM1*FB1(I,J)+RM2*FB2(I,J)+RM3*FB3(I,J)+RM4*FB4(I,J)
529      SB(J,I)=SB(I,J)
530      SG(I,J)=RM4*FB5(I,J)
531      SG(J,I)=SG(I,J)
532      I1=I+10
533      J1=J+10
534      SB(I1,J1)=RM1*FT1(I,J)+RM2*FT2(I,J)+RM3*FT3(I,J)+RM4*FT4(I,J)
535      SB(J1,I1)=SB(I1,J1)
536      SG(I1,J1)=RM4*FT5(I,J)
537      SG(J1,I1)=SG(I1,J1)
538      DO 110 I=1,7
539      DO 110 J=1,7
540      I4=I+4
541      J4=J+4
542      SB(I4,J4)=RM5*WB1(I,J)+RM6*WB2(I,J)+RM7*WB3(I,J)+RM8*WB4(I,J)
543      SB(J4,I4)=SB(I4,J4)
544      SG(I4,J4)=RM8*WB5(I,J)
545      SG(J4,I4)=SG(I4,J4)
546      110 SB(5,5)=SB(5,5)+RM1*FB1(5,5)+RM2*FB2(5,5)+RM3*FB3(5,5)+RM4*FB4(5,5)
547      &
548      SG(5,5)=SG(5,5)+RM4*FB5(5,5)
549      SB(11,11)=SB(11,11)+RM1*FT1(1,1)+RM2*FT2(1,1)+RM3*FT3(1,1)+RM4*FT4
550      &(1,1)
551      SG(11,11)=SG(11,11)+RM4*FT5(1,1)
552      C
553      C
554      C      INVERT THE BENDING STIFFNESS MATRIX (SB INVERSE = SBI)
555      C
556      120 CALL INVERT(N,SB,SBI)
557      C
558      C
559      C      MULTIPLY THE GEOMETRIC STIFFNESS MATRIX BY THE
560      C      BENDING STIFFNESS MATRIX. (ABC = SBI X SG)
561      C
562      CALL MULT(ABC,SBI,SG,N)
563      C
564      C

```



```

565 C
566 C      CALL EIGEN TO PERFORM MATRIX ITERATION FOR CURRENT
567 C      VALUE OF M.
568 C
569 C      CALL EIGEN(ABC,SG,EVAL,IG,SVEC,&160)
570 X(M)=EVAL
571 DO 130 I=1,N
572 130 XVEC(I,M)=SVEC(I)
573 C
574 C
575 C      SELECT THE VALUE OF M CORRESPONDING TO THE LOWEST
576 C      ENERGY SHAPE IN THE LONGITUDINAL DIRECTION. THE
577 C      LOWEST ENERGY SHAPE OF A CROSS-SECTION HAS BEEN
578 C      DETERMINED FROM MATRIX ITERATION
579 C
580 C      IF (X(M).GT.XMIN) GO TO 140
581 XMIN=X(M)
582 MM1=M
583 140 CONTINUE
584 150 EIGV=X(MM1)
585 RETURN
586 160 RETURN 1
587 END
588 C
589 C
590 C
591 C      SUBROUTINE 7
592 C
593 C
594 C
595 C
596 C      SUBROUTINE EXPO
597 C
598 C      THIS ROUTINE CALCULATES THE INTEGRAL OF
599 C      A POWER H BETWEEN THE LIMITS A AND B.
600 C
601 C
602 C      SUBROUTINE EXPO(B,A,H,NH)
603 IMPLICIT REAL*8(A-H,D-Z)
604 DIMENSION H(18)
605 DO 10 I=1,NH
606 E=DFLOAT(I)
607 H(I)=(B**I-A**I)/E
608 10 CONTINUE
609 RETURN
610 END
611 C
612 C
613 C
614 C      SUBROUTINE 8
615 C
616 C
617 C
618 C
619 C      SUBROUTINE FBINAB
620 C
621 C      THIS SUBROUTINE FORMULATES THE STIFFNESS
622 C      SUBMATRICES, FB1, FB2, FB3, FB4, FB5, FOR A
623 C      BOTTOM (TENSION) FLANGE WHEN THE W-SHAPE IS
624 C      SUBJECTED TO A CONSTANT AXIAL LOAD AND A
625 C      VARIABLE SUPERIMPOSED BENDING MOMENT.
626 C
627 C
628 C      SUBROUTINE FBINAB(ST,EAPP)
629 IMPLICIT REAL*8(A-H,D-Z)
630 DIMENSION F1(7,7),F1Y(7,7),F1F1(7,7),F1Y(7,7),EF1(7,7)
631 COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
632 COMMON /BLK2/CB(7,9),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5),FB5(5,5)
633 COMMON /BLK4/FA,FB,FC,FD,FE
634 COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
635 COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
636 COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
637 COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
638 C
639 C
640 C      INITIALISE THE STIFFNESS SUBMATRICES TO ZERO
641 C
642 DO 10 J=1,5
643 DO 10 I=1,5
644 FB1(I,J)=0.0D0
645 FB2(I,J)=0.0D0
646 FB3(I,J)=0.0D0
647 FB4(I,J)=0.0D0
648 FB5(I,J)=0.0D0
649 10 C
650 C
651 C      ADJUST FLANGE THICKNESS TO ACCOUNT FOR
652 C      WEB-TO-FLANGE FILLETS
653 C
654 TSAVE=TF1
655 TF1=BF1+TF1/(BF1-2.0D0*TW)
656 C
657 C
658 C      INITIALISE STRAINS AND STRAIN LIMITS, T1, T2, T3,
659 C      T4.
660 C
661 RST=R*ST
662 EARST=EAPP-RST
663 STEA=ST+EAPP
664 RSTEA=RST-EAPP
665 STR=1.0D0/STEA
666 C
667 C
668 C      IF THE LOWER FLANGE IS IN COMPRESSION GO TO THE
669 C      SECOND PART OF THE SUBROUTINE, OTHERWISE CONTINUE
670 C
671 IF (EARST.GT.0.0D0) GO TO 200
672 T1=EYBF-ERTB
673 T2=EYBF+ERCB
674 T3=ESH-ERTB
675 T4=ESH+ERCB
676 AD1=ERTB+ERCB
677 C

```



```

678      C
679      C      EVALUATE PARAMETERS ALFB AND ALFBP WHICH INDICATE
680      C      EXTENTS OF YIELDING AND STRAIN HARDENING,
681      C      RESPECTIVELY.
682      C
683      IF (RSTEA.GT.T1) GO TO 20
684      ALFB=0.ODO
685      ALFBP=0.ODO
686      GO TO 70
687      20  IF (RSTEA.GT.T2) GO TO 40
688      IF (AD1.EQ.0.ODO) GO TO 30
689      AN1=RSTEA-T1
690      ALFB=AN1/AD1
691      ALFBP=0.ODO
692      GO TO 70
693      30  ALFB=1.ODO
694      ALFBP=0.ODO
695      GO TO 70
696      40  IF (RSTEA.GT.T3) GO TO 50
697      ALFB=1.ODO
698      ALFBP=0.ODO
699      GO TO 70
700      50  IF (RSTEA.GT.T4) GO TO 60
701      IF (AD1.EQ.0.ODO) GO TO 60
702      AN1=RSTEA-T3
703      ALFBP=AN1/AD1
704      ALFB=1.ODO
705      GO TO 70
706      60  ALFB=1.ODO
707      ALFBP=1.ODO
708      70  CALL FWCALC(BCL,BF1,TF1,E)
709      C
710      C
711      C      CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
712      C      TO FAC6.
713      C
714      FAC1=(RSTEA+ERTB)*E*FE*STR
715      FAC2=AD1*E*FE*STR
716      FAC3=(YSFB-(T1-RSTEA)*E1)*FE*STR
717      FAC4=AD1*E1*FE*STR
718      FAC5=(YSFB+(ESH-EYBF)*E1-(T3-RSTEA)*E2)*FE*STR
719      FAC6=AD1*E2*FE*STR
720      C
721      C
722      C      FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION.
723      C
724      A=-1.ODO
725      B=-ALFB
726      IF (DABS(A-B).LT.1.OD-5) GO TO 90
727      CALL PHI(B,A,FI,CB,5,6,11)
728      CALL PHIYY(B,A,FIYY,CB,5,6,7)
729      CALL PHIPHI(B,A,FIPI,CB,5,6,9)
730      CALL PHIV(B,A,FIY,CB,5,6,5)
731      CALL EPHI(B,A,EPI,CB,5,6,12)
732      DO 80 I=1,5
733      DO 80 J=1,5
734      FB1(I,J)=FA*FI(I,J)
735      FB2(I,J)=FB*FIYY(I,J)
736      FB3(I,J)=-FC*FIPI(I,J)
737      FB4(I,J)=FD*FIY(I,J)
738      FB5(I,J)=-FAC1*FI(I,J)-FAC2*EPI(I,J)
739      C
740      C
741      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION.
742      C
743      90  A=-ALFB
744      B=-ALFBP
745      IF (DABS(A-B).LT.1.OD-5) GO TO 110
746      CALL FWCALC(BCL,BF1,TF1,E1)
747      CALL PHI(B,A,FI,CB,5,6,11)
748      CALL PHIYY(B,A,FIYY,CB,5,6,7)
749      CALL PHIPHI(B,A,FIPI,CB,5,6,9)
750      CALL PHIV(B,A,FIY,CB,5,6,5)
751      CALL EPHI(B,A,EPI,CB,5,6,12)
752      DO 100 I=1,5
753      DO 100 J=1,5
754      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
755      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
756      FB3(I,J)=FB3(I,J)-FC*FIPI(I,J)
757      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
758      FB5(I,J)=FB5(I,J)-FAC3*FI(I,J)-FAC4*EPI(I,J)
759      C
760      C
761      C      FORMULATE STIFFNESS SUBMATRICES FOR
762      C      STRAIN-HARDENED REGION
763      C
764      110  A=-ALFBP
765      B=0.ODO
766      IF (DABS(A-B).LT.1.OD-5) GO TO 130
767      CALL FWCALC(BCL,BF1,TF1,E2)
768      CALL PHI(B,A,FI,CB,5,6,11)
769      CALL PHIYY(B,A,FIYY,CB,5,6,7)
770      CALL PHIPHI(B,A,FIPI,CB,5,6,9)
771      CALL PHIV(B,A,FIY,CB,5,6,5)
772      CALL EPHI(B,A,EPI,CB,5,6,12)
773      DO 120 I=1,5
774      DO 120 J=1,5
775      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
776      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
777      FB3(I,J)=FB3(I,J)-FC*FIPI(I,J)
778      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
779      FB5(I,J)=FB5(I,J)-FAC5*FI(I,J)-FAC6*EPI(I,J)
780      120  A=0.ODO
781      130  B=ALFBP
782      IF (DABS(A-B).LT.1.OD-5) GO TO 150
783      CALL PHI(B,A,FI,CB,5,6,11)
784      CALL PHIYY(B,A,FIYY,CB,5,6,7)
785      CALL PHIPHI(B,A,FIPI,CB,5,6,9)
786      CALL PHIV(B,A,FIY,CB,5,6,5)
787      CALL EPHI(B,A,EPI,CB,5,6,12)
788      DO 140 I=1,5
789      DO 140 J=1,5
790      FB1(I,J)=FB1(I,J)+FA*FI(I,J)

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761      FB2(I,J):FB2(I,J)+FB*FIYY(I,J)
762      FB3(I,J):FB3(I,J)+FC*FIFI(I,J)
763      FB4(I,J):FB4(I,J)+FD*FIY(I,J)
764      FB5(I,J):FB5(I,J)+FAC5*FI(I,J)+FAC6*EF1(I,J)
765
766      C
767      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
768      C
769      C
770      150      A:ALFBP
771      B:ALFB
772      IF (DABS(A-B).LT.1.OD-5) GO TO 170
773      CALL FWCALC(BCL,BF1,TF1,E1)
774      CALL PHI(B,A,FI,CB,5,6,11)
775      CALL PHIYY(B,A,FIYY,CB,5,6,7)
776      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
777      CALL PHII(B,A,FIY,CB,5,6,9)
778      CALL EPHI(B,A,EFI,CB,5,6,12)
779      DO 160 I=1,5
780      DO 160 J=1,5
781      FB1(I,J):FB1(I,J)+FA*FI(I,J)
782      FB2(I,J):FB2(I,J)+FB*FIYY(I,J)
783      FB3(I,J):FB3(I,J)+FC*FIFI(I,J)
784      FB4(I,J):FB4(I,J)+FD*FIY(I,J)
785      FB5(I,J):FB5(I,J)+FAC3*FI(I,J)+FAC4*EF1(I,J)
786
787      160      C
788      C      FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION
789      C
790      C
791      170      A:ALFB
792      B:1.ODO
793      IF (DABS(A-B).LT.1.OD-5) GO TO 190
794      CALL FWCALC(BCL,BF1,TF1,E)
795      CALL PHI(B,A,FI,CB,5,6,11)
796      CALL PHIYY(B,A,FIYY,CB,5,6,7)
797      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
798      CALL PHII(B,A,FIY,CB,5,6,9)
799      CALL EPHI(B,A,EFI,CB,5,6,12)
800      DO 180 I=1,5
801      DO 180 J=1,5
802      FB1(I,J):FB1(I,J)+FA*FI(I,J)
803      FB2(I,J):FB2(I,J)+FB*FIYY(I,J)
804      FB3(I,J):FB3(I,J)+FC*FIFI(I,J)
805      FB4(I,J):FB4(I,J)+FD*FIY(I,J)
806      FB5(I,J):FB5(I,J)+FAC1*FI(I,J)+FAC2*EF1(I,J)
807
808      180      C
809      C      RETURN
810      C
811      C
812      C      PERFORM SUBMATRIX FORMULATION FOR THE CASE WHEN
813      C      THE LOWER (TENSION) FLANGE IS IN COMPRESSION
814      C      INITIALISE STRAIN LIMITS T1, T2, T3, T4 AND
815      C      INTEGRATION LIMITS ALFT AND ALFP
816      C
817      C
818      200      T1:EYBF-ERCB
819      T2:EYBF+ERTB
820      T3:ESH-ERCB
821      T4:ESH+ERTB
822      ADEN:ERCB+ERTB
823      IF (EARST.GT.T1) GO TO 210
824      ALFT=1 ODO
825      ALFP=1 ODO
826      GO TO 260
827
828      210      IF (EARST.GT.T2) GO TO 230
829      IF (ADEN.EQ.O.ODO) GO TO 220
830      ANUM=T2-EARST
831      ALFT=ANUM/ADEN
832      ALFP=1 ODO
833      GO TO 260
834
835      220      ALFT=0 ODO
836      ALFP=1.ODO
837      GO TO 260
838
839      230      IF (EARST.GT.T3) GO TO 240
840      ALFT=0 ODO
841      ALFP=1.ODO
842      GO TO 260
843
844      240      IF (EARST.GT.T4) GO TO 250
845      IF (ADEN.EQ.O.ODO) GO TO 250
846      ANUM=T4-EARST
847      ALFP=ANUM/ADEN
848      ALFT=0 ODO
849      GO TO 260
850
851      250      ALFP=0 ODO
852      ALFT=0 ODO
853
854      C
855      C      CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
856      C      TO FACE
857      C
858      260      CALL FWCALC(BCL,BF1,TF1,E2)
859      FAC1:(YSFB+(ESH-EYBF)*E1+(EARST-T4)*E2)*FE*STP
860      FAC2:ADEN+E2*FE*STR
861      FAC3:(YSFB+(EARST-T2)*E1)*FE*STR
862      FAC4:ADEN+E1*FE*STR
863      FAC5:(IRSTEA+ERTB)*E*FE*STR
864      FAC6:ADEN+E*FE*STR
865
866      C
867      C      FORMULATE STIFFNESS SUBMATRICES FOR
868      C      STRAIN-HARDENED REGION
869      C
870      C
871      A=-1.ODO
872      B:-ALFP
873      IF (DABS(A-B).LT.1.OD-5) GO TO 280
874      CALL PHI(B,A,FI,CB,5,6,11)
875      CALL PHIYY(B,A,FIYY,CB,5,6,7)
876      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
877      CALL PHII(B,A,FIY,CB,5,6,9)
878      CALL EPHI(B,A,EFI,CB,5,6,12)
879      DO 270 I=1,5
880      DO 270 J=1,5
881      FB1(I,J):FA*FI(I,J)
882      FB2(I,J):FB*FIYY(I,J)
883      FB3(I,J):-FC*FIFI(I,J)

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904      FB4(I,J)=FD*FIY(I,J)
905      270  FB5(I,J)=FAC1*FI(I,J)-FAC2*EF1(I,J)
906      C
907      C
908      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
909      C
910      280  A=-ALFP
911      B=-ALFT
912      IF (DABS(A-B).LT.1.0D-5) GO TO 300
913      CALL FWCALC(BCL,BF1,TF1,E1)
914      CALL PHI(B,A,FI,CB,5,6,11)
915      CALL PHIYY(B,A,FIYY,CB,5,6,7)
916      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
917      CALL PHIY(B,A,FIY,CB,5,6,8)
918      CALL EPHI(B,A,EFI,CB,5,6,12)
919      DO 290 I=1,5
920      DO 290 J=1,5
921      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
922      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
923      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
924      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
925      290  FB5(I,J)=FB5(I,J)+FAC3*FI(I,J)-FAC4*EF1(I,J)
926      C
927      C
928      C      FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION
929      C
930      300  A=-ALFT
931      B=0.000
932      IF (DABS(A-B).LT.1.0D-5) GO TO 320
933      CALL FWCALC(BCL,BF1,TF1,E)
934      CALL PHI(B,A,FI,CB,5,6,11)
935      CALL PHIYY(B,A,FIYY,CB,5,6,7)
936      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
937      CALL PHIY(B,A,FIY,CB,5,6,8)
938      CALL EPHI(B,A,EFI,CB,5,6,12)
939      DO 310 I=1,5
940      DO 310 J=1,5
941      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
942      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
943      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
944      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
945      310  FB5(I,J)=FB5(I,J)+FAC5*FI(I,J)-FAC6*EF1(I,J)
946      320  A=0.000
947      B=ALFT
948      IF (DABS(A-B).LT.1.0D-5) GO TO 340
949      CALL FWCALC(BCL,BF1,TF1,E)
950      CALL PHI(B,A,FI,CB,5,6,11)
951      CALL PHIYY(B,A,FIYY,CB,5,6,7)
952      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
953      CALL PHIY(B,A,FIY,CB,5,6,8)
954      CALL EPHI(B,A,EFI,CB,5,6,12)
955      DO 330 I=1,5
956      DO 330 J=1,5
957      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
958      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
959      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
960      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
961      330  FB5(I,J)=FB5(I,J)+FAC5*FI(I,J)+FAC6*EF1(I,J)
962      C
963      C
964      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
965      C
966      340  A=ALFT
967      B=ALFP
968      IF (DABS(A-B).LT.1.0D-5) GO TO 360
969      CALL FWCALC(BCL,BF1,TF1,E1)
970      CALL PHI(B,A,FI,CB,5,6,11)
971      CALL PHIYY(B,A,FIYY,CB,5,6,7)
972      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
973      CALL PHIY(B,A,FIY,CB,5,6,8)
974      CALL EPHI(B,A,EFI,CB,5,6,12)
975      DO 350 I=1,5
976      DO 350 J=1,5
977      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
978      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
979      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
980      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
981      350  FB5(I,J)=FB5(I,J)+FAC3*FI(I,J)+FAC4*EF1(I,J)
982      C
983      C
984      C      FORMULATE STIFFNESS SUBMATRICES FOR
985      C      STRAIN-HARDENED REGION.
986      C
987      360  A=ALFP
988      B=1.000
989      IF (DABS(A-B).LT.1.0D-5) GO TO 380
990      CALL FWCALC(BCL,BF1,TF1,E2)
991      CALL PHI(B,A,FI,CB,5,6,11)
992      CALL PHIYY(B,A,FIYY,CB,5,6,7)
993      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
994      CALL PHIY(B,A,FIY,CB,5,6,8)
995      CALL EPHI(B,A,EFI,CB,5,6,12)
996      DO 370 I=1,5
997      DO 370 J=1,5
998      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
999      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1000     FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1001     FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1002     370  FB5(I,J)=FB5(I,J)+FAC1*FI(I,J)+FAC2*EF1(I,J)
1003     380  TF1=TSAVE
1004     RETURN
1005     END
1006     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1007     C
1008     C
1009     C      SUBROUTINE 9
1010     C
1011     C
1012     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1013     C
1014     C      SUBROUTINE FBINBE
1015     C
1016     C      THIS ROUTINE FORMULATES THE STIFFNESS
1017     C

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1017 C          SUBMATRICES FB1, FB2, FB3, FB4 AND FB5 FOR THE C
1018 C          BOTTOM (TENSION) FLANGE WHEN THE SPECIMEN IS C
1019 C          SUBJECTED TO BENDING STRESSES. C
1020 C C
1021 C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1022 C SUBROUTINE FBINBE(ST)
1023 C IMPLICIT REAL*8(A-H,O-Z)
1024 C DIMENSION F1(7,7),F1YY(7,7),F1F1(7,7),F1Y(7,7),EF1(7,7)
1025 C COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
1026 C COMMON /BLK2/CB(7,5),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5),FB5(5,5)
1027 C COMMON /BLK4/FA,FB,FC,FD,FE
1028 C COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
1029 C COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
1030 C COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
1031 C COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
1032 C
1033 C
1034 C          INITIALISE THE STIFFNESS SUBMATRICES TO ZERO.
1035 C
1036 C          DO 10 I=1,5
1037 C          DO 10 J=1,5
1038 C          FB1(I,J)=0.0DO
1039 C          FB2(I,J)=0.0DO
1040 C          FB3(I,J)=0.0DO
1041 C          FB4(I,J)=0.0DO
1042 C          FB5(I,J)=0.0DO
1043 C
1044 C
1045 C          ADJUST FLANGE THICKNESS TO ACCOUNT FOR
1046 C          WEB-TO-FLANGE FILLETS
1047 C
1048 C          TSAVE=TF1
1049 C          TF1=BF1*TF1/(BF1-2.0DO*TW)
1050 C
1051 C
1052 C          INITIALISE STRAINS AND STRAIN LIMITS, T1, T2, T3,
1053 C          T4.
1054 C
1055 C          STR=1.0DO/ST
1056 C          RST=R*ST
1057 C          T1=EYBF-ERTB
1058 C          T2=EYBF+ERCB
1059 C          T3=ESH-ERTB
1060 C          T4=ESH+ERCB
1061 C          AD1=ERTB+ERCB
1062 C
1063 C
1064 C          EVALUATE PARAMETERS ALFB AND ALFBP WHICH INDICATE
1065 C          EXTENTS OF YIELDING AND STRAIN HARDENING,
1066 C          RESPECTIVELY
1067 C
1068 C          IF (RST.GT.T1) GO TO 20
1069 C          ALFB=0.0DO
1070 C          ALFBP=0.0DO
1071 C          GO TO 70
1072 C          IF (RST.GT.T2) GO TO 40
1073 C          IF (AD1.EQ.0.0DO) GO TO 30
1074 C          AN1=RST-T1
1075 C          ALFB=AN1/AD1
1076 C          ALFBP=0.0DO
1077 C          GO TO 70
1078 C          ALFB=1.0DO
1079 C          ALFBP=0.0DO
1080 C          GO TO 70
1081 C          IF (RST.GT.T3) GO TO 50
1082 C          ALFB=1.0DO
1083 C          ALFBP=0.0DO
1084 C          GO TO 70
1085 C          IF (RST.GT.T4) GO TO 60
1086 C          IF (AD1.EQ.0.0DO) GO TO 60
1087 C          AN1=RST-T3
1088 C          ALFBP=AN1/AD1
1089 C          ALFB=1.0DO
1090 C          GO TO 70
1091 C          ALFB=1.0DO
1092 C          ALFBP=1.0DO
1093 C
1094 C
1095 C          CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
1096 C          TO FAC6.
1097 C
1098 C          CALL FWCALC(BCL,BF1,TF1,E)
1099 C          FAC1=(RST+ERTB)*E*FE
1100 C          FAC2=AD1*E*FE
1101 C          FAC3=(YSFB-(T1-RST)*E1)*FE
1102 C          FAC4=AD1*E1*FE
1103 C          FAC5=(YSFB+(ESH-EYBF)*E1-(T3-RST)*E2)*FE
1104 C          FAC6=AD1*E2*FE
1105 C
1106 C
1107 C          FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION
1108 C
1109 C          A=-1.0DO
1110 C          B=-ALFB
1111 C          IF (DABS(A-B).LT.1.0D-5) GO TO 90
1112 C          CALL PH1Y(B,A,F1,CB,5,6,11)
1113 C          CALL PH1Y(B,A,F1YY,CB,5,6,7)
1114 C          CALL PH1PH1(B,A,F1F1,CB,5,6,9)
1115 C          CALL PH1Y(B,A,F1Y,CB,5,6,9)
1116 C          CALL EPHI(B,A,EF1,CB,5,6,12)
1117 C          DO 80 I=1,5
1118 C          DO 80 J=1,5
1119 C          FB1(I,J)=FA*F1(I,J)
1120 C          FB2(I,J)=FB*F1YY(I,J)
1121 C          FB3(I,J)=-FC*F1F1(I,J)
1122 C          FB4(I,J)=FD*F1Y(I,J)
1123 C          FB5(I,J)=-FAC1*F1(I,J)-FAC2*EF1(I,J)
1124 C
1125 C
1126 C          FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1127 C
1128 C          A=-ALFB
1129 C          B=-ALFBP

```



```

1130 IF (DABS(A-B).LT.1.0D-5) GO TO 110
1131 CALL FWCALC(BCL,BF1,TF1,E1)
1132 CALL PHI(B,A,F1,CB,5,6,11)
1133 CALL PHIYY(B,A,F1YY,CB,5,6,7)
1134 CALL PHIPHI(B,A,F1F1,CB,5,6,9)
1135 CALL PHIY(B,A,F1Y,CB,5,6,9)
1136 CALL EPHI(B,A,EF1,CB,5,6,12)
1137 DO 100 I=1,5
1138 DO 100 J=1,5
1139 FB1(I,J)=FB1(I,J)+FA*F1(I,J)
1140 FB2(I,J)=FB2(I,J)+FB*F1YY(I,J)
1141 FB3(I,J)=FB3(I,J)-FC*F1F1(I,J)
1142 FB4(I,J)=FB4(I,J)+FD*F1Y(I,J)
1143 100 FB5(I,J)=FB5(I,J)-FAC3*F1(I,J)-FAC4*EF1(I,J)
1144 C
1145 C
1146 FORMULATE STIFFNESS SUBMATRICES FOR
1147 STRAIN-HARDENED REGION.
1148 C
1149 110 A=-ALFBP
1150 B=0.0D0
1151 IF (DABS(A-B).LT.1.0D-5) GO TO 130
1152 CALL FWCALC(BCL,BF1,TF1,E2)
1153 CALL PHI(B,A,F1,CB,5,6,11)
1154 CALL PHIYY(B,A,F1YY,CB,5,6,7)
1155 CALL PHIPHI(B,A,F1F1,CB,5,6,9)
1156 CALL PHIY(B,A,F1Y,CB,5,6,9)
1157 CALL EPHI(B,A,EF1,CB,5,6,12)
1158 DO 120 I=1,5
1159 DO 120 J=1,5
1160 FB1(I,J)=FB1(I,J)+FA*F1(I,J)
1161 FB2(I,J)=FB2(I,J)+FB*F1YY(I,J)
1162 FB3(I,J)=FB3(I,J)-FC*F1F1(I,J)
1163 FB4(I,J)=FB4(I,J)+FD*F1Y(I,J)
1164 120 FB5(I,J)=FB5(I,J)-FAC5*F1(I,J)-FAC6*EF1(I,J)
1165 130 A=0.0D0
1166 B=ALFBP
1167 IF (DABS(A-B).LT.1.0D-5) GO TO 150
1168 CALL PHI(B,A,F1,CB,5,6,11)
1169 CALL PHIYY(B,A,F1YY,CB,5,6,7)
1170 CALL PHIPHI(B,A,F1F1,CB,5,6,9)
1171 CALL PHIY(B,A,F1Y,CB,5,6,9)
1172 CALL EPHI(B,A,EF1,CB,5,6,12)
1173 DO 140 I=1,5
1174 DO 140 J=1,5
1175 FB1(I,J)=FB1(I,J)+FA*F1(I,J)
1176 FB2(I,J)=FB2(I,J)+FB*F1YY(I,J)
1177 FB3(I,J)=FB3(I,J)-FC*F1F1(I,J)
1178 FB4(I,J)=FB4(I,J)+FD*F1Y(I,J)
1179 140 FB5(I,J)=FB5(I,J)-FAC5*F1(I,J)+FAC6*EF1(I,J)
1180 C
1181 C
1182 FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1183 C
1184 150 A=ALFBP
1185 B=ALFB
1186 IF (DABS(A-B).LT.1.0D-5) GO TO 170
1187 CALL FWCALC(BCL,BF1,TF1,E1)
1188 CALL PHI(B,A,F1,CB,5,6,11)
1189 CALL PHIYY(B,A,F1YY,CB,5,6,7)
1190 CALL PHIPHI(B,A,F1F1,CB,5,6,9)
1191 CALL PHIY(B,A,F1Y,CB,5,6,9)
1192 CALL EPHI(B,A,EF1,CB,5,6,12)
1193 DO 160 I=1,5
1194 DO 160 J=1,5
1195 FB1(I,J)=FB1(I,J)+FA*F1(I,J)
1196 FB2(I,J)=FB2(I,J)+FB*F1YY(I,J)
1197 FB3(I,J)=FB3(I,J)-FC*F1F1(I,J)
1198 FB4(I,J)=FB4(I,J)+FD*F1Y(I,J)
1199 160 FB5(I,J)=FB5(I,J)-FAC3*F1(I,J)+FAC4*EF1(I,J)
1200 C
1201 C
1202 FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION
1203 C
1204 170 A=ALFB
1205 B=1.0D0
1206 IF (DABS(A-B).LT.1.0D-5) GO TO 190
1207 CALL FWCALC(BCL,BF1,TF1,E)
1208 CALL PHI(B,A,F1,CB,5,6,11)
1209 CALL PHIYY(B,A,F1YY,CB,5,6,7)
1210 CALL PHIPHI(B,A,F1F1,CB,5,6,9)
1211 CALL PHIY(B,A,F1Y,CB,5,6,9)
1212 CALL EPHI(B,A,EF1,CB,5,6,12)
1213 DO 180 I=1,5
1214 DO 180 J=1,5
1215 FB1(I,J)=FB1(I,J)+FA*F1(I,J)
1216 FB2(I,J)=FB2(I,J)+FB*F1YY(I,J)
1217 FB3(I,J)=FB3(I,J)-FC*F1F1(I,J)
1218 FB4(I,J)=FB4(I,J)+FD*F1Y(I,J)
1219 180 FB5(I,J)=FB5(I,J)-FAC1*F1(I,J)+FAC2*EF1(I,J)
1220 190 TF1=TSAVE
1221 RETURN
1222 END
1223 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1224 C
1225 C
1226 SUBROUTINE 10
1227 C
1228 C
1229 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1230 C
1231 SUBROUTINE FBINAX
1232 C
1233 THIS ROUTINE FORMULATES STIFFNESS
1234 SUBMATRICES FB1, FB2, FB3, FB4, AND FB5 FOR THE
1235 BOTTOM (TENSION) FLANGE WHEN THE SPECIMEN IS
1236 SUBJECTED TO AXIAL STRESSES.
1237 C
1238 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1239 SUBROUTINE FBINAX(ST)
1240 IMPLICIT REAL*8(A-H,O-Z)
1241 DIMENSION F1(7,7),F1YY(7,7),F1F1(7,7),F1Y(7,7),EF1(7,7)
1242 COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND

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1243 COMMON /BLK2/CB(7,9),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5),FB5(5,5)
1244 COMMON /BLK4/FA,FB,FC,FD,FE
1245 COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
1246 COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
1247 COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
1248
1249 C
1250 C INITIALISE THE STIFFNESS SUBMATRICES TO ZERO
1251 C
1252 DO 10 I=1,5
1253 DO 10 J=1,5
1254 FB1(I,J)=0.0DO
1255 FB2(I,J)=0.0DO
1256 FB3(I,J)=0.0DO
1257 FB4(I,J)=0.0DO
1258 FB5(I,J)=0.0DO
1259 10 C
1260 C
1261 C ADJUST FLANGE THICKNESS TO ACCOUNT FOR
1262 C WEB-TO-FLANGE FILLETS
1263 C
1264 TSAVE=TF1
1265 TF1=BF1*TF1/(BF1-2.0DO*TW)
1266 C
1267 C
1268 C INITIALISE STRAINS AND STRAIN LIMITS, T1, T2, T3,
1269 C T4.
1270 C
1271 STR=1.0DO/ST
1272 T1=EYBF-ERCB
1273 T2=EYBF+ERTB
1274 T3=ESH-ERCB
1275 T4=ESH+ERTB
1276 C
1277 C
1278 C EVALUATE PARAMETERS ALFT AND ALFP WHICH INDICATE
1279 C EXTENTS OF YIELDING AND STRAIN HARDENING,
1280 C RESPECTIVELY
1281 C
1282 ADEN=ERCB+ERTB
1283 IF (ST.GT.T1) GO TO 20
1284 ALFT=1.0DO
1285 ALFP=1.0DO
1286 GO TO 70
1287 20 IF (ST.GT.T2) GO TO 40
1288 IF (ADEN.EQ.0.0DO) GO TO 30
1289 ANUM=T2-ST
1290 ALFT=ANUM/ADEN
1291 ALFP=1.0DO
1292 GO TO 70
1293 30 ALFT=0.0DO
1294 ALFP=1.0DO
1295 GO TO 70
1296 40 IF (ST.GT.T3) GO TO 50
1297 ALFT=0.0DO
1298 ALFP=1.0DO
1299 GO TO 70
1300 50 IF (ST.GT.T4) GO TO 60
1301 IF (ADEN.EQ.0.0DO) GO TO 60
1302 ANUM=T4-ST
1303 ALFT=ANUM/ADEN
1304 ALFP=0.0DO
1305 GO TO 70
1306 60 ALFP=0.0DO
1307 ALFT=0.0DO
1308 C
1309 C
1310 C CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
1311 C TO FAC6.
1312 C
1313 70 CALL FWCALC(BCL,BF1,TF1,E2)
1314 FAC1=(YSFB+(ESH-EYBF)*E1+(ST-T4)*E2)*FE
1315 FAC2=ADEN*E2*FE
1316 FAC3=(YSFB+(ST-T2)*E1)*FE
1317 FAC4=ADEN*E1*FE
1318 FAC5=(ST-ERTB)*E*FE
1319 FAC6=ADEN*E*FE
1320 C
1321 C
1322 C FORMULATE STIFFNESS SUBMATRICES FOR
1323 C STRAIN-HARDENED REGION
1324 C
1325 A=-1.0DO
1326 B=-ALFP
1327 IF (DABS(A-B).LT.1.0D-5) GO TO 90
1328 CALL PHI(E,A,F1,CB,5,6,11)
1329 CALL PH1YY(B,A,F1YY,CB,5,6,7)
1330 CALL PH1PH1(B,A,F1F1,CB,5,6,9)
1331 CALL PH1Y(B,A,F1Y,CB,5,6,9)
1332 CALL EPH1(B,A,EF1,CB,5,6,12)
1333 DO 80 I=1,5
1334 DO 80 J=1,5
1335 FB1(I,J)=FA*F1(I,J)
1336 FB2(I,J)=FB*F1YY(I,J)
1337 FB3(I,J)=FC*F1F1(I,J)
1338 FB4(I,J)=FD*F1Y(I,J)
1339 FB5(I,J)=FAC1*F1(I,J)-FAC2*EF1(I,J)
1340 80 C
1341 C
1342 C FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1343 C
1344 90 A=-ALFP
1345 B=-ALFT
1346 IF (DABS(A-B).LT.1.0D-5) GO TO 110
1347 CALL FWCALC(BCL,BF1,TF1,E1)
1348 CALL PHI(B,A,F1,CB,5,6,11)
1349 CALL PH1YY(B,A,F1YY,CB,5,6,7)
1350 CALL PH1PH1(B,A,F1F1,CB,5,6,9)
1351 CALL PH1Y(B,A,F1Y,CB,5,6,9)
1352 CALL EPH1(B,A,EF1,CB,5,6,12)
1353 DO 100 I=1,5
1354 DO 100 J=1,5
1355 FB1(I,J)=FB1(I,J)+FA*F1(I,J)

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1356      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1357      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1358      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1359      FB5(I,J)=FB5(I,J)+FAC3*FI(I,J)-FAC4*EFI(I,J)
1360
1361      C
1362      C      FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION.
1363      C
1364      110      A=ALFT
1365      B=0.000
1366      IF (DABS(A-B).LT.1.0D-5) GO TO 130
1367      CALL FWCALC(BCL,BF1,TF1,E)
1368      CALL PHI(B,A,FI,CB,5,6,11)
1369      CALL PHIYY(B,A,FIYY,CB,5,6,7)
1370      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
1371      CALL PHIY(B,A,FIY,CB,5,6,8)
1372      CALL EPHI(B,A,EFI,CB,5,6,12)
1373      DO 120 I=1,5
1374      DO 120 J=1,5
1375      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
1376      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1377      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1378      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1379      FB5(I,J)=FB5(I,J)+FAC5*FI(I,J)-FAC6*EFI(I,J)
1380      A=0.000
1381      130      B=ALFT
1382      IF (DABS(A-B).LT.1.0D-5) GO TO 150
1383      CALL FWCALC(BCL,BF1,TF1,E)
1384      CALL PHI(B,A,FI,CB,5,6,11)
1385      CALL PHIYY(B,A,FIYY,CB,5,6,7)
1386      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
1387      CALL PHIY(B,A,FIY,CB,5,6,8)
1388      CALL EPHI(B,A,EFI,CB,5,6,12)
1389      DO 140 I=1,5
1390      DO 140 J=1,5
1391      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
1392      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1393      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1394      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1395      FB5(I,J)=FB5(I,J)+FAC5*FI(I,J)+FAC6*EFI(I,J)
1396
1397      C
1398      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1399      C
1400      150      A=ALFT
1401      B=ALFP
1402      IF (DABS(A-B).LT.1.0D-5) GO TO 170
1403      CALL FWCALC(BCL,BF1,TF1,E)
1404      CALL PHI(B,A,FI,CB,5,6,11)
1405      CALL PHIYY(B,A,FIYY,CB,5,6,7)
1406      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
1407      CALL PHIY(B,A,FIY,CB,5,6,8)
1408      CALL EPHI(B,A,EFI,CB,5,6,12)
1409      DO 160 I=1,5
1410      DO 160 J=1,5
1411      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
1412      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1413      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1414      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1415      FB5(I,J)=FB5(I,J)+FAC3*FI(I,J)+FAC4*EFI(I,J)
1416
1417      C
1418      C      FORMULATE STIFFNESS SUBMATRICES FOR
1419      C      STRAIN-HARDENED REGION.
1420      C
1421      170      A=ALFP
1422      B=1.000
1423      IF (DABS(A-B).LT.1.0D-5) GO TO 190
1424      CALL FWCALC(BCL,BF1,TF1,E2)
1425      CALL PHI(B,A,FI,CB,5,6,11)
1426      CALL PHIYY(B,A,FIYY,CB,5,6,7)
1427      CALL PHIPHI(B,A,FIFI,CB,5,6,9)
1428      CALL PHIY(B,A,FIY,CB,5,6,8)
1429      CALL EPHI(B,A,EFI,CB,5,6,12)
1430      DO 180 I=1,5
1431      DO 180 J=1,5
1432      FB1(I,J)=FB1(I,J)+FA*FI(I,J)
1433      FB2(I,J)=FB2(I,J)+FB*FIYY(I,J)
1434      FB3(I,J)=FB3(I,J)-FC*FIFI(I,J)
1435      FB4(I,J)=FB4(I,J)+FD*FIY(I,J)
1436      FB5(I,J)=FB5(I,J)+FAC1*FI(I,J)+FAC2*EFI(I,J)
1437      180      TF1=TSAVE
1438      190      RETURN
1439      END
1440
1441      C
1442      C      SUBROUTINE 11
1443      C
1444      C
1445      C
1446      C      SUBROUTINE FBFORM
1447      C
1448      C      THIS ROUTINE FORMULATES THE BENDING AND
1449      C      GEOMETRIC STIFFNESS SUBMATRICES FB1, FB2, FB3,
1450      C      FB4, AND FB5 FOR THE THE BOTTOM FLANGE IN THE
1451      C      ELASTIC RANGE.
1452      C
1453      C
1454      C      SUBROUTINE FBFORM
1455      C      IMPLICIT REAL*8(A-H,O-Z)
1456      C      DIMENSION FI(7,7),FIYY(7,7),FIFI(7,7),FIY(7,7),EFI(7,7)
1457      C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
1458      C      COMMON /BLK2/CB(7,9),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5),FB5(5,5)
1459      C      COMMON /BLK4/FA,FB,FC,FD,FE
1460      C      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
1461      C      COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
1462      C      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
1463
1464      C
1465      C      ADJUST FLANGE THICKNESS TO ACCOUNT FOR
1466      C      WEB-TD-FLANGE FILLETS
1467
1468

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1469 C
1470 IF (ITYPE.EQ.7) TSAVE=TF1
1471 IF (ITYPE.EQ.7) TF1=BF1*TF1/(BF1-2.0DO*TW)
1472 C
1473 C
1474 C CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
1475 C AND FAC2.
1476 C
1477 CALL FWCALC(BCL,BF1,TF1,E)
1478 IF (ITYPE.EQ.7) TF1=TSAVE
1479 FAC1=E*(ERTB+ERCB)*FE
1480 FAC2=E*ERTB*FE
1481 C
1482 C
1483 C DEFINE LIMITS OF INTEGRATION, A AND B AND
1484 C FORMULATE ELASTIC STIFFNESS SUBMATRICES FOR
1485 C W-SHAPES.
1486 C
1487 B=1.0DO
1488 A=-1.0DO
1489 CALL PHI(B,A,F1,CB,5,6,11)
1490 CALL PHIY(B,A,FIY,CB,5,6,7)
1491 CALL PHIPHI(B,A,FIF1,CB,5,6,9)
1492 CALL PHIV(B,A,FIV,CB,5,6,8)
1493 IF (ITYPE.LT.7) GO TO 30
1494 CALL EPHI(0.0DO,A,EF1,CB,5,6,12)
1495 DO 10 I=1,5
1496 DO 10 J=1,5
1497 FB1(I,J)=FA*F1(I,J)
1498 FB2(I,J)=FB*FIY(I,J)
1499 FB3(I,J)=FC*FIF1(I,J)
1500 10 FB4(I,J)=FD*FIV(I,J)+FAC2*F1(I,J)+FAC1*EF1(I,J)
1501 CALL EPHI(B,0.0DO,EF1,CB,5,6,12)
1502 DO 20 I=1,5
1503 DO 20 J=1,5
1504 20 FB4(I,J)=FB4(I,J)-FAC1*EF1(I,J)
1505 GO TO (50,70,90,110),LTYPE
1506 C
1507 C
1508 C FORMULATE STIFFNESS SUBMATRICES FOR SINGLE PLATES
1509 C
1510 30 DO 40 I=1,5
1511 DO 40 J=1,5
1512 FB1(I,J)=FA*F1(I,J)
1513 FB2(I,J)=FB*FIY(I,J)
1514 FB3(I,J)=FC*FIF1(I,J)
1515 40 FB4(I,J)=FD*FIV(I,J)
1516 GO TO (50,70,90,110),LTYPE
1517 C
1518 C FORMULATE GEOMETRIC STIFFNESS SUBMATRICES
1519 C
1520 C
1521 C AXIAL LOAD
1522 C
1523 50 DO 60 I=1,5
1524 DO 60 J=1,5
1525 60 FB5(I,J)=E*FE*FI(I,J)
1526 RETURN
1527 C
1528 C
1529 C BENDING MOMENT.
1530 C
1531 70 RF=E*R*FE
1532 DO 80 I=1,5
1533 DO 80 J=1,5
1534 80 FB5(I,J)=RF*FI(I,J)
1535 RETURN
1536 C
1537 C
1538 C AXIAL LOAD PLUS BENDING MOMENT.
1539 C
1540 90 FEAL=E*FE*ALAR
1541 RF=E*R*FE
1542 DO 100 I=1,5
1543 DO 100 J=1,5
1544 100 FB4(I,J)=FB4(I,J)-FEAL*FI(I,J)
1545 FB5(I,J)=RF*FI(I,J)
1546 RETURN
1547 C
1548 C
1549 C ECCENTRIC AXIAL LOAD.
1550 C
1551 110 R1=AREA*ECC*HW
1552 P2=2.0DO*(ERTBAR-APEA+ECC*Y1)
1553 RFF=((R2-R1)/(R2+R1))*FE=E
1554 DO 120 I=1,5
1555 DO 120 J=1,5
1556 120 FB5(I,J)=RFF*FI(I,J)
1557 RETURN
1558 END
1559 C
1560 C
1561 C SUBROUTINE 12
1562 C
1563 C
1564 C
1565 C
1566 C
1567 C SUBROUTINE FTINAB
1568 C
1569 C THIS SUBROUTINE FORMULATES THE STIFFNESS
1570 C SUBMATRICES, FT1, FT2, FT3, FT4, FT5, FOR A
1571 C TOP (COMPRESSION) FLANGE WHEN A W-SHAPE IS
1572 C SUBJECTED TO A CONSTANT AXIAL LOAD AND A
1573 C VARIABLE SUPERIMPOSED BENDING MOMENT
1574 C
1575 C
1576 C SUBROUTINE FTINAB(ST,EAPP)
1577 IMPLICIT REAL*8(A-H,O-Z)
1578 DIMENSION FI(7,7),FIY(7,7),FIF1(7,7),FIV(7,7),EF1(7,7)
1579 COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
1580 COMMON /BLK4/FA,FB,FC,FD,FE
1581 COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF

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1582 COMMON /BLK8/CT(7,8),FT1(5,5),FT2(5,5),FT3(5,5),FT4(5,5),FT5(5,5)
1583 COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
1584 COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRGM,SRIT,SRCB,SRCT
1585
1586 C
1587 C INITIALISE THE STIFFNESS SUBMATRICES TO ZERO
1588 C
1589 DO 10 I=1,5
1590 DO 10 J=1,5
1591 FT1(I,J)=0.0DO
1592 FT2(I,J)=0.0DO
1593 FT3(I,J)=0.0DO
1594 FT4(I,J)=0.0DO
1595 FT5(I,J)=0.0DO
1596 10
1597 C
1598 C ADJUST FLANGE THICKNESS TO ACCOUNT FOR
1599 C WEB-TO-FLANGE FILLETS
1600 C
1601 TSAVE=TF2
1602 TF2=BF2*(TF2/(BF2-2.0DO+TW))
1603 C
1604 C
1605 C INITIALISE STRAINS AND STRAIN LIMITS, T1, T2, T3,
1606 C T4
1607 C
1608 STEA=ST+EAPP
1609 STR=1.0DO/STEA
1610 T1=EYTF-ERCT
1611 T2=EYTF+ERTT
1612 T3=ESH-ERCT
1613 T4=ESH+ERTT
1614 ADEN=ERCT+ERTT
1615 C
1616 C
1617 C EVALUATE PARAMETERS ALFT AND ALFP WHICH INDICATE
1618 C EXTENTS OF YIELDING AND STRAIN HARDENING,
1619 C RESPECTIVELY
1620 C
1621 IF (STEA.GT.T1) GO TO 20
1622 ALFT=1.0DO
1623 ALFP=1.0DO
1624 GO TO 70
1625 20 IF (STEA.GT.T2) GO TO 40
1626 IF (ADEN.EQ.0.0DO) GO TO 30
1627 ANUM=T2-STEA
1628 ALFT=ANUM/ADEN
1629 ALFP=1.0DO
1630 GO TO 70
1631 30 ALFT=0.0DO
1632 ALFP=1.0DO
1633 GO TO 70
1634 40 IF (STEA.GT.T3) GO TO 50
1635 ALFT=0.0DO
1636 ALFP=1.0DO
1637 GO TO 70
1638 50 IF (STEA.GT.T4) GO TO 60
1639 IF (ADEN.EQ.0.0DO) GO TO 50
1640 ANUM=T4-STEA
1641 ALFP=ANUM/ADEN
1642 ALFT=0.0DO
1643 GO TO 70
1644 60 ALFP=0.0DO
1645 ALFT=0.0DO
1646 C
1647 C
1648 C CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
1649 C AND FAC6.
1650 C
1651 70 CALL FWCALC(BCL,BF2,TF2,E2)
1652 FAC1=(YSFT+(ESH-EYTF)*E1+(STEA-T4)*E2)*FE*STR
1653 FAC2=ADEN+E2*FE*STR
1654 FAC3=(YSFT+(STEA-T2)*E1)*FE*STR
1655 FAC4=ADEN+E1*FE*STR
1656 FAC5=(STEA-ERTT)*E*FE*STR
1657 FAC6=ADEN+E*FE*STR
1658 C
1659 C
1660 C FORMULATE STIFFNESS SUBMATRICES FOR
1661 C STRAIN-HARDENED REGION.
1662 C
1663 A=-1.0DO
1664 B=-ALFP
1665 IF (DABS(A-B).LT.1.0D-5) GO TO 90
1666 CALL PH1(B,A,F1,CT,5,6,11)
1667 CALL PH1Y(B,A,F1Y,CT,5,6,7)
1668 CALL PH1PH1(B,A,F1F1,CT,5,6,9)
1669 CALL PH1Y(B,A,F1Y,CT,5,6,9)
1670 CALL EPH1(B,A,EF1,CT,5,6,12)
1671 DO 80 I=1,5
1672 DO 80 J=1,5
1673 FT1(I,J)=FA*F1(I,J)
1674 FT2(I,J)=FB*F1Y(I,J)
1675 FT3(I,J)=FC*F1F1(I,J)
1676 FT4(I,J)=FD*F1Y(I,J)
1677 FT5(I,J)=FAC1*F1(I,J)-FAC2*EF1(I,J)
1678 80
1679 C
1680 C FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1681 C
1682 90 A=-ALFP
1683 B=-ALFT
1684 IF (DABS(A-B).LT.1.0D-5) GO TO 110
1685 CALL FWCALC(BCL,BF2,TF2,E1)
1686 CALL PH1(B,A,F1,CT,5,6,11)
1687 CALL PH1Y(B,A,F1Y,CT,5,6,7)
1688 CALL PH1PH1(B,A,F1F1,CT,5,6,9)
1689 CALL PH1Y(B,A,F1Y,CT,5,6,9)
1690 CALL EPH1(B,A,EF1,CT,5,6,12)
1691 DO 100 I=1,5
1692 DO 100 J=1,5
1693 FT1(I,J)=FT1(I,J)+FA*F1(I,J)
1694 FT2(I,J)=FT2(I,J)+FB*F1Y(I,J)

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1606 DD 10 J=1,5
1608 FT1(I,J)=0.0DO
1610 FT2(I,J)=0.0DO
1611 FT3(I,J)=0.0DO
1612 FT4(I,J)=0.0DO
1613 FT5(I,J)=0.0DO
1614
1615
1616 ADJUST FLANGE THICKNESS TO ACCOUNT FOR
1617 WEB-TO-FLANGE FILLETS
1618
1619 TSAVE=TF2
1620 TF2=BF2*TF2/(BF2-2.0DO*TW)
1621
1622
1623 INITIALISE STRAINS AND STRAIN LIMITS, T1, T2, T3,
1624 T4.
1625
1626 STR=1.0DO/ST
1627 T1=EYTF-ERCT
1628 T2=EYTF+ERTT
1629 T3=ESH-ERCT
1630 T4=ESH+ERTT
1631
1632
1633 EVALUATE PARAMETERS ALFT AND ALFP WHICH INDICATE
1634 EXTENTS OF YIELDING AND STRAIN HARDENING,
1635 RESPECTIVELY.
1636
1637 ADEN=ERCT+ERTT
1638 IF (ST.GT.T1) GO TO 20
1639 ALFT=1.0DO
1640 ALFP=1.0DO
1641 GO TO 70
1642 IF (ST.GT.T2) GO TO 40
1643 IF (ADEN.EQ.0.0DO) GO TO 30
1644 ANUM=T2-ST
1645 ALFT=ANUM/ADEN
1646 ALFP=1.0DO
1647 GO TO 70
1648 IF (ST.GT.T3) GO TO 50
1649 ALFT=0.0DO
1650 ALFP=1.0DO
1651 GO TO 70
1652 IF (ST.GT.T4) GO TO 60
1653 IF (ADEN.EQ.0.0DO) GO TO 60
1654 ANUM=T4-ST
1655 ALFP=ANUM/ADEN
1656 ALFT=0.0DO
1657 GO TO 70
1658 ALFP=0.0DO
1659 ALFT=0.0DO
1660
1661
1662 CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
1663 AND FAC6.
1664
1665 CALL FWCALC(BCL,BF2,TF2,E2)
1666 FAC1=(YSFT+(ESH-EYTF)*E1+(ST-T4)*E2)=FE
1667 FAC2=ADEN+E2=FE
1668 FAC3=(YSFT+(ST-T2)*E1)*FE
1669 FAC4=ADEN+E1=FE
1670 FAC5=(ST-ERTT)*E*FE
1671 FAC6=ADEN+E*FE
1672
1673
1674 FORMULATE STIFFNESS SUBMATRICES FOR
1675 STRAIN-HARDENED REGION.
1676
1677
1678 A=-1.0DO
1679 B=-ALFP
1680 IF (DABS(A-B).LT.1.0D-5) GO TO 90
1681 CALL PHI(B,A,FI,CT,5,6,11)
1682 CALL PHIYY(B,A,FIYY,CT,5,6,7)
1683 CALL PHIPHI(B,A,FI,CT,5,6,9)
1684 CALL PHIV(B,A,FI,CT,5,6,8)
1685 CALL EPHI(B,A,EFI,CT,5,6,12)
1686 DO 80 I=1,5
1687 DO 80 J=1,5
1688 FT1(I,J)=FA*FI(I,J)
1689 FT2(I,J)=FB*FIYY(I,J)
1690 FT3(I,J)=FC*FI(I,J)
1691 FT4(I,J)=FD*FI(I,J)
1692 FT5(I,J)=FAC1*FI(I,J)-FAC2*EFI(I,J)
1693
1694 FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1695
1696
1697 A=-ALFP
1698 B=-ALFT
1699 IF (DABS(A-B).LT.1.0D-5) GO TO 110
1700 CALL FWCALC(BCL,BF2,TF2,E1)
1701 CALL PHI(B,A,FI,CT,5,6,11)
1702 CALL PHIYY(B,A,FIYY,CT,5,6,7)
1703 CALL PHIPHI(B,A,FI,CT,5,6,9)
1704 CALL PHIV(B,A,FI,CT,5,6,8)
1705 CALL EPHI(B,A,EFI,CT,5,6,12)
1706 DO 100 I=1,5
1707 DO 100 J=1,5
1708 FT1(I,J)=FT1(I,J)+FA*FI(I,J)
1709 FT2(I,J)=FT2(I,J)+FB*FIYY(I,J)
1710 FT3(I,J)=FT3(I,J)+FC*FI(I,J)
1711 FT4(I,J)=FT4(I,J)+FD*FI(I,J)
1712 FT5(I,J)=FT5(I,J)+FAC3*FI(I,J)-FAC4*EFI(I,J)
1713
1714 FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION.
1715
1716
1717 A=-ALFT
1718 B=0.0DO
1719
1720

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1921      IF (DABS(A-B).LT.1.0D-5) GO TO 130
1922      CALL FWCALC(BCL,BF2,TF2,E)
1923      CALL PHI(B,A,F1,CT,5,6,11)
1924      CALL PHIYY(B,A,FIYY,CT,5,6,7)
1925      CALL PHIPHI(B,A,FIFI,CT,5,6,9)
1926      CALL PHIY(B,A,FIY,CT,5,6,9)
1927      CALL EPHI(B,A,EFI,CT,5,6,12)
1928      DO 120 I=1,5
1929      DO 120 J=1,5
1930      FT1(I,J)=FT1(I,J)+FA*FI(I,J)
1931      FT2(I,J)=FT2(I,J)+FB*FIYY(I,J)
1932      FT3(I,J)=FT3(I,J)-FC*FIFI(I,J)
1933      FT4(I,J)=FT4(I,J)+FD*FIY(I,J)
1934      120 FT5(I,J)=FT5(I,J)+FAC5*FI(I,J)-FAC6*EFI(I,J)
1935      130 A=0.0D0
1936      B=ALFT
1937      IF (DABS(A-B).LT.1.0D-5) GO TO 150
1938      CALL FWCALC(BCL,BF2,TF2,E)
1939      CALL PHI(B,A,F1,CT,5,6,11)
1940      CALL PHIYY(B,A,FIYY,CT,5,6,7)
1941      CALL PHIPHI(B,A,FIFI,CT,5,6,9)
1942      CALL PHIY(B,A,FIY,CT,5,6,9)
1943      CALL EPHI(B,A,EFI,CT,5,6,12)
1944      DO 140 I=1,5
1945      DO 140 J=1,5
1946      FT1(I,J)=FT1(I,J)+FA*FI(I,J)
1947      FT2(I,J)=FT2(I,J)+FB*FIYY(I,J)
1948      FT3(I,J)=FT3(I,J)-FC*FIFI(I,J)
1949      FT4(I,J)=FT4(I,J)+FD*FIY(I,J)
1950      140 FT5(I,J)=FT5(I,J)+FAC5*FI(I,J)+FAC6*EFI(I,J)
1951      C
1952      C
1953      C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
1954      C
1955      150 A=ALFT
1956      B=ALFP
1957      IF (DABS(A-B).LT.1.0D-5) GO TO 170
1958      CALL FWCALC(BCL,BF2,TF2,E1)
1959      CALL PHI(B,A,F1,CT,5,6,11)
1960      CALL PHIYY(B,A,FIYY,CT,5,6,7)
1961      CALL PHIPHI(B,A,FIFI,CT,5,6,9)
1962      CALL PHIY(B,A,FIY,CT,5,6,9)
1963      CALL EPHI(B,A,EFI,CT,5,6,12)
1964      DO 160 I=1,5
1965      DO 160 J=1,5
1966      FT1(I,J)=FT1(I,J)+FA*FI(I,J)
1967      FT2(I,J)=FT2(I,J)+FB*FIYY(I,J)
1968      FT3(I,J)=FT3(I,J)-FC*FIFI(I,J)
1969      FT4(I,J)=FT4(I,J)+FD*FIY(I,J)
1970      160 FT5(I,J)=FT5(I,J)+FAC3*FI(I,J)+FAC4*EFI(I,J)
1971      C
1972      C
1973      C      FORMULATE STIFFNESS SUBMATRICES FOR
1974      C      STRAIN-HARDENED REGION.
1975      C
1976      170 A=ALFP
1977      B=1.0D0
1978      IF (DABS(A-B).LT.1.0D-5) GO TO 190
1979      CALL FWCALC(BCL,BF2,TF2,E2)
1980      CALL PHI(B,A,F1,CT,5,6,11)
1981      CALL PHIYY(B,A,FIYY,CT,5,6,7)
1982      CALL PHIPHI(B,A,FIFI,CT,5,6,9)
1983      CALL PHIY(B,A,FIY,CT,5,6,9)
1984      CALL EPHI(B,A,EFI,CT,5,6,12)
1985      DO 180 I=1,5
1986      DO 180 J=1,5
1987      FT1(I,J)=FT1(I,J)+FA*FI(I,J)
1988      FT2(I,J)=FT2(I,J)+FB*FIYY(I,J)
1989      FT3(I,J)=FT3(I,J)-FC*FIFI(I,J)
1990      FT4(I,J)=FT4(I,J)+FD*FIY(I,J)
1991      180 FT5(I,J)=FT5(I,J)+FAC1*FI(I,J)+FAC2*EFI(I,J)
1992      190 TF2=TSAVE
1993      RETURN
1994      END
1995      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
1996      C
1997      C
1998      C      SUBROUTINE 14
1999      C
2000      C
2001      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
2002      C
2003      C      SUBROUTINE FTINAX
2004      C
2005      C      THIS ROUTINE FORMULATES THE STIFFNESS
2006      C      SUBMATRICES FT1, FT2, FT3, FT4, AND FT5 FOR
2007      C      THE TOP FLANGE WHEN THE SPECIMEN IS SUBJECTED
2008      C      TO AXIAL STRESSES.
2009      C
2010      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
2011      C      SUBROUTINE FTINAX(ST)
2012      C      IMPLICIT REAL*8(A-H,O-Z)
2013      C      DIMENSION F1(7,7),FIYY(7,7),FIFI(7,7),FIY(7,7),EFI(7,7)
2014      C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
2015      C      COMMON /BLK4/FA,FB,FC,FD,FE
2016      C      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
2017      C      COMMON /BLK6/CT(7,8),FT1(5,5),FT2(5,5),FT3(5,5),FT4(5,5),FT5(5,5)
2018      C      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
2019      C      COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
2020      C
2021      C
2022      C      INITIALISE THE STIFFNESS SUBMATRICES TO ZERO.
2023      C
2024      DO 10 I=1,5
2025      DO 10 J=1,5
2026      FT1(I,J)=0.0D0
2027      FT2(I,J)=0.0D0
2028      FT3(I,J)=0.0D0
2029      FT4(I,J)=0.0D0
2030      10 FT5(I,J)=0.0D0
2031      C
2032      C
2033      C      ADJUST FLANGE THICKNESS TO ACCOUNT FOR

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2034 C      WEB-TO-FLANGE FILLETS
2035 C
2036 TSAVE=TF2
2037 TF2=BF2*TF2/(BF2-2 ODO*TW)
2038 C
2039 C      INITIAL SE STRAINS AND STRAIN LIMITS, T1, T2, T3,
2040 C      T4.
2041 C
2042 C
2043 STR=1 ODO/ST
2044 T1=EYTF-ERCT
2045 T2=EYTF+ERTT
2046 T3=ESH-ERCT
2047 T4=ESH+ERTT
2048 C
2049 C
2050 C      EVALUATE PARAMETERS ALFT AND ALFP WHICH INDICATE
2051 C      EXTENTS OF YIELDING AND STRAIN HARDENING,
2052 C      RESPECTIVELY.
2053 C
2054 ADEN=ERCT+ERTT
2055 IF (ST GT T1) GO TO 20
2056 ALFT=1 ODO
2057 ALFP=1 ODO
2058 GO TO 70
2059 20 IF (ST GT T2) GO TO 40
2060 IF (ADEN EQ 0 ODO) GO TO 30
2061 ANUM=T2-ST
2062 ALFT=ANUM/ADEN
2063 ALFP=1 ODO
2064 GO TO 70
2065 30 ALFT=0 ODO
2066 ALFP=1 ODO
2067 GO TO 70
2068 40 IF (ST GT T3) GO TO 50
2069 ALFT=0 ODO
2070 ALFP=1 ODO
2071 GO TO 70
2072 50 IF (ST GT T4) GO TO 60
2073 IF (ADEN EQ 0 ODO) GO TO 60
2074 ANUM=T4-ST
2075 ALFP=ANUM/ADEN
2076 ALFT=0 ODO
2077 GO TO 70
2078 60 ALFP=0 ODO
2079 ALFT=0 ODO
2080 C
2081 C
2082 C      CALCULATE GEOMETRIC SUBMATRIX COEFFICIENTS, FAC1
2083 C      AND FAC6.
2084 C
2085 70 CALL FWCALC(BCL,BF2,TF2,E2)
2086 FAC1=(YSFT+ESH-EYTF)*E1*(ST-T4)*E2)=FE
2087 FAC2=ADEN+E2*FE
2088 FAC3=(YSFT*(ST-T2)*E1)=FE
2089 FAC4=ADEN+E1*FE
2090 FAC5=(ST-ERTT)*E*FE
2091 FAC6=ADEN+E*FE
2092 C
2093 C
2094 C      FORMULATE STIFFNESS SUBMATRICES FOR
2095 C      STRAIN-HARDENED REGION.
2096 C
2097 A=-1 ODO
2098 B=-ALFP
2099 IF (DABS(A-B).LT.1.OE-5) GO TO 90
2100 CALL PHI(B,A,F1,CT,5,6,11)
2101 CALL PHIYY(B,A,F1YY,CT,5,6,7)
2102 CALL PHIPHI(B,A,F1F1,CT,5,6,9)
2103 CALL PHIY(B,A,F1Y,CT,5,6,8)
2104 CALL EPHI(B,A,EF1,CT,5,6,12)
2105 DO 80 I=1,5
2106 DO 80 J=1,5
2107 FT1(I,J)=FA*F1(I,J)
2108 FT2(I,J)=FB*F1YY(I,J)
2109 FT3(I,J)=-FC*F1F1(I,J)
2110 FT4(I,J)=FD*F1Y(I,J)
2111 80 FT5(I,J)=FAC1*F1(I,J)-FAC2*EF1(I,J)
2112 C
2113 C
2114 C      FORMULATE STIFFNESS SUBMATRICES FOR YIELDED REGION
2115 C
2116 90 A=-ALFP
2117 B=-ALFT
2118 IF (DABS(A-B).LT.1.OE-5) GO TO 110
2119 CALL FWCALC(BCL,BF2,TF2,E1)
2120 CALL PHI(B,A,F1,CT,5,6,11)
2121 CALL PHIYY(B,A,F1YY,CT,5,6,7)
2122 CALL PHIPHI(B,A,F1F1,CT,5,6,9)
2123 CALL PHIY(B,A,F1Y,CT,5,6,8)
2124 CALL EPHI(B,A,EF1,CT,5,6,12)
2125 DO 100 J=1,5
2126 DO 100 I=1,5
2127 FT1(I,J)=FT1(I,J)+FA*F1(I,J)
2128 FT2(I,J)=FT2(I,J)+FB*F1YY(I,J)
2129 FT3(I,J)=FT3(I,J)-FC*F1F1(I,J)
2130 FT4(I,J)=FT4(I,J)+FD*F1Y(I,J)
2131 100 FT5(I,J)=FT5(I,J)+FAC3*F1(I,J)-FAC4*EF1(I,J)
2132 C
2133 C
2134 C      FORMULATE STIFFNESS SUBMATRICES FOR ELASTIC REGION.
2135 C
2136 110 A=-ALFT
2137 B=0 ODO
2138 IF (DABS(A-B).LT.1.OE-5) GO TO 130
2139 CALL FWCALC(BCL,BF2,TF2,E)
2140 CALL PHI(B,A,F1,CT,5,6,11)
2141 CALL PHIYY(B,A,F1YY,CT,5,6,7)
2142 CALL PHIPHI(B,A,F1F1,CT,5,6,9)
2143 CALL PHIY(B,A,F1Y,CT,5,6,8)
2144 CALL EPHI(B,A,EF1,CT,5,6,12)
2145 DO 120 J=1,5
2146 DO 120 I=1,5

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2260      CALL PHIPH1(B,A,FI1,CT,5,6,9)
2261      CALL PH1Y(E,A,FIY,CT,5,6,9)
2262      IF (ITYPE.LT.7) GO TO 30
2263      CALL EPH1(O,ODO,A,EF1,CT,5,6,12)
2264      DO 10 I=1,5
2265      DO 10 J=1,5
2266      FT1(I,J)=FA*FI(I,J)
2267      FT2(I,J)=FB*FIY(I,J)
2268      FT3(I,J)=-FC*FI(I,J)
2269      FT4(I,J)=FD*FIY(I,J)+FAC2*FI(I,J)+FAC1*EF1(I,J)
2270 10      CALL EPH1(B,O,ODO,EF1,CT,5,6,12)
2271      DO 20 I=1,5
2272      DO 20 J=1,5
2273      FT4(I,J)=FT4(I,J)-FAC1*EF1(I,J)
2274 20      GO TO (50,50,70,50),LTYPE
2275      C
2276      C
2277      C      CALCULATE BENDING STIFFNESS MATRICES FOR SINGLE
2278      C      PLATES.
2279      C
2280      C
2281 30      DO 40 I=1,5
2282      DO 40 J=1,5
2283      FT1(I,J)=FA*FI(I,J)
2284      FT2(I,J)=FB*FIY(I,J)
2285      FT3(I,J)=-FC*FI(I,J)
2286      FT4(I,J)=FD*FIY(I,J)
2287      C
2288      C
2289      C      CALCULATE GEOMETRIC STIFFNESS SUBMATRICES
2290      C
2291      C      GO TO (50,50,70,50),LTYPE
2292      C
2293      C
2294      C      AXIAL LOAD OR BENDING OR COMBINED
2295      C
2296 50      DO 60 I=1,5
2297      DO 60 J=1,5
2298      FT5(I,J)=E*FE*FI(I,J)
2299      RETURN
2300      C
2301      C
2302      C      AXIAL LOAD AT SPECIFIED ECCENTRICITY.
2303      C
2304 70      FEAL=E*FE*ALAR
2305      DO 80 I=1,5
2306      DO 80 J=1,5
2307      FT4(I,J)=FT4(I,J)-FEAL*FI(I,J)
2308      FT5(I,J)=E*FE*FI(I,J)
2309      RETURN
2310      C
2311      C
2312      C
2313      C      SUBROUTINE 16
2314      C
2315      C
2316      C
2317      C
2318      C      SUBROUTINE FWALC
2319      C
2320      C
2321      C      THIS ROUTINE CALCULATES THE PLATE
2322      C      BUCKLING FACTORS FOR THE ELASTIC, INELASTIC,
2323      C      AND STRAIN-HARDENING RANGES
2324      C
2325      C      SUBROUTINE FWALC(BPL,WPL,TPL,EX)
2326      C      IMPLICIT REAL*8(A-H,D-Z)
2327      C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,1KIND
2328      C      COMMON /BLK4/FA,FB,FC,FD,FE
2329      C      COMMON /BLK10/HW,TW,BCL,BF1,BF2,TF1,BF2,TF2
2330      C      COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCLM,SRTT,SRCLB,SRCT
2331      C      P=DARCO5(-1.ODO)
2332      C
2333      C
2334      C      CALCULATE EFFECTIVE MODULI FOR PLANE STRESS.
2335      C
2336 10      EEX=E+3.ODO*EX
2337      EEY=4.ODO*E*EX/EEY
2338      V1=2.ODO*(1.ODO+V)
2339      V2=2.ODO*V-1.ODO
2340      EXE=EX*V2+E
2341      G1=E/V1
2342      G2=G1
2343      IF (ITYPE.NE.7) GO TO 8
2344      GST=2.ODO*G1/(1.ODO+E/(4.ODO*E2*(1.ODO+V)))
2345      IF (EX.EQ.E1 OR EX.EQ.E2) G=GST
2346 8      VX=EXE/(2.ODO+E)
2347      VV=2.ODO*EXE/EEY
2348      VDEN=1.ODO-VX*VV
2349      IF (VDEN.EQ.ODO) GO TO 20
2350      C
2351      C
2352      C      CALCULATE PLATE BUCKLING STIFFNESS FACTORS DX, DY,
2353      C      DXY
2354      C
2355      C      DX=EX/VDEN
2356      C      DY=EEY/VDEN
2357      C      GO TO 30
2358      C
2359 20      DX=ODO
2360      DY=ODO
2361 30      DXY=VX*DY
2362      IF (ICLAMP.EQ.1) GO TO 40
2363      IF (ICLAMP.EQ.2.AND.HW.EQ.WPL.AND.TW.EQ.TPL) GO TO 40
2364      C
2365      C
2366      C      CALCULATE PLATE BUCKLING FACTORS FOR PINNED ENDS
2367      C
2368      C      FL=DX*TPL**3*P]**4*WPL/BPL**3/48.ODO
2369      C      FB=DY*TPL**3*BP/3.ODO/WPL**3
2370      C      FC=2.ODO*DXY*TPL**3*P]**2/24.ODO/BPL/WPL
2371      C      FD=G*TPL**3*P]**2/3.ODO/BPL/WPL
2372      C      FE=WPL*TPL*P]**2/4.ODO/BPL
2373      C      RETURN

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2373 C
2374 C
2375 C      CALCULATE PLATE BUCKLING FACTORS FOR OTHER THAN
2376 C      PINNED ENDS
2377 C
2378 40  FA=DX*TPL**3=WPL/24.ODO
2379      FB=DY*TPL**3*2 ODO/3 ODO/WPL**3
2380      FC=-DXY*TPL**3/6 ODO/WPL
2381      FD=G*TPL**3*2 ODO/(3 ODO*WPL)
2382      FE=O SDO*TPL=WPL
2383      RETURN
2384      END
2385 C
2386 C
2387 C
2388 C      SUBROUTINE 17
2389 C
2390 C
2391 C
2392 C
2393 C      SUBROUTINE GETST
2394 C
2395 C      THIS SUBROUTINE USES AN ITERATIVE TECHNIQUE
2396 C      (REGULI FALS) TO DETERMINE INELASTIC AXIAL
2397 C      STRAIN, EAX, WHEN THE LOAD, AXLO, IS KNOWN IT
2398 C      INCLUDES THE EFFECTS OF RESIDUAL STRAINS
2399 C
2400 C
2401 C      SUBROUTINE GETST(AXLO,EAX,*)
2402 C      IMPLICIT REAL*8(A-H,O-Z)
2403 C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
2404 C      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
2405 C      COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
2406 C      COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
2407 C
2408 C
2409 C      CALCULATE STRAIN LIMITS T1, T2, T3 FOR WEB.
2410 C
2411 C      T1=EYTF-ERCT
2412 C      T2=EYBF-ERCB
2413 C      T3=EYW-ERCM
2414 C      TM=DMIN1(T1,T2,T3)
2415 C
2416 C
2417 C      CALCULATE LOAD AT BEGINNING OF YIELD (PBOY)
2418 C
2419 C      PBOY=TM*E*AREA
2420 C
2421 C
2422 C      INITIALISE STARTING STRAINS ET1 AT BEGINNING OF
2423 C      YIELD (PROPORTIONAL LIMIT) AND ET2 AT LOAD EQUAL TO
2424 C      AXLO ASSUMING LINEAR BEHAVIOUR
2425 C
2426 C      ET1=TM
2427 C      P1=PBOY
2428 C      ET2=(AXLO/AREA)/E
2429 C      CALL PSCALC(ET2,P2,PPY,SAVE)
2430 C      NCOUNT=0
2431 C      EDEN=P2-P1
2432 C      IF (EDEN.EQ.O.OODO) GO TO 30
2433 C
2434 C
2435 C      PREDICT STRAIN ET3 USING METHOD OF FALSE POSITION
2436 C
2437 C      ENUM=ET2*(AXLO-P1)-ET1*(AXLO-P2)
2438 C      ET3=ENUM/EDEN
2439 C      CALL PSCALC(ET3,P3,PPY,SAVE)
2440 C      PDIF1=DABS((P3-P2)/P3)
2441 C      PDIF2=DABS((ET3-ET2)/ET3)
2442 C      NCOUNT=NCOUNT+1
2443 C
2444 C
2445 C      CHECK CONVERGENCE NOTE: THE PURPOSE IS TO GET THE
2446 C      DIFFERENCE BETWEEN APPLIED AXIAL LOAD, AXLO, AND AN
2447 C      ASSUMED LOAD, P3, ON THE NON-LINEAR LOAD-STRAIN
2448 C      CURVE SUCH THAT AXLO-P3=0. THE CORRESPONDING STRAIN
2449 C      IS THE REQUIRED VALUE
2450 C
2451 C      IF (PDIF1.LT.O.OO1DO.AND.PDIF2.LT.O.OO1DO) GO TO 20
2452 C      IF (NCOUNT.GT.50) GO TO 40
2453 C      ET1=ET2
2454 C      P1=P2
2455 C      ET2=ET3
2456 C      P2=P3
2457 C      GO TO 10
2458 C      20  EAX=ET3
2459 C      RETURN
2460 C      30  WRITE (6,50)
2461 C      RETURN 1
2462 C      40  WRITE (6,60)
2463 C      RETURN 1
2464 C      50  FORMAT ('-',10X,'DIVISION BY ZERO IN SUBROUTINE GETST')
2465 C      60  FORMAT ('-',10X,'SUBROUTINE GETST FAILED TO CONVERGE IN 50 ITERATI
2466 C      8ONS')
2467 C      END
2468 C
2469 C
2470 C
2471 C      SUBROUTINE 18
2472 C
2473 C
2474 C
2475 C
2476 C      SUBROUTINE INVERT
2477 C
2478 C      THIS ROUTINE CALCULATES THE INVERSE OF THE
2479 C      BENDING STIFFNESS MATRIX AND RETURNS IT IN SB1
2480 C      THE GAUSS-JORDAN METHOD IS USED
2481 C
2482 C
2483 C      SUBROUTINE INVERT(N,SB,SB1)
2484 C      IMPLICIT REAL*8(A-H,O-Z)
2485 C      DIMENSION SB(15,15),SB1(15,15)

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2486 C
2487 C
2488 C INITIALISE THE INVERTED MATRIX
2489 C
2490 C DD 10 J=1,N
2491 C DD 10 I=1,N
2492 C SBI(1,J)=0.0DO
2493 C IF (1.EQ.J) SBI(1,J)=1.0DO
2494 C 10 CONTINUE
2495 C N1=N-1
2496 C DD 70 K=1,N
2497 C RP=1.0DO/SB(K,K)
2498 C K1=K+1
2499 C DD 30 J=K1,N
2500 C SBJ=SB(K,J)*RP
2501 C SBIJ=SBI(K,J)*RP
2502 C DD 20 I=1,N
2503 C SBI(1,J)=SBI(1,J)-SBJ*SB(1,K)
2504 C SBI(I,J)=SBI(I,J)-SBIJ*SB(1,K)
2505 C 20 SE(K,J)=SBJ
2506 C 30 SBI(K,J)=SBIJ
2507 C RP=1.0DO/SB(N,N)
2508 C DD 60 J=1,K
2509 C SBIJ=SBI(K,J)*RP
2510 C DD 50 I=1,N
2511 C SBI(1,J)=SBI(1,J)-SBIJ*SB(1,K)
2512 C 60 SBI(K,J)=SBIJ
2513 C 70 CONTINUE
2514 C RETURN
2515 C END
2516 C
2517 C
2518 C SUBROUTINE 19
2519 C
2520 C
2521 C
2522 C
2523 C
2524 C SUBROUTINE MULT
2525 C
2526 C THIS ROUTINE MULTIPLIES TWO MATRICES SBI
2527 C AND SG AND RETURNS THE RESULTING MATRIX ABC.
2528 C
2529 C
2530 C SUBROUTINE MULT(ABC,SBI,SG,N)
2531 C IMPLICIT REAL*8(A-H,O-Z)
2532 C DIMENSION ABC(15,15),SBI(15,15),SG(15,15)
2533 C DD 20 J=1,N
2534 C DD 20 I=1,N
2535 C SUM=0.0DO
2536 C DD 10 K=1,N
2537 C SUM=SUM+SBI(I,K)*SG(K,J)
2538 C 20 ABC(I,J)=SUM
2539 C RETURN
2540 C END
2541 C
2542 C
2543 C SUBROUTINE 20
2544 C
2545 C
2546 C
2547 C
2548 C
2549 C SUBROUTINE MCALC
2550 C
2551 C THIS SUBROUTINE CALCULATES THE ELASTIC OR
2552 C INELASTIC BENDING MOMENT ON A W-SHAPE WHEN THE
2553 C STRAIN DISTRIBUTION ON A CROSS-SECTION IS KNOWN
2554 C AND THE SECTION IS SUBJECTED TO A KNOWN AXIAL
2555 C STRAIN.
2556 C
2557 C
2558 C SUBROUTINE MCALC(EB,EAPP,BM,BMY,BMP,BMMP,DNA,RDNA)
2559 C IMPLICIT REAL*8(A-H,O-Z)
2560 C COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
2561 C COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
2562 C COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
2563 C COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
2564 C COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCT,SRCT
2565 C IF (EAPP.EQ.0.0DO) ALAR=0.0DO
2566 C
2567 C
2568 C CALCULATE YIELD MOMENT, YM, AND THE REDUCED PLASTIC
2569 C MOMENT, PM.
2570 C
2571 C Y1=YB-0.5DO*HW
2572 C DT=0.5DO*HW-Y1
2573 C DE=0.5DO*HW+Y1
2574 C YM1=YSFT*ERTBAR/(DT+0.5DO*TF2)
2575 C YM2=YSFB*ERTBAR/(DB+0.5DO*TF1)
2576 C YM=DMIN1(YM1,YM2)
2577 C
2578 C
2579 C DETERMINE THE LOCATION OF THE PLASTIC NEUTRAL AXIS.
2580 C
2581 C F1=YSFB*BF1+TF1
2582 C F2=YSFT*BF2+TF2
2583 C FW=YSW*TW*HW
2584 C PY=F1+F2+FW
2585 C P=ALAR*PY
2586 C D=HW+C.5DO*(TF1+TF2)
2587 C APPY=(1.0DO+ALAR)*PY
2588 C AMPY=(1.0DO-ALAR)*PY
2589 C AMPY1=AMPY-2.0DO*F1
2590 C F22=2.0DO*F2
2591 C FW2=2.0DO*FW
2592 C F12=2.0DO*F1
2593 C IF (APPY.GT.0.0DO.AND.APPY.LE.F22) GO TO 10
2594 C IF (AMPY1.GE.0.0DO.AND.AMPY1.LT.FW2) GO TO 20
2595 C IF (AMPY.GE.0.0DO.AND.AMPY.LT.F12) GO TO 30
2596 C
2597 C
2598 C PLASTIC NEUTRAL AXIS IN TOP (COMPRESSION) FLANGE.

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2599      C
2600      10      DSO=D**2
2601              DN=D-O.5DO*(P+PY)*TF2/F2
2602              DNSO=DN**2
2603              DMT2SO=(D-TF2)**2
2604              T1SO=TF1**2
2605              CPM1=O.5DO*YSFT*BF2*(DSO-2.ODO*DNSO+DMT2SO)
2606              CPM2=O.5DO*YSW*TW*(DMT2SO-T1SO)
2607              CPM3=O.5DO*YSFB*BF1*T1SO
2608              PM=CPM1+CPM2+CPM3-P*(DB+O.5DO*TF1)
2609              GO TO 40
2610      C
2611      C
2612      C          PLASTIC NEUTRAL AXIS IN THE WEB
2613      C
2614      20      DSO=D**2
2615              DC=D-TF1-TF2
2616              PFP=PY-F12-P
2617              DN=TF1+O.5DO*PFP*DC/FW
2618              DNSO=DN**2
2619              DMT2SO=(D-TF2)**2
2620              T1SO=TF1**2
2621              B2T2=BF2*TF2
2622              CPM1=O.5DO*YSFT*B2T2*(2.ODO*D-TF2)
2623              CPM2=O.5DO*YSW*TW*(DMT2SO-2.ODO*DNSO+T1SO)
2624              CPM3=O.5DO*YSFB*BF1*T1SO
2625              PM=CPM1+CPM2+CPM3-P*(DB+O.5DO*TF1)
2626              GO TO 40
2627      C
2628      C
2629      C          PLASTIC NEUTRAL AXIS IN BOTTOM (TENSION) FLANGE.
2630      C
2631      30      DSO=D**2
2632              DN=O.5DO*TF1*(PY-P)/F1
2633              DNSO=DN**2
2634              DMT2SO=(D-TF2)**2
2635              T1SO=TF1**2
2636              B2T2=BF2*TF2
2637              CPM1=O.5DO*YSFT*B2T2*(2.ODO*D-TF2)
2638              CPM2=O.5DO*YSW*TW*(DMT2SO-T1SO)
2639              CPM3=O.5DO*YSFB*BF1*(T1SO-2.ODO*DNSO)
2640              PM=CPM1+CPM2+CPM3-P*(DB+O.5DO*TF1)
2641      C
2642      C
2643      C          DEFINE STRAINS AND STRAIN LIMITS T1, T2, T3, T4 FOR
2644      C          TOP (COMPRESSION) FLANGE.
2645      C
2646      40      EAPB=EB+EAPP
2647              EC=(2.ODO*Y1/([2.ODO*Y1-HW]))=EB
2648              REB=R*EB
2649              EARST=EAPP-REB
2650              RSTED=REB-EAPP
2651              T1=EYTF-ERTT
2652              T2=EYTF+ERTT
2653              T3=ESH-ERTT
2654              T4=ESH+ERTT
2655      C
2656      C
2657      C          CALCULATE TOTAL FORCE IN TOP (COMPRESSION) FLANGE
2658      C
2659      ADEN=ERTT+ERTT
2660      IF (EAPB.GT.T1) GO TO 50
2661      C
2662      C
2663      C          DEFINE LIMITS OF INTEGRATION ALFT AND ALFP.
2664      C
2665      ALFT=1.ODO
2666      ALFP=1.ODO
2667      GO TO 100
2668      50      IF (EAPB.GT.T2) GO TO 70
2669              IF (ADEN.EQ.O.ODO) GO TO 60
2670              ANUM=T2-EAPB
2671              ALFT=ANUM/ADEN
2672              IF (ALFT.GT.1.ODO) ALFT=1.ODO
2673              IF (ALFT.LT.O.ODO) ALFT=O.ODO
2674              ALFP=1.ODO
2675              GO TO 100
2676      60      ALFT=O.ODO
2677              ALFP=1.ODO
2678              GO TO 100
2679      70      IF (EAPB.GT.T3) GO TO 80
2680              ALFT=O.ODO
2681              ALFP=1.ODO
2682              GO TO 100
2683      80      IF (EAPB.GT.T4) GO TO 90
2684              IF (ADEN.EQ.O.ODO) GO TO 90
2685              ANUM=T4-EAPB
2686              ALFP=ANUM/ADEN
2687              IF (ALFP.GT.1.ODO) ALFP=1.ODO
2688              IF (ALFP.LT.O.ODO) ALFP=O.ODO
2689              ALFT=O.ODO
2690              GO TO 100
2691      90      ALFP=O.ODO
2692              ALFT=O.ODO
2693      C
2694      C
2695      C          ELASTIC RANGE.
2696      C
2697      100      A1=O.5DO*BF2*TF2*ALFT
2698              A2=2.ODO*(EAPB-ERTT)*E
2699              A3=ADEN*E*ALFT
2700              F21=A1*(A2+A3)
2701      C
2702      C
2703      C          YIELDED RANGE
2704      C
2705      A1=O.5DO*BF2*TF2*(ALFP+ALFT)
2706      A2=2.ODO*(YSFT+(EAPB-T2)*E1)
2707      A3=ADEN*E1*(ALFP+ALFT)
2708      F22=A1*(A2+A3)
2709      C
2710      C
2711      C          STRAIN-HARDENED REGION.

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2712 C
2713 A1=C.5DO*BF2*TF2*(1.ODO-ALFP)
2714 A2=2.ODO*(1-SFT*(ESH-EYTF)*E1+(EAPB-T4)*E2)
2715 A3=ADEN+E2*(1.ODO+ALFP)
2716 F23=A1*(A2+A3)
2717 F2=F21+F22+F23
2718 C
2719 C
2720 C
2721 C
2722 C
2723 C
2724 IF (EARST.GT.O.ODO) GO TO 170
2725 C
2726 C
2727 C
2728 C
2729 C
2730 T1=EYBF-ERTB
2731 T2=EYBF+ERCB
2732 T3=ESH-ERTB
2733 T4=ESH+ERCB
2734 AD1=ERTB+ERCB
2735 C
2736 C
2737 C
2738 C
2739 C
2740 IF (RSTEA.GT.T1) GO TO 110
2741 ALFB=O.ODO
2742 ALFBP=O.ODO
2743 CO TO 160
2744 110 IF (RSTEA.GT.T2) GO TO 130
2745 IF (AD1.EQ.O.ODO) GO TO 120
2746 AN1=RSTEA-T1
2747 ALFB=AN1/AD1
2748 IF (ALFB.GT.1.ODO) ALFB=1.ODO
2749 IF (ALFB.LT.O.ODO) ALFB=O.ODO
2750 ALFBP=O.ODO
2751 GO TO 160
2752 120 ALFB=1.ODO
2753 ALFBP=O.ODO
2754 GO TO 160
2755 130 IF (RSTEA.GT.T3) GO TO 140
2756 ALFB=1.ODO
2757 ALFBP=O.ODO
2758 GO TO 160
2759 140 IF (RSTEA.GT.T4) GO TO 150
2760 IF (AD1.EQ.O.ODO) GO TO 150
2761 AN1=RSTEA-T3
2762 ALFBP=AN1/AD1
2763 IF (ALFBP.GT.1.ODO) ALFBP=1.ODO
2764 IF (ALFBP.LT.O.ODO) ALFBP=O.ODO
2765 ALFB=1.ODO
2766 GO TO 160
2767 150 ALFB=1.ODO
2768 ALFBP=1.ODO
2769 C
2770 C
2771 C
2772 C
2773 160 A1=O.5DO*BF1*TF1*ALFBP
2774 A2=2.ODO*(-YSFB-(ESH-EYBF)*E1+(T3-RSTEA)*E2)
2775 A3=AD1+E2*ALFBP
2776 F11=A1*(A2+A3)
2777 C
2778 C
2779 C
2780 C
2781 C
2782 C
2783 C
2784 C
2785 C
2786 C
2787 C
2788 C
2789 C
2790 C
2791 C
2792 C
2793 C
2794 C
2795 C
2796 C
2797 C
2798 C
2799 C
2800 170 T1=EYBF-ERCB
2801 T2=EYBF+ERTB
2802 T3=ESH-ERCB
2803 T4=ESH+ERTB
2804 C
2805 C
2806 C
2807 C
2808 C
2809 C
2810 C
2811 C
2812 C
2813 180 IF (EARST.GT.T1) GO TO 200
2814 IF (ADEN.EQ.O.ODO) GO TO 190
2815 ANUM=T2-EARST
2816 ALFT=ANUM/ADEN
2817 IF (ALFT.GT.1.ODO) ALFT=1.ODO
2818 IF (ALFT.LT.O.ODO) ALFT=O.ODO
2819 ALFP=1.ODO
2820 GO TO 230
2821 190 ALFT=O.ODO
2822 ALFP=1.ODO
2823 GO TO 230
2824 200 IF (EARST.GT.T3) GO TO 210

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2825      ALFT=0.0DO
2826      ALFP=1.0DO
2827      GO TO 230
2828  210  IF (EARST.GT.T4) GO TO 220
2829      IF (ADEN.EQ.0.0DO) GO TO 220
2830      ANUM=T4-EARST
2831      ALFP=ANUM/ADEN
2832      IF (ALFP.GT.1.0DO) ALFP=1.0DO
2833      IF (ALFP.LT.0.0DO) ALFP=0.0DO
2834      ALFT=0.0DO
2835      GO TO 230
2836  220  ALFP=0.0DO
2837      ALFT=0.0DO
2838      C
2839      C
2840      C      ELASTIC REGION
2841      C
2842  230  A1=0.5DO*BF1*TF1*ALFT
2843      A2=2.0DO*(EARST-ERTB)*E
2844      A3=ADEN+E*ALFT
2845      F11=A1*(A2+A3)
2846      C
2847      C
2848      C      YIELDED REGION.
2849      C
2850      A1=C.5DO*BF1*TF1*(ALFP-ALFT)
2851      A2=2.0DO*(YSFB+(EARST-T2)*E1)
2852      A3=ADEN+E1*(ALFP+ALFT)
2853      F12=A1*(A2+A3)
2854      C
2855      C
2856      C      STRAIN-HARDENED REGION.
2857      C
2858      A1=C.5DO*BF1*TF1*(1.0DO-ALFP)
2859      A2=2.0DO*(YSFB+(ESH-EYBF)*E1+(EARST-T4)*E2)
2860      A3=ADEN+E2*(1.0DO+ALFP)
2861      F13=A1*(A2+A3)
2862      F1=F11+F12+F13
2863      C
2864      C
2865      C      CALCULATE BENDING MOMENT FROM LOWER (TENSION)
2866      C      PORTION OF WEB. DEFINE LIMITS OF INTEGRATION T1,
2867      C      T2, T3, T4, T5, T6.
2868      C
2869  240  T1=ERCM+EC
2870      T2=EYW-ERTB
2871      T3=ESH-ERTB
2872      T4=EYW-ERTT
2873      T5=ESH-ERTT
2874      T6=T1+EAPP
2875      C
2876      C
2877      C      DEFINE LIMITS OF INTEGRATION BET1, BETP1, BET3,
2878      C      BETP3.
2879      C
2880      AD1=REB+T1+ERTB
2881      AD2=EB-T1-ERTT
2882      IF (T6.GT.EYW) GO TO 250
2883      BET3=0.0DO
2884      BETP3=0.0DO
2885      GO TO 290
2886  250  IF (T6.GT.ESH) GO TO 270
2887      IF (AD1.EQ.0.0DO) GO TO 260
2888      BET3=(T6-EYW)/AD1
2889      IF (BET3.GT.1.0DO) BET3=1.0DO
2890      IF (BET3.LT.0.0DO) BET3=0.0DO
2891      BETP3=0.0DO
2892      GO TO 290
2893  260  BET3=1.0DO
2894      BETP3=0.0DO
2895      GO TO 290
2896  270  IF (AD1.EQ.0.0DO) GO TO 280
2897      BET3=(T6-EYW)/AD1
2898      BETP3=(T6-ESH)/AD1
2899      IF (BET3.GT.1.0DO) BET3=1.0DO
2900      IF (BET3.LT.0.0DO) BET3=0.0DO
2901      IF (BETP3.GT.1.0DO) BETP3=1.0DO
2902      IF (BETP3.LT.0.0DO) BETP3=0.0DO
2903      GO TO 290
2904  280  BET3=1.0DO
2905      BETP3=1.0DO
2906  290  IF (RSTEA.GT.T2) GO TO 300
2907      BET1=1.0DO
2908      BETP1=1.0DO
2909      GO TO 340
2910  300  IF (RSTEA.GT.T3) GO TO 320
2911      IF (AD1.EQ.0.0DO) GO TO 310
2912      BET1=(T6+EYW)/AD1
2913      IF (BET1.GT.1.0DO) BET1=1.0DO
2914      IF (BET1.LT.0.0DO) BET1=0.0DO
2915      BETP1=1.0DO
2916      GO TO 340
2917  310  BET1=1.0DO
2918      BETP1=1.0DO
2919      GO TO 340
2920  320  IF (AD1.EQ.0.0DO) GO TO 330
2921      BET1=(T6+EYW)/AD1
2922      BETP1=(T6+ESH)/AD1
2923      IF (BET1.GT.1.0DO) BET1=1.0DO
2924      IF (BET1.LT.0.0DO) BET1=0.0DO
2925      IF (BETP1.GT.1.0DO) BETP1=1.0DO
2926      IF (BETP1.LT.0.0DO) BETP1=0.0DO
2927      GO TO 340
2928  330  BET1=1.0DO
2929      BETP1=1.0DO
2930      C
2931      C
2932      C      CALCULATE MOMENT FROM LOWER
2933      C      STRAIN-HARDENING REGION
2934      C
2935  340  B1=TW*HW**2/24.0DO
2936      B2=3.0DO*(-YSW*(ESH-EYW)*E1+(T6+ESH)*E2)
2937      B3=BETP1*BETP1-1.0DO

```



```

2838      B4:=2 ODO*(REB+T1+ERTB)*E2*(1.ODO-BETP1**3)
2839      WM1:=B1*(B2+B3+B4)
2840
2841      C
2842      C      CALCULATE MOMENT FROM YIELDED REGION.
2843      C
2844      B2:=3 ODO*(-YSW*(T6+EYW)*E1)*(BET1**2-BETP1**2)
2845      B3:=2 ODO*(REB+T1+ERTB)*E1*(BET1**3-BETP1**3)
2846      WM2:=B1*(B2-B3)
2847
2848      C
2849      C      CALCULATE MOMENT FROM ELASTIC REGION.
2850      C
2851      B2:=3 ODO*T6*E*(BET3**2-BET1**2)
2852      B3:=2 ODO*(REB+ERTB+T1)*E*(BET3**3-BET1**3)
2853      WM3:=B1*(B2-B3)
2854
2855      C
2856      C      CALCULATE MOMENT FROM UPPER YIELD REGION.
2857      C
2858      B2:=3 ODO*(YSW*(T6+EYW)*E1)*(BETP3**2-BET3**2)
2859      B3:=2 ODO*(REB+ERTB+T1)*E1*(BETP3**3-BET3**3)
2860      WM4:=B1*(B2-B3)
2861
2862      C
2863      C      CALCULATE MOMENT FROM UPPER
2864      C      STRAIN-HARDENED REGION
2865      C
2866      B2:=3 ODO*(YSW*(ESH+EYW)*E1+(T6-ESH)*E2)*(-BETP3**2)
2867      B3:=2 ODO*(REB+ERTB+T1)*E2*BETP3**3
2868      WM5:=B1*(B2+B3)
2869
2870      C
2871      C      CALCULATE MOMENT FROM TOP HALF (COMPRESSION)
2872      C      REGION OF WEB.
2873      C
2874      C      CASE I - MIDDLE OF WEB ELASTIC.
2875      C      CASE II - MIDDLE OF WEB YIELDED
2876      C      CASE III - MIDDLE OF WEB STRAIN-HARDENED.
2877      C
2878      IF (T6.GT.EYW) GO TO 400
2879      IF (EAPEB.GT.T4) GO TO 350
2880
2881      C
2882      C      CASE I
2883      C
2884      C      DEFINE LIMITS OF INTEGRATION BET2 AND BETP2.
2885      C
2886      C
2887      BET2:=1.ODO
2888      BETP2:=1.ODO
2889      GO TO 350
2890
2891      350 IF (EAPEB.GT.T5) GO TO 370
2892      IF (AD2.EQ.O.ODO) GO TO 360
2893      BET2:=(EYW-T6)/AD2
2894      IF (BET2.GT.1.ODO) BET2:=1.ODO
2895      IF (BET2.LT.O.ODO) BET2:=O.ODO
2896      BETP2:=1.ODO
2897      GO TO 390
2898
2899      360 BET2:=O.ODO
2900      BETP2:=1.ODO
2901      GO TO 390
2902
2903      370 IF (AD2.EQ.O.ODO) GO TO 380
2904      BET2:=(EYW-T6)/AD2
2905      BETP2:=(ESH-T6)/AD2
2906      IF (BET2.GT.1.ODO) BET2:=1.ODO
2907      IF (BET2.LT.O.ODO) BET2:=O.ODO
2908      IF (BETP2.GT.1.ODO) BETP2:=1.ODO
2909      IF (BETP2.LT.O.ODO) BETP2:=O.ODO
2910      GO TO 390
2911
2912      380 BET2:=O.ODO
2913      BETP2:=O.ODO
2914
2915      C
2916      C      CALCULATE MOMENT FROM ELASTIC REGION
2917      C
2918      390 B1:=TW*HW**2/24 ODO
2919      B2:=3 ODO*T6*E*BET2**2
2920      B3:=2 ODO*(EB-ERTT-T1)*E*(BET2**3-BETP2**3)
2921      WM6:=B1*(B2+B3)
2922
2923      C
2924      C      CALCULATE MOMENT FROM YIELDED REGION.
2925      C
2926      B2:=2 ODO*(YSW*(T6+EYW)*E1)*(BETP2**2-BET2**2)
2927      B3:=2 ODO*(EB-ERTT-T1)*E1*(BETP2**3-BET2**3)
2928      WM7:=B1*(B2+B3)
2929
2930      C
2931      C      CALCULATE MOMENT FROM STRAIN-HARDENED REGION
2932      C
2933      B2:=3 ODO*(YSW*(ESH+EYW)*E1+(T6-ESH)*E2)*(1.ODO-BETP2**2)
2934      B3:=2 ODO*(EB-ERTT-T1)*E2*(1.ODO-BETP2**3)
2935      WM8:=B1*(B2+B3)
2936      GO TO 510
2937
2938      400 IF (T6.GT.ESH) GO TO 460
2939
2940      C
2941      C      CASE II
2942      C
2943      C      DEFINE LIMITS OF INTEGRATION BET2 AND BETP2
2944      C
2945      IF (EAPEB.GT.T4) GO TO 420
2946      IF (AD2.EQ.O.ODO) GO TO 410
2947      BET2:=(EYW-T6)/AD2
2948      IF (BET2.GT.1.ODO) BET2:=1.ODO
2949      IF (BET2.LT.O.ODO) BET2:=O.ODO
2950      BETP2:=1.ODO
2951      GO TO 450
2952
2953      410 BET2:=1.ODO
2954      BETP2:=1.ODO
2955      GO TO 450
2956
2957      420 IF (EAPEB.GT.T5) GO TO 430
2958      BET2:=1.ODO

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[illegible]



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3991      8481(' ',J')
3992      END
3993      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
3994      C
3995      SUBROUTINE 26
3996      C
3997      C
3998      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
3999      C
4000      SUBROUTINE PLATE2
4001      C
4002      THIS SUBROUTINE CALCULATES THE ELASTIC
4003      CRITICAL STRESS FOR A FULLY SUPPORTED RECTANGULAR
4004      PLATE HAVING VARIOUS BOUNDARY CONDITIONS AND
4005      SUBJECTED TO VARIOUS TYPES OF LOADS.
4006      C
4007      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
4008      SUBROUTINE PLATE2
4009      IMPLICIT REAL*8(A-H,O-Z)
4010      DIMENSION XVEC(15,20)
4011      COMMON /BLK1/E,V,N5,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
4012      COMMON /BLK4/FA,FB,FC,FD,FE
4013      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
4014      COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
4015      COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
4016      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
4017      C
4018      C
4019      READ IN PLATE DIMENSIONS, CALCULATE SHAPE FUNCTION
4020      COEFFICIENTSAND ECHO CHECK INPUT DATA.
4021      C
4022      READ (5,20) BCL,HW,TW,YSW
4023      CALL COEFW
4024      IF (LTYPE.EQ.3) READ (5,30) ALAR
4025      IF (LTYPE.EQ.3) ALAR=ALAR+YSW
4026      IF (LTYPE.EQ.3) AXLO=ALAR+HW+TW
4027      IF (LTYPE.EQ.4) READ (5,30) ECC
4028      IF (ITYPE.EQ.3) WRITE (6,60) NS
4029      IF (ITYPE.EQ.4) WRITE (6,60) NS
4030      IF (ITYPE.EQ.5) WRITE (6,70) NS
4031      IF (ITYPE.EQ.6) WRITE (6,80) NS
4032      IF (LTYPE.EQ.1) WRITE (6,90)
4033      IF (LTYPE.EQ.2) WRITE (6,100)
4034      IF (LTYPE.EQ.3) WRITE (6,110) ALAR
4035      IF (LTYPE.EQ.4) WRITE (6,120) ECC
4036      WRITE (6,130) BCL,HW,TW,YSW
4037      AREA=HW*TW
4038      Y1=O.OOO
4039      ERTBAR=TW*HW**3/12.OOO
4040      C
4041      C
4042      CALL WBFORM TO FORMULATE STIFFNESSES AND ESFORM TO
4043      SOLVE FOR THE EIGENPAIR.
4044      C
4045      CALL WBFORM
4046      CALL ESFORM(EIGV,MM1,XVEC,810)
4047      EIGV=EIGV+E
4048      PCR=HW*TW+EIGV
4049      SP=HW/TW
4050      WAVE=BCL/MM1
4051      C
4052      C
4053      WRITE OUT RESULTS
4054      C
4055      C
4056      SP          = PLATE SLENDERNESS
4057      EIGV        = CRITICAL STRESS
4058      PCR         = CRITICAL LOAD
4059      WAVE        = WAVELENGTH
4060      XVEC(1,MM1) = BUCKLED SHAPE
4061      C
4062      WRITE (6,140) SP,PCR
4063      WRITE (6,150) EIGV,WAVE
4064      WRITE (6,40) XVEC(7,MM1),XVEC(6,MM1),XVEC(5,MM1),XVEC(4,MM1),XVEC(
4065      &3,MM1),XVEC(1,MM1),XVEC(2,MM1)
4066      RETURN
4067      C
4068      C
4069      FORMATS
4070      C
4071      FORMAT (4F8.5)
4072      FORMAT (F8.5)
4073      FORMAT ('//T31,'//T31,'| U = O OOOOO'/T31,'| U='//F8.5/'
4074      &T31,'| U='//F8.5/(T31,'| //),T31,'| U='//F8.5/T31,'| U='//
4075      &F8.5/T31,'| U='//F8.5/(T31,'| //),T31,'| U = O OOOOO'/T31,'
4076      &3| U='//F8.5/T31,'| U='//F8.5/T31,'//)
4077      C
4078      FORMAT ('//O'/O',10X,'PLATE NUMBER',I3/O',10X,'PINNED AT THE T
4079      &OP AND BOTTOM')
4080      C
4081      FORMAT ('//O'/O',10X,'PLATE NUMBER',I3/O',10X,'PINNED AT THE T
4082      &OP AND FIXED AT THE BOTTOM')
4083      C
4084      FORMAT ('//O'/O',10X,'PLATE NUMBER',I3/O',10X,'FIXED AT THE TO
4085      &OP AND PINNED AT THE BOTTOM')
4086      C
4087      FORMAT ('//O'/O',10X,'PLATE NUMBER',I3/O',10X,'FIXED AT THE TO
4088      &P AND BOTTOM')
4089      C
4090      FORMAT ('O',10X,'SUBJECTED TO AXIAL LOAD ONLY.')
4091      C
4092      FORMAT ('O',10X,'SUBJECTED TO PURE BENDING.')
4093      C
4094      FORMAT ('O',10X,'SUBJECTED TO AN APPLIED STRESS OF',F10.4,' KSI' PL
4095      &US BENDING.')
4096      C
4097      FORMAT ('O',10X,'SUBJECTED TO A LOAD AT AN ECCENTRICITY OF',F8.3,'
4098      & INCHES ')
4099      C
4100      FORMAT ('O'/O',10X,'LENGTH OF PLATE              ='//F9.4,' IN
4101      &CHES'/O',10X,'WIDTH OF PLATE                      ='//F9.5,' INCHES'/
4102      &O',10X,'THICKNESS OF PLATE                          ='//F9.5,' INCHES'/O',
4103      &10X,'YIELD STRESS OF MATERIAL                        ='//F9.4,' KSI')
4104      C
4105      FORMAT ('O',10X,'SLENDERNESS RATIO                ='//F9.4/O',10X,
4106      &'CRITICAL LOAD                                      ='//F9.4,' KIPS')
4107      C
4108      FORMAT ('O',10X,'CRITICAL STRESS ='//F10.6,' KSI'/O',10X,'HALF WAV
4109      &E LENGTH ='//F8.4/'//)
4110      END
4111      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
4112      C
4113      SUBROUTINE 27
4114      C

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3503 C
3504 C
3505 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
3506 C
3507 C SUBROUTINE PLATE3
3508 C
3509 C THIS SUBROUTINE CALCULATES THE CRITICAL
3510 C STRESS, CRITICAL LOAD, AND BUCKLED CONFIGURATION
3511 C FOR A W-SHAPE SUBJECTED TO AN AXIAL LOAD
3512 C ELASTIC OR INELASTIC ANALYSIS MAY BE PERFORMED
3513 C FOR VARIOUS END CONDITIONS AND RESIDUAL STRESSES
3514 C MAY ALSO BE INCLUDED
3515 C
3516 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
3517 C SUBROUTINE PLATE3
3518 C IMPLICIT REAL*8(A-H,O-Z)
3519 C DIMENSION XVEC(15,20)
3520 C COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
3521 C COMMON /BLK2/CB(7,9),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5),FB5(5,5)
3522 C COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
3523 C COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
3524 C COMMON /BLK8/CT(7,9),FT1(5,5),FT2(5,5),FT3(5,5),FT4(5,5),FT5(5,5)
3525 C COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
3526 C COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
3527 C COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
3528 C READ (5,170) YSW,YSFB,YSFT,GAM1,GAM2,GAM3
3529 C READ (5,180) BCL,HW,TW,BF1,TF1,BF2,TF2
3530 C READ (5,120) E1,E2,ESH
3531 C
3532 C
3533 C CALCULATE SHAPE FUNCTION COEFFICIENT MATRICES
3534 C
3535 C CALL CDEFB(BF1)
3536 C CALL COEFT(BF2)
3537 C CALL COEFW
3538 C
3539 C
3540 C CALCULATE THE FOLLOWING:
3541 C
3542 C EYW = YIELD STRAIN IN THE WEB
3543 C EYBF = YIELD STRAIN IN THE BOTTOM FLANGE
3544 C EYTF = YIELD STRAIN IN THE TOP FLANGE
3545 C AREA = CROSS-SECTIONAL AREA
3546 C ERTB = RESIDUAL TENSILE STRAIN AT WEB BOTTOM
3547 C ERCM = RESIDUAL COMPRESSIVE STRAIN AT WEB
3548 C MIDDLE
3549 C ERTT = RESIDUAL COMPRESSIVE STRAIN AT WEB
3550 C TOP
3551 C ERCB = RESIDUAL COMPRESSIVE STRAIN AT
3552 C BOTTOM FLANGE
3553 C ERCT = RESIDUAL COMPRESSIVE STRAIN AT TOP
3554 C FLANGE
3555 C YB = DISTANCE TO CENTROID FROM BOTTOM OF
3556 C SECTION
3557 C ERTBAR = MOMENT OF INERTIA
3558 C Y1 = DISTANCE TO THE CENTROID FROM
3559 C MID-HEIGHT
3560 C R = RATIO OF STRAIN IN COMPRESSION
3561 C FLANGE/TENSION
3562 C FLANGE (BENDING)
3563 C
3564 C EYW=YSW/E
3565 C EYBF=YSFB/E
3566 C EYTF=YSFT/E
3567 C AF1=BF1*TF1
3568 C AF2=BF2*TF2
3569 C AFS=AF1+AF2
3570 C AW=HW*TW
3571 C AREA=AFS+AW
3572 C ERTB=GAM1+EYW
3573 C ERCM=GAM2+EYW
3574 C ERTT=GAM3+EYW
3575 C FA1=AW/(12.0DO*AF1)
3576 C FA2=AW/(12.0DO*AF2)
3577 C SER=6.0DO*ERCM
3578 C FERT=5.0DO*ERTT
3579 C FERB=5.0DO*ERCB
3580 C ERCB=ERTB-FA1*(SER-ERTT-FERB)
3581 C ERCT=ERTT+FA2*(SER-FERT-ERTB)
3582 C SRTB=ERTB+E
3583 C SPCM=ERCM+E
3584 C SRTT=ERTT+E
3585 C SRCB=ERCB+E
3586 C SRCT=ERCT+E
3587 C YB=(0.5DO*AW+AF2)*HW/AREA
3588 C ERT1=TW*HW**3/12.0DO
3589 C ERT2=BF2*TF2**3/12.0DO
3590 C ERT3=BF1*TF1**3/12.0DO
3591 C EPT4=AFS*HW*HW/4.0DO
3592 C EPT1A=ERT1+EPT2+EPT3+EPT4
3593 C Y1=YB-HW/2.0DO
3594 C ERTBAR=EPT1A+AREA*Y1**2
3595 C DEN=0.5DO*HW-Y1
3596 C IF (DEN.EQ.0.0DO) GO TO 10
3597 C RNUM=0.5DO*HW+Y1
3598 C R=RNUM/DEN
3599 C GO TO 20
3600 C 10 R=1.0DO
3601 C
3602 C
3603 C ECHD CHECK INPUT DATA.
3604 C
3605 C 20 WRITE (6,190) NS
3606 C IF (INEL.EQ.0) WRITE (6,130)
3607 C IF (INEL.EQ.1) WRITE (6,140)
3608 C WRITE (6,200) BCL,HW,TW,BF1,TF1,BF2,TF2
3609 C WRITE (6,160) AREA,YB,ERTBAR
3610 C WRITE (6,210) YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
3611 C WRITE (6,150) E1,E2,ESH
3612 C
3613 C
3614 C CALCULATE ELASTIC CRITICAL AXIAL STRESS.
3615 C

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3616      CALL FBFORM
3617      CALL FTFORM
3618      CALL WBFORM
3619      CALL ESFORM(EIGV,MM1,XVEC,&80)
3620      EIGV=EIGV/E
3621      T1=EYTF-ERCT
3622      T2=EYBF-ERCB
3623      T3=EYW-ERCM
3624      TM=DMIN1(T1,T2,T3)
3625
3626      C
3627      C
3628      C      IF SECTION BUCKLES ELASTICALLY OR IF INELASTIC
3629      C      ANALYSIS NOT REQUIRED WRITE RESULTS FOR ELASTIC
3630      C      BUCKLING. OTHERWISE PROCEED WITH INELASTIC ANALYSIS
3631      C
3632      CRST=EIGV/E
3633      IF (CRST.LE.TM) GO TO 70
3634      IF ((INEL.EQ.0) GO TO 70
3635      NSTOP=0
3636      EA1=TM
3637      EA2=EA1
3638
3639      C
3640      C      CALCULATE EIGENVALUE FOR FIRST ASSUMED STRAIN (EA1)
3641      C
3642      CALL FBINAX(EA1)
3643      CALL FTINAX(EA1)
3644      CALL WBINAX(EA1)
3645      CALL ESFORM(S1,MM1,XVEC,&80)
3646      S1=S1-1.0DO
3647      IF (S1.LT.0.0DO) GO TO 24
3648      GO TO 26
3649      NSTOP=NSTOP+1
3650      IF (NSTOP.GT.20) GO TO 115
3651      EA1=0.5DO*EA1
3652      GO TO 22
3653      NSTOP=0
3654      EA2=2.0*EA2
3655      NSTOP=NSTOP+1
3656      IF (NSTOP.GT.20) GO TO 110
3657
3658      C
3659      C      CALCULATE EIGENVALUE FOR SECOND ASSUMED STRAIN
3660      C      (EA2)
3661      C
3662      CALL FBINAX(EA2)
3663      CALL FTINAX(EA2)
3664      CALL WBINAX(EA2)
3665      CALL ESFORM(S2,MM1,XVEC,&80)
3666      S2=S2-1.0DO
3667      IF (S2.GE.0.0DO) GO TO 40
3668      NCOUNT=0
3669      GO TO 50
3670      EA1=EA2
3671      S1=S2
3672      GO TO 30
3673
3674      C
3675      C      BEGIN ITERATION USING METHOD OF BISECTION.
3676      C      NOTE: S3 IS THE EIGENVALUE STRAIN CALCULATED FOR A
3677      C      SECTION HAVING AN ASSUMED STRAIN (EA3). CONVERGENCE
3678      C      IS REACHED PERFECTLY WHEN S3=EA3 (I.E. S3-EA3=0.0)
3679      C
3680      C      50
3681      EA3=0.5DO*(EA1+EA2)
3682      CALL FBINAX(EA3)
3683      CALL FTINAX(EA3)
3684      CALL WBINAX(EA3)
3685      CALL ESFORM(S3,MM1,XVEC,&80)
3686      S3=S3-EA3
3687      NCOUNT=NCOUNT+1
3688      DS1=DABS(EA3-EA1)/EA1
3689      DS2=DABS(EA3-EA2)/EA2
3690
3691      C
3692      C      CHECK CONVERGENCE.
3693      C
3694      TL=0.001DO
3695      IF (DS1.LT.TL.OR.DS2.LT.TL) GO TO 50
3696      IF (NCOUNT.GT.50) GO TO 100
3697      S12=S1*S3
3698      IF (S12.LT.0.0DO) GO TO 60
3699      EA1=EA3
3700      S1=S3
3701      GO TO 50
3702      EA2=EA3
3703      S2=S3
3704      GO TO 50
3705
3706      C
3707      C      CALCULATION OF CRITICAL LOAD AND PLATE SLENDERNESS
3708      C      FOR ELASTIC CASES.
3709      C
3710      C      70
3711      CALL PSCALC(CRST,P,PPY,SAVE)
3712      SB1=0.5DO*BF1/TF1
3713      SE2=0.5DO*BF2/TF2
3714      SW1=(HW-0.5DO*(TF1+TF2))/TW
3715      SB1C=SB1*DSORT(Y5FB)
3716      SB2C=SE2*DSORT(Y5FT)
3717      SW1C=SW1*DSORT(Y5W)
3718      WRITE (6,250) P,PPY,SAVE,SB1,SB2,SW1,SB1C,SB2C,SW1C
3719      WAVE=BCL/MM1
3720      WRITE (6,220) EIGV,WAVE
3721      WRITE (6,240) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
3722      &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
3723      &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
3724      &MM1),XVEC(5,MM1),XVEC(4,MM1)
3725      RETURN
3726
3727      C
3728      C      CALCULATION OF CRITICAL AXIAL LOAD AND PLATE
3729      C      SLENDERNESSES FOR INELASTIC CASES
3730      C
3731      C      90
3732      CALL PSCALC(EA3,P,PPY,SAVE)

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3842 C
3843 C
3844 C      READ IN INPUT DATA.
3845 C
3846 READ (5,150) YSW,YSFB,YSFT,GAM1,GAM2,GAM3
3847 READ (5,160) BCL,HW,TW,BF1,TF1,BF2,TF2
3848 READ (5,120) E1,E2,ESH
3849 C
3850 C      CALCULATE SHAPE FUNCTION COEFFICIENT MATRICES
3851 C
3852 CALL COEFB(BF1)
3853 CALL COEFT(BF2)
3854 CALL COEFW
3855 C
3856 C
3857 C      CALCULATE THE FOLLOWING
3858 C
3859 C      EYW = YIELD STRAIN IN THE WEB
3860 C      EYBF = YIELD STRAIN IN THE BOTTOM FLANGE
3861 C      EYTF = YIELD STRAIN IN THE TOP FLANGE
3862 C      AREA = CROSS-SECTIONAL AREA
3863 C      ERTB = RESIDUAL TENSILE STRAIN AT WEB BOTTOM
3864 C      ERCM = RESIDUAL COMPRESSIVE STRAIN AT WEB
3865 C      MIDDLE
3866 C      ERTT = RESIDUAL COMPRESSIVE STRAIN AT WEB
3867 C      TOP
3868 C      ERCB = RESIDUAL COMPRESSIVE STRAIN AT BOTTOM
3869 C      FLANGE
3870 C      ERCT = RESIDUAL COMPRESSIVE STRAIN AT TOP
3871 C      FLANGE
3872 C      YB = DISTANCE TO CENTROID FROM BOTTOM OF
3873 C      SECTION
3874 C      ERTBAR = MOMENT OF INERTIA
3875 C      Y1 = DISTANCE TO THE CENTROID FROM
3876 C      MID-HEIGHT
3877 C      R = RATIO OF STRAIN IN COMPRESSION
3878 C      FLANGE/TENSION FLANGE (BENDING)
3879 C
3880 C      EYW=YSW/E
3881 C      EYBF=YSFB/E
3882 C      EYTF=YSFT/E
3883 C      AF1=BF1*TF1
3884 C      AF2=BF2*TF2
3885 C      AFS=AF1+AF2
3886 C      AW=HW*TW
3887 C      AREA=AFS+AW
3888 C      ERTB=GAM1+EYW
3889 C      ERCM=GAM2+EYW
3890 C      ERTT=GAM3+EYW
3891 C      FA1=AW/((12.0DO*AF1))
3892 C      FA2=AW/((12.0DO*AF2))
3893 C      SER=6.0DO*EPCW
3894 C      FERT=5.0DO*ERTT
3895 C      FERB=5.0DO*ERTB
3896 C      ERCB=ERTB-FA1*((SER-ERTT-FERB))
3897 C      ERCT=ERTT-FA2*((SER-FERT-ERTB))
3898 C      SRTB=ERTB+E
3899 C      SRCM=ERCM+E
3900 C      SRTT=ERTT+E
3901 C      SRCB=ERCB+E
3902 C      SRCT=ERCT+E
3903 C      YB=(0.5DO*AW+AF2)*HW/AREA
3904 C      EPT1=TW*HW**3/12.0DO
3905 C      EPT2=BF2*TF2**3/12.0DO
3906 C      EPT3=BF1*TF1**3/12.0DO
3907 C      EPT4=AFS*HW*HW**3/12.0DO
3908 C      ERTIA=ERT1+ERT2+ERT3+ERT4
3909 C      Y1=YB*HW/2.0DO
3910 C      ERTBAR=ERTIA+AREA*Y1**2
3911 C      PDEN=C.5DO*HW*Y1
3912 C      IF (RDEN.EQ.0.0DO) GO TO 10
3913 C      RNUM=C.5DO*HW*Y1
3914 C      R=RNUM/RDEN
3915 C      GO TO 20
3916 C      R=1.0DO
3917 C
3918 C
3919 C      ECHO CHECK INPUT DATA.
3920 C
3921 C
3922 C      WRITE (6,190) NS
3923 C      IF (INEL.EQ.0) WRITE (6,130)
3924 C      IF (INEL.EQ.1) WRITE (6,140)
3925 C      WRITE (6,200) BCL,HW,TW,BF1,TF1,BF2,TF2
3926 C      WRITE (6,180) AREA,YB,ERTBAR
3927 C      WRITE (6,210) YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
3928 C      WRITE (6,170) E1,E2,ESH
3929 C
3930 C
3931 C      CALCULATE ELASTIC CRITICAL STRESS
3932 C
3933 C      CALL FBFORM
3934 C      CALL FTFORM
3935 C      CALL WBFORM
3936 C      CALL ESFORM(EIGV,MM1,XVEC,&BO)
3937 C      EIGV=EIGV/E
3938 C      CRST=EIGV/E
3939 C      CRST1=CRST
3940 C      ECT1=2.0DO*Y1/((12.0DO*Y1*HW**3)+CRST1)
3941 C      T1=EYTF-ERCT
3942 C      T2=EYBF-ERTB
3943 C      T3=EYW-ERCM
3944 C      T4=EYW-ERTB
3945 C      T5=EYW-ERTT
3946 C
3947 C      IF SECTION BUCKLES ELASTICALLY OR IF INELASTIC
3948 C      ANALYSIS IS NOT REQUIRED WRITE OUT RESULTS AND
3949 C      RETURN TO MAIN SUBROUTINE FOR NEXT SPECIMEN
3950 C      OTHERWISE CONTINUE WITH INELASTIC ANALYSIS
3951 C
3952 C      TM1=DMIN1(T1,T3,T5)
3953 C      TM2=DMIN1(T2,T4)
3954 C      IF (CRST1 LE TM1 AND CPST1 LE TM2 AND EC LE T3 GO TO 70

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3555      IF (INEL.EQ 0) GO TO 70
3556      C
3557      C
3558      C      ASSUME A VALUE OF STRAIN, EA1, AND CALCULATE THE
3559      C      CORRESPONDING EIGENVALUE STRAIN, S1, SUCH THAT
3560      C      S1-EA1 > 0. SUBROUTINE SATEQ RESTORES EQUILIBRIUM
3561      C      TO THE SECTION
3562      C
3563      NSTOP=0
3564      EA1=DMIN1(TM1,TM2)
3565      22  EA2=EA1
3566      CALL SATEQ(EA1,0.0DO,0.0DO)
3567      CALL FBINBE(EA1)
3568      CALL FTINBE(EA1)
3569      CALL WBINBE(EA1)
3570      CALL ESFORM(S1,MM1,XVEC,&80)
3571      S1=S1-1.0DO
3572      IF (S1.LT.0.0DO) GO TO 24
3573      GO TO 26
3574      24  NSTOP=NSTOP+1
3575      IF (NSTOP.GT.20) GO TO 105
3576      EA1=0.5DO*EA1
3577      GO TO 22
3578      26  NSTOP=0
3579      C
3580      C
3581      C      DETERMINE A VALUE OF STRAIN, EA2, SUCH THAT
3582      C      S2-EA2 < 0
3583      C
3584      30  EA2=1.2DO*EA1+AA=EA2
3585      CALL SATEQ(EA2,0.0DO,0.0DO)
3586      NSTOP=NSTOP+1
3587      IF (NSTOP.GT.20) GO TO 110
3588      CALL FBINBE(EA2)
3589      CALL FTINBE(EA2)
3590      CALL WBINBE(EA2)
3591      CALL ESFORM(S2,MM1,XVEC,&80)
3592      S2=S2-1.0DO
3593      IF (S2.GE.0.0DO) GO TO 40
3594      NCOUNT=0
3595      GO TO 50
3596      40  EA1=EA2
3597      S1=S2
3598      GO TO 30
3599      C
4000      C
4001      C      DETERMINE EA3 BY THE METHOD OF BISECTION AND USE
4002      C      SATEQ TO FIND THE EQUILIBRIUM LOCATION OF THE
4003      C      NEUTRAL AXIS FOR THIS VALUE OF STRAIN. CALCULATE
4004      C      THE CORRESPONDING EIGENVALUE, S3, AND CHECK TO SEE
4005      C      IF S3-EA3 IS LESS THAN OR EQUAL TO THE TOLERANCE,
4006      C      TL.
4007      C
4008      50  EA3=0.5DO*(EA1+EA2)
4009      CALL SATEQ(EA3,0.0DO,0.0DO)
4010      CALL FBINBE(EA3)
4011      CALL FTINBE(EA3)
4012      CALL WBINBE(EA3)
4013      CALL ESFORM(S3,MM1,XVEC,&80)
4014      S3=S3-EA3
4015      NCOUNT=NCOUNT+1
4016      DS1=DABS((EA1-EA3)/EA1)
4017      DS2=DABS((EA2-EA3)/EA2)
4018      C
4019      C
4020      C      IF CONVERGENCE HAS BEEN REACHED WRITE OUT RESULTS
4021      C      OTHERWISE CONTINUE ITERATIONS UP TO A MAXIMUM OF
4022      C      50. IF CONVERGENCE ISNOT REACHED WRITE OUT MESSAGE
4023      C      AND RETURN FOR A NEW SPECIMEN.
4024      C
4025      TL=0.001DO
4026      IF (DS1.LT.TL OR DS2.LT.TL) GO TO 90
4027      IF (NCOUNT.GT.50) GO TO 100
4028      S13=S1*S3
4029      IF (S13.LT.0.0DO) GO TO 60
4030      EA1=EA3
4031      S1=S3
4032      GO TO 50
4033      60  EA2=EA3
4034      S2=S3
4035      GO TO 50
4036      C
4037      C
4038      C      CALCULATE PLATE SLENDERNESSES (ELASTIC CASE)
4039      C
4040      70  SB1=0.5DO*BF1/TF1
4041      SB2=0.5DO*BF2/TF2
4042      SW1=(HW-0.5DO*(TF1+TF2))/TW
4043      SB1C=SB1*DSORT(YSEB)
4044      SB2C=SB2*DSORT(YSFT)
4045      SW1C=SW1*DSORT(YSW)
4046      C
4047      C
4048      C      CALCULATE CRITICAL BENDING MOMENT (ELASTIC CASE)
4049      C
4050      CALL MCALC(CRST,0.0DO,BM,BMY,BMP,BMMP,DNA,RDNA)
4051      C
4052      C
4053      C      WRITE OUT RESULTS (ELASTIC CASE)
4054      C
4055      WRITE (6,260) BM,BMY,BMMP,DNA,RDNA,SB1,SB2,SW1,SB1C,SB2C,SW1C
4056      WAVE=BCL/MM1
4057      WRITE (6,220) EIGV,WAVE
4058      WRITE (6,240) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4059      &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4060      &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4061      &,MM1),XVEC(5,MM1),XVEC(4,MM1)
4062      80  RETURN
4063      C
4064      C
4065      C      CALCULATE PLATE SLENDERNESSES (INELASTIC CASE)
4066      C
4067      90  SB1=0.5DO*BF1/TF1

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4068      SE2=0.5DO*BF2/TF2
4069      SW1=(HW-O.5DO*(TF1+TF2))/TW
4070      SB1C=SB1*DSQRT(YSF8)
4071      SE2C=SB2*DSORT(YSFT)
4072      SW1C=SW1*DSORT(YSW)
4073
4074      C
4075      C
4076      C
4077      C      CALCULATE CRITICAL BENDING MOMENT (INELASTIC CASE)
4078
4079      C      CALL MCALC(EA3,0.0DO,BM,BMY,BMP,BMPP,DNA,RDNA)
4080
4081      C      WRITE OUT RESULTS (INELASTIC CASE).
4082
4083      C      WRITE (6,260) BM,BMY,BMPP,DNA,RDNA,SB1,SB2,SW1,SB1C,SB2C,SW1C
4084      C      WAVE=BCL/MM1
4085      C      WRITE (6,230) EA3,WAVE
4086      C      WRITE (6,240) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4087      C      &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4088      C      &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4089      C      &,MM1),XVEC(5,MM1),XVEC(4,MM1)
4090      C      RETURN
4091      C
4092      C
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4096      C
4097      C
4098      C
4099      C      FORMAT STATEMENTS
4100      C
4101      C      FORMAT (3F12.6)
4102      C      FORMAT ('O',10X,'ELASTIC ANALYSIS.')
4103      C      FORMAT ('O',10X,'INELASTIC ANALYSIS.')
4104      C      FORMAT (6F8.4)
4105      C      FORMAT (7F8.5)
4106      C      FORMAT ('O',10X,'YIELD MODULUS' ,F9.4,' KSI')
4107      C      &STRAIN HARDENING MODULUS ,F9.4,' KSI')
4108      C      &STRAIN HARDENING STRAIN ,F9.5)
4109      C      FORMAT ('O',10X,'AREA OF CROSS-SECTION' ,F9.4,' SQ IN
4110      C      &'O',10X,'DISTANCE TO CENTROID' ,F9.4,' IN')
4111      C      &'O',10X,'FROM CENTER OF BOTTOM FL
4112      C      &ANGE' ,F9.4,' INCHES')
4113      C      &'O',10X,'CENTROIDAL MOMENT OF INERTIA
4114      C      &'O',10X,'INCHES')
4115      C      &'O',10X,'WIDE FLANGE SECTION',13,'O',10X,'SUBJECTE
4116      C      &D TO PURE BENDING.')
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4995      C
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4997      C
4998      C
4999      C
5000      C

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4181      IMPLICIT REAL*8(A-H,D-Z)
4182      DIMENSION XVEC(15,20)
4183      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
4184      COMMON /BLK2/CB(7,9),FB1(5,5),FB2(5,5),FB3(5,5),FB4(5,5) FB5(5,5)
4185      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
4186      COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
4187      COMMON /BLK8/CT(7,9),FT1(5,5),FT2(5,5),FT3(5,5),FT4(5,5),FT5(5,5)
4188      COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
4189      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
4190      COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCE,SRCT
4191      C
4192      C
4193      C      READ IN PLATE DIMENSIONS AND MATERIAL PROPERTIES
4194      C
4195      10 READ (5,310) YSW,YSFB,YSFT,GAM1,GAM2,GAM3
4196      READ (5,320) BCL,HW,TW,BF1,TF1,BF2,TF2
4197      READ (5,290) E1,E2,ESH
4198      C
4199      C
4200      C      FORMULATE MATRICES OF COEFFICIENTS OF SHAPE
4201      C      FUNCTIONS
4202      C
4203      CALL COEFB(BF1)
4204      CALL COEFT(BF2)
4205      CALL COEFW
4206      ICASE=1
4207      C
4208      C
4209      C      CALCULATE THE FOLLOWING
4210      C
4211      C      EYW      = YIELD STRAIN IN THE WEB
4212      C      EYBF     = YIELD STRAIN IN THE BOTTOM FLANGE
4213      C      EYTF     = YIELD STRAIN IN THE TOP FLANGE
4214      C      AREA     = CROSS-SECTIONAL AREA
4215      C      ERTB     = RESIDUAL TENSILE STRAIN AT WEB BOTTOM
4216      C      ERCM     = RESIDUAL COMPRESSIVE STRAIN AT WEB
4217      C      MIDDLE
4218      C      ERTT     = RESIDUAL COMPRESSIVE STRAIN AT WEB
4219      C      TOP
4220      C      ERCB     = RESIDUAL COMPRESSIVE STRAIN AT BOTTOM
4221      C      FLANGE
4222      C      ERCT     = RESIDUAL COMPRESSIVE STRAIN AT TOP
4223      C      FLANGE
4224      C      YB       = DISTANCE TO CENTROID FROM BOTTOM OF
4225      C      SECTION
4226      C      ERTBAR   = MOMENT OF INERTIA
4227      C      Y1       = DISTANCE TO THE CENTROID FROM
4228      C      MID-HEIGHT
4229      C      R        = RATIO OF STRAIN IN COMPRESSION
4230      C      FLANGE/TENSION FLANGE (BENDING)
4231      C
4232      AF1=BF1*TF1
4233      AF2=BF2*TF2
4234      AFS=AF1+AF2
4235      AW=HW*TW
4236      AREA=AFS+AW
4237      EYW=YSW/E
4238      EYBF=YSFB/E
4239      EYTF=YSFT/E
4240      ERTB=GAM1+EYW
4241      ERCM=GAM2+EYW
4242      ERTT=GAM3+EYW
4243      FA1=AW/(12.0DO*AF1)
4244      FA2=AW/(12.0DO*AF2)
4245      SER=6.0DO*ERCM
4246      FERT=5.0DO*ERTT
4247      FERB=5.0DO*ERTB
4248      ERCB=ERTB-FA1*(SER-ERTT-FERB)
4249      ERCT=ERTT-FA2*(SER-FERT-ERTB)
4250      SRTB=ERTB+E
4251      SRCM=ERCM+E
4252      SRTT=ERTT+E
4253      SRCB=ERCB+E
4254      SRCT=ERCT+E
4255      YB=(0.5DO*AW+AF2)*HW/AREA
4256      ERT1=TW*HW**3/12.0DO
4257      ERT2=BF2*TF2**3/12.0DO
4258      ERT3=BF1*TF1**3/12.0DO
4259      ERT4=AFS*HW*HW*0.25DO
4260      ERT1A=ERT1+ERT2+ERT3+ERT4
4261      Y1=YB-HW/2.0DO
4262      ERTBAR=ERT1A+AREA*Y1**2
4263      RDEN=0.5DO*HW*Y1
4264      IF (RDEN.EQ.0.0DO) GO TO 20
4265      RNUM=0.5DO*HW*Y1
4266      R=RNUM/RDEN
4267      GO TO 30
4268      20 R=1.0DO
4269      30 READ (5,300) ALAR
4270      AX1=YSFT*AF2
4271      AX2=YSFB*AF1
4272      AX3=YSW*AW
4273      AXLD=ALAR*(AX1+AX2+AX3)
4274      C
4275      C
4276      C      ECHO CHECK INPUT DATA
4277      C
4278      IF (INEL.EQ.1) WRITE (6,390) NS
4279      IF (INEL.EQ.0) WRITE (6,330) NS,AXLD
4280      IF (INEL.EQ.0) WRITE (6,400)
4281      IF (INEL.EQ.1) WRITE (6,410)
4282      WRITE (6,340) ALAR
4283      WRITE (6,420) BCL,HW,TW,BF1,TF1,BF2,TF2
4284      WRITE (6,450) AREA,YB,ERTBAR
4285      WRITE (6,430) YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCE,SRCT
4286      WRITE (6,440) E1,E2,ESH
4287      IF (INEL.EQ.1) GO TO 50
4288      LKEEP=LTYPE
4289      C
4290      C
4291      C      CALCULATE ELASTIC CRITICAL AXIAL LOAD
4292      C
4293      LTYPE=1

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4294      CALL FBFORM
4295      CALL FTFORM
4296      CALL WBFORM
4297      CALL ESFORM(EIGV,MM1,XVEC,&210)
4298      EIGV=EIGV/E
4299      CRST=EIGV/E
4300      APPS=AXLO/AREA
4301      C
4302      C
4303      C      IF APPLIED LOAD EXCEEDS CRITICAL LOAD WRITE MESSAGE
4304      C      AND INFORMATION FOR AXIAL BUCKLING OTHERWISE
4305      C      CALCULATE CRITICAL SUPERIMPOSED BENDING MOMENT
4306      C
4307      IF (APPS GE EIGV) GO TO 40
4308      LTYPE=LKEEP
4309      CALL FBFORM
4310      CALL FTFORM
4311      CALL WBFORM
4312      CALL ESFORM(EIGV,MM1,XVEC,&210)
4313      EIGV=EIGV/E
4314      CRST1=EIGV/E
4315      AXAR=AXLO/AREA
4316      EAPP=EAPP/E
4317      C
4318      C
4319      C      CALCULATE PLATE SLENDERNESSES
4320      C
4321      SB1=C 500*BF1/TF1
4322      SB2=C 500*BF2/TF2
4323      SW1=(HW-C 500*(TF1+TF2))/TW
4324      SB1C=SB1*DSORT(YSF8)
4325      SB2C=SB2*DSORT(YSFT)
4326      SW1C=SW1*DSORT(YSW)
4327      C
4328      C
4329      C      CALCULATE BENDING MOMENT AND WRITE OUT RESULTS FOR
4330      C      AXIAL BUCKLING
4331      C
4332      CALL MCALC(CRST1,EAPP,BM,BMY,BMP,BMPP,DNA,RDNA)
4333      WRITE (6,530) BM,BMY,BMP,BMPP,DNA,RDNA,SB1,SB2,SW1,SB1C,SB2C,SW1C
4334      WAVE=BCL/MM1
4335      WRITE (6,470) EIGV,AXAR,WAVE
4336      WRITE (6,520) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4337      &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4338      &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4339      &,MM1),XVEC(5,MM1),XVEC(4,MM1)
4340      RETURN
4341      40 C
4342      WAVE=BCL/MM1
4343      C
4344      C
4345      C      CALCULATE PLATE SLENDERNESSES
4346      C
4347      SB1=C 500*BF1/TF1
4348      SB2=C 500*BF2/TF2
4349      SW1=(HW-C 500*(TF1+TF2))/TW
4350      SB1C=SB1*DSORT(YSF8)
4351      SB2C=SB2*DSORT(YSFT)
4352      SW1C=SW1*DSORT(YSW)
4353      C
4354      C
4355      C      WRITE OUT RESULTS FOR ELASTIC BUCKLING DUE TO AXIAL
4356      C      PLUS BENDING STRESSES AND RETURN TO MAIN PROGRAM
4357      C
4358      CALL PSCALC(CRST,P,PPY,SAVE)
4359      WRITE (6,540) P,PPY,SAVE,SB1,SB2,SW1,SB1C,SB2C,SW1C
4360      WRITE (6,460) APPS,EIGV,WAVE
4361      WRITE (6,520) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4362      &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4363      &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4364      &,MM1),XVEC(5,MM1),XVEC(4,MM1)
4365      RETURN
4366      C
4367      C
4368      C      CALCULATE CRITICAL AXIAL LOAD FOR INELASTIC
4369      C      ANALYSIS SEE PLATE3 FOR STEPS
4370      C
4371      50 LKEEP=LTYPE
4372      LTYPE=L1
4373      T1=EN*F-ERC*
4374      T2=EN*F-ERCB
4375      T3=EN*F-ERCM
4376      TM=DM/N*(T1-T2-T3)
4377      CALL FBFORM
4378      CALL FTFORM
4379      CALL WBFORM
4380      CALL ESFORM(EIGV,MM1,XVEC,&240)
4381      EIGV=EIGV/E
4382      CRST=EIGV/E
4383      IF (CRST LE TM) GO TO 100
4384      NSTOP=C
4385      EAL=TM
4386      EAL=EAL
4387      CALL FBINAX(EA1)
4388      CALL FTINAX(EA1)
4389      CALL WBINAX(EA1)
4390      CALL ESFORM(S1,MM1,XVEC,&240)
4391      S1=S1-1 ODO
4392      IF (S1 LT 0 ODO) GO TO 54
4393      GO TO 56
4394      54 NSTOP=NSTOP+1
4395      IF (NSTOP GT 20) GO TO 285
4396      EAL=C 500*EAL
4397      GO TO 52
4398      56 NSTOP=C
4399      60 EAL=1 200*EAL+AA*EAL
4400      NSTOP=NSTOP+1
4401      IF (NSTOP GT 20) GO TO 270
4402      CALL FBINAX(EA2)
4403      CALL FTINAX(EA2)
4404      CALL WBINAX(EA2)
4405      CALL ESFORM(S2,MM1,XVEC,&240)
4406      S2=S2-1 ODO
4407      IF (S2 GE 0 ODO) GO TO 70

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4407      NCOUNT=0
4408      GO TO 80
4409      70    EA1=EA2
4410          S1=S2
4411          GO TO 80
4412      80    EA3=O SDO=(EA1+EA2)
4413          CALL FBINAX(EA3)
4414          CALL FTINAX(EA3)
4415          CALL WBINAX(EA3)
4416          CALL ESFORM(S3,MM1,XVEC,&240)
4417          S3=S3-1.ODO
4418          NCOUNT=NCOUNT+1
4419          DS1=DABS((EA1-EA3)/EA1)
4420          DS2=DABS((EA2-EA3)/EA2)
4421          TL=O.OO1DO
4422          IF (DS1.LT.TL OR DS2.LT.TL) GO TO 110
4423          IF (NCOUNT.GT.50) GO TO 200
4424          S13=S1+S3
4425          IF (S13.LT.O ODO) GO TO 90
4426          EA1=EA3
4427          S1=S3
4428          GO TO 80
4429      90    EA2=EA3
4430          S2=S3
4431          GO TO 80
4432      100   STRAIN=CRST
4433          GO TO 120
4434      110   STRAIN=EA3
4435      120   CALL PSCALC(STRAIN,P,PPY,SAVE)
4436          SB1=O SDO=BF1/TF1
4437          SB2=O SDO=BF2/TF2
4438          SW1=(HW-O SDO*(TF1+TF2))/TW
4439          SB1C=SB1*DSORT(YSFB)
4440          SB2C=SB2*DSORT(YSFT)
4441          SW1C=SW1*DSORT(YSW)
4442          WRITE (6,540) P,PPY,SAVE,SB1,SB2,SW1,SB1C,SB2C,SW1C
4443          WAVE=BCL/MM1
4444          WRITE (6,510) STRAIN,WAVE
4445          WRITE (6,520) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4446          &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4447          &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4448          &,MM1),XVEC(5,MM1),XVEC(4,MM1)
4449      C
4450      C
4451      C      IF THE APPLIED AXIAL LOAD IS GREATER THAN THE
4452      C      CRITICAL AXIAL LOAD, WRITE OUT MESSAGE. OTHERWISE
4453      C      CONTINUE
4454      C
4455      130   IF (AXLO GE.P) GO TO 230
4456          WRITE (6,350) ICASE,AXLO
4457          WRITE (6,340) ALAR
4458      C
4459      C
4460      C      DETERMINE THE STRAIN CORRESPONDING TO THE APPLIED
4461      C      AXIAL LOAD
4462      C
4463      PBOY=TM+E*AREA
4464      IF (AXLO.LE.PBOY) GO TO 140
4465      CALL GETST(AXLO,EAPP,&210)
4466      GO TO 150
4467      140   EAPP=AXLO/AREA/E
4468      150   LTYPE=LKEEP
4469          WRITE (6,360) STRAIN,EAPP
4470      C
4471      C
4472      C      BEGIN CALCULATION OF CRITICAL BENDING STRAIN.
4473      C      ASSUME INITIAL STRAIN, ESB1, AND RESTORE
4474      C      EQUILIBRIUM TO THE SECTION USING SUBROUTINE SATEO
4475      C      CALCULATE THE CORRESPONDING EIGENVALUE STRAIN SAB1
4476      C      (IDEALLY EQUAL TO ESB1+EAPP, WHERE EAPP IS THE
4477      C      APPLIED AXIAL STRAIN).
4478      C
4479      NSTOP=O
4480      ESB1=O.1DO*(STRAIN-EAPP)
4481      152   ESB2=ESB1
4482          CALL SATEO(ESB1,EAPP,AXLO)
4483          CALL FBINAR(ESB1,EAPP)
4484          CALL FTINAB(ESB1,EAPP)
4485          CALL WBINAB(ESB1,EAPP)
4486          CALL ESFORM(SAB1,MM1,XVEC,&240)
4487          SAB1=SAB1-ESB1-EAPP
4488          IF (SAB1.LT.O ODO) GO TO 154
4489          GO TO 156
4490      154   NSTOP=NSTOP+1
4491          IF (NSTOP.GT.20) GO TO 255
4492          ESB1=O SDO=ESB1
4493      156   NSTOP=O
4494      160   ESB2=1.2DO*ESB1+44*ESB2
4495          CALL SATEO(ESB2,EAPP,AXLO)
4496          NSTOP=NSTOP+1
4497          IF (NSTOP.GT.20) GO TO 260
4498      C
4499      C
4500      C      DETERMINE THE VALUE OF ESB2 SUCH THAT
4501      C      SAB2-ESB2-EAPP < O
4502      C
4503          CALL FBINAB(ESB2,EAPP)
4504          CALL FTINAB(ESB2,EAPP)
4505          CALL WBINAB(ESB2,EAPP)
4506          CALL ESFORM(SAB2,MM1,XVEC,&240)
4507          SAB2=SAB2-ESB2-EAPP
4508          IF (SAB2 GE.O ODO) GO TO 170
4509          NCOUNT=O
4510          GO TO 180
4511      170   ESB1=ESB2
4512          SAB1=SAB2
4513          GO TO 160
4514      180   ESB3=O SDO=(ESB1+ESB2)
4515      C
4516      C
4517      C      DETERMINE ESB3 BY THE METHOD OF BISECTION AND
4518      C      CONTINUE ITERATIONS UP TO A MAXIMUM OF 50
4519      C      CONVERGENCE IS REACHED WHEN SAB3-EAB3-EAPP IS CLOSE

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4520 C          ENOUGH TO ZERO
4521 C
4522 CALL SATEO(ESB3,EAPP,AXLD)
4523 CALL FBINAB(ESB3,EAPP)
4524 CALL FTINAB(ESB3,EAPP)
4525 CALL WBINAB(ESB3,EAPP)
4526 CALL ESFORM(SAB3,MM1,XVEC,240)
4527 SAB3=SAB3-ESB3-EAPP
4528 NCOUNT=NCOUNT+1
4529 DS1=DABS((ESB3-ESB1)/ESB1)
4530 DS2=DABS((ESB2-ESB3)/ESB2)
4531 TL=0.00100
4532 IF (DS1.LT.TL OR DS2.LT.TL) GO TO 220
4533 IF (NCOUNT.GT.50) GO TO 250
4534 SAB13=SAB1+SAB3
4535 IF (SAB13.LT.0.000) GO TO 190
4536 ESB1=ESB3
4537 SAB1=SAB3
4538 GO TO 180
4539 ESB2=ESB3
4540 SAB2=SAB3
4541 GO TO 180
4542 200 WRITE (6,480)
4543 210 RETURN
4544 C
4545 C
4546 C          CALCULATE PLATE SLENDERNESSES
4547 C
4548 220 SB1=0.500*BF1/TF1
4549 SB2=0.500*BF2/TF2
4550 SW1=(HW-0.500*(TF1+TF2))/TW
4551 SB1C=SB1*DSORT(Y5FB)
4552 SB2C=SB2*DSORT(Y5FB)
4553 SW1C=SW1*DSORT(Y5W)
4554 C
4555 C
4556 C          CALCULATE CRITICAL BENDING MOMENT CORRESPONDING TO
4557 C          CRITICAL STRAIN, ESB3, AND WRITE OUT RESULTS
4558 C
4559 CALL MCALC(ESB3,EAPP,BM,BMY,BMP,BMPP,DNA,RDNA)
4560 WRITE (6,530) BM,BMY,BMP,BMPP,DNA,RDNA,SB1,SB2,SW1,SB1C,SB2C,SW1C
4561 WAVE=BCL/MM1
4562 RATIO=ESB3/EYTF
4563 WRITE (6,570) RATIO
4564 WRITE (6,510) ESB3,WAVE
4565 WRITE (6,520) XVEC(12,MM1),XVEC(14,MM1),XVEC(13,MM1),XVEC(11,MM1),
4566 &XVEC(15,MM1),XVEC(11,MM1),XVEC(10,MM1),XVEC(9,MM1),XVEC(8,MM1),
4567 &XVEC(7,MM1),XVEC(5,MM1),XVEC(6,MM1),XVEC(1,MM1),XVEC(3,MM1),XVEC(2
4568 &MM1),XVEC(5,MM1),XVEC(4,MM1)
4569 GO TO 240
4570 230 WRITE (6,380) ICASE,AXLD
4571 WRITE (6,340) ALAR
4572 WRITE (6,490)
4573 C
4574 C
4575 C          READ IN NEXT VALUE OF ALAR = P/PY WHERE P IS THE
4576 C          APPLIED AXIAL LOAD AND PY IS THE YIELD LOAD
4577 C          CONTINUE WITH CALCULATIONS OF CRITICAL BENDING
4578 C          STRESS FOR THIS AXIAL LOAD.
4579 C
4580 240 READ (5,300) ALAR
4581 ICASE=ICASE+1
4582 ALAR1=0.000
4583 ALAR2=-1.000
4584 IF (ALAR.LT.ALAR2) RETURN
4585 IF (ALAR.LT.ALAR1) GO TO 10
4586 AXLD=ALAR*(AX1+AX2+AX3)
4587 IF (AXLD.GE.P) GO TO 230
4588 GO TO 130
4589 250 WRITE (6,500)
4590 RETURN
4591 260 WRITE (6,560)
4592 STOP
4593 270 WRITE (6,550)
4594 STOP
4595 285 WRITE (6,580)
4596 STOP
4597 285 WRITE (6,570)
4598 STOP
4599 C
4600 C
4601 C          FORMAT STATEMENTS
4602 C
4603 280 FORMAT (3F12.6)
4604 300 FORMAT (F12.6)
4605 310 FORMAT (6F12.6)
4606 320 FORMAT (7F10.6)
4607 330 FORMAT ('11//5//10X,'WIDE FLANGE SECTION',13//10X,'SUBJECT
4608 &ED TO AN APPLIED AXIAL LOAD OF',F10.4,' KIPS PLUS BENDING.')
4609 340 FORMAT ('0//0//10X,'P(APPLIED)/P(YIELD)',F9.4)
4610 350 FORMAT ('1//0//10X,'BEAM-COLUMN CASE NO.',13//0//10X,'SUBJECTE
4611 &D TO AN APPLIED AXIAL LOAD OF',F12.6,' KIPS PLUS BENDING.')
4612 360 FORMAT ('0//10X,'CRITICAL AXIAL STRAIN',F12.6//0//10X
4613 &,'APPLIED AXIAL STRAIN',F12.6)
4614 370 FORMAT ('0//10X,'CRIT. BEND. STRAIN/YIELD STRAIN',F9.5)
4615 380 FORMAT ('0//0//10X,'BEAM-COLUMN CASE NO.',13//0//10X,'SUBJECTE
4616 &D TO AN APPLIED AXIAL LOAD OF',F12.6,' KIPS PLUS BENDING.')
4617 390 FORMAT ('1//0//10X,'WIDE FLANGE SECTION',13//0//10X,'CALCULAT
4618 &ION OF CRITICAL AXIAL LOAD.')
4619 400 FORMAT ('0//10X,'ELASTIC ANALYSIS.')
4620 410 FORMAT ('0//10X,'INELASTIC ANALYSIS.')
4621 420 FORMAT ('0//0//10X,'LENGTH OF WIDE FLANGE',F9.4,' IN
4622 &CHES//0//10X,'WEB DEPTH',F9.5,' INCHES//
4623 &0//10X,'WEB THICKNESS',F9.5,' INCHES//0//
4624 &10X,'BOTTOM FLANGE WIDTH',F9.5,' INCHES//0//10X,'B
4625 &OTTOM FLANGE THICKNESS',F9.5,' INCHES//0//10X,'TOP FLA
4626 &NGE WIDTH',F9.5,' INCHES//0//10X,'TOP FLANGE TH
4627 &ICKNESS',F9.5,' INCHES')
4628 430 FORMAT ('0//10X,'WEB YIELD STRESS',F9.4,' KSI//
4629 &0//10X,'BOTTOM FLANGE YIELD STRESS',F9.4,' KSI//0//10X,'T
4630 &OP FLANGE YIELD STRESS',F9.4,' KSI//0//10X,'RES TENS
4631 &S STRESS (BOTTL.)',F9.5,' KSI//0//10X,'RES COMP STRESS
4632 &(MID WEB)',F9.5,' KSI//0//10X,'RES TENS STRESS (TOP)

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[illegible]



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4746      ALFP=1.ODO
4747      GO TO 60
4748 20     ALFT=0.ODO
4749      ALFP=1.ODO
4750      GO TO 60
4751 30     IF (AXST.GT.T3) GO TO 40
4752      ALFT=0.ODO
4753      ALFP=1.ODO
4754      GO TO 60
4755 40     IF (AXST.GT.T4) GO TO 50
4756      IF (ADEN.EQ.0.ODO) GO TO 50
4757      ANUM=T4-AXST
4758      ALFP=ANUM/ADEN
4759      ALFT=0.ODO
4760      GO TO 60
4761 50     ALFP=0.ODO
4762      ALFT=0.ODO
4763      C
4764      C
4765      C      CALCULATE FORCES IN BOTTOM FLANGE FOR ELASTIC,
4766      C      YIELDING, AND STRAIN-HARDENING REGIONS.
4767      C
4768 60     A1=0.5DO*BF1*TF1*ALFT
4769      A2=2.ODO*(AXST-ERTB)*E
4770      A3=ADEN*E*ALFT
4771      F11=A1*(A2+A3)
4772      A1=0.5DO*BF1*TF1*(ALFP-ALFT)
4773      A2=2.ODO*(YSFB*(AXST-T2)*E1)
4774      A3=ADEN*E1*(ALFP+ALFT)
4775      F12=A1*(A2+A3)
4776      A1=0.5DO*BF1*TF1*(1.ODO-ALFP)
4777      A2=2.ODO*(YSFB*(ESH-EYBF)*E1+(AXST-T4)*E2)
4778      A3=ADEN*E2*(1.ODO+ALFP)
4779      F13=A1*(A2+A3)
4780      F1=F11+F12+F13
4781      C
4782      C
4783      C      DEFINE STRAIN LIMITS AND LIMITS OF INTEGRATION FOR
4784      C      THE TOP COMPRESSION FLANGE.
4785      C
4786      T1=EYTF-ERCT
4787      T2=EYTF+ERTT
4788      T3=ESH+ERCT
4789      T4=ESH+ERTT
4790      ADEN=ERCT+ERTT
4791      IF (AXST.GT.T1) GO TO 70
4792      ALFT=1.ODO
4793      ALFP=1.ODO
4794      GO TO 120
4795 70     IF (AXST.GT.T2) GO TO 90
4796      IF (ADEN.EQ.0.ODO) GO TO 80
4797      ANUM=T2-AXST
4798      ALFT=ANUM/ADEN
4799      ALFP=1.ODO
4800      GO TO 120
4801 80     ALFT=0.ODO
4802      ALFP=1.ODO
4803      GO TO 120
4804 90     IF (AXST.GT.T3) GO TO 100
4805      ALFT=0.ODO
4806      ALFP=1.ODO
4807      GO TO 120
4808 100    IF (AXST.GT.T4) GO TO 110
4809      IF (ADEN.EQ.0.ODO) GO TO 110
4810      ANUM=T4-AXST
4811      ALFP=ANUM/ADEN
4812      ALFT=0.ODO
4813      GO TO 120
4814 110    ALFP=0.ODO
4815      ALFT=0.ODO
4816      C
4817      C
4818      C      CALCULATE FORCES IN TOP COMPRESSION FLANGE FOR
4819      C      ELASTIC, YIELDING, AND STRAIN-HARDENING REGIONS.
4820      C
4821 120    A1=0.5DO*BF2*TF2*ALFT
4822      A2=2.ODO*(AXST-ERTT)*E
4823      A3=ADEN*E*ALFT
4824      F21=A1*(A2+A3)
4825      A1=0.5DO*BF2*TF2*(ALFP-ALFT)
4826      A2=2.ODO*(YSFT*(AXST-T2)*E1)
4827      A3=ADEN*E1*(ALFP+ALFT)
4828      F22=A1*(A2+A3)
4829      A1=0.5DO*BF2*TF2*(1.ODO-ALFP)
4830      A2=2.ODO*(YSFT*(ESH-EYTF)*E1+(AXST-T4)*E2)
4831      A3=ADEN*E2*(1.ODO+ALFP)
4832      F23=A1*(A2+A3)
4833      F2=F21+F22+F23
4834      C
4835      C
4836      C      DEFINE STRAIN LIMITS AND LIMITS OF INTEGRATION FOR
4837      C      THE WEB FORCES.
4838      C
4839      T1=EYW-ERCM
4840      T2=EYW+ERTB
4841      T3=ESH-ERCM
4842      T4=ESH+ERTB
4843      T5=EYW+ERTT
4844      T6=ESH+ERTT
4845      AD1=ERTB+ERCM
4846      AD2=ERTT+ERCM
4847      IF (AXST.GT.T1) GO TO 130
4848      BETB=0.ODO
4849      BETBP=0.ODO
4850      GO TO 180
4851 130    IF (AXST.GT.T2) GO TO 150
4852      IF (AD1.EQ.0.ODO) GO TO 140
4853      AN1=AXST-T1
4854      BETB=AN1/AD1
4855      BETBP=0.ODO
4856      GO TO 180
4857 140    BETB=1.ODO
4858      BETBP=0.ODO

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4859      GO TO 180
4860      IF (AXST.GT.T3) GO TO 160
4861      BETB=1.0DO
4862      BETTP=0.0DO
4863      GO TO 180
4864      IF (AXST.GT.T4) GO TO 170
4865      IF (AD1.EQ.0.0DO) GO TO 170
4866      AN1=AXST-T3
4867      BETB=1.0DO
4868      BETTP=AN1/AD1
4869      GO TO 180
4870      BETB=1.0DO
4871      BETTP=1.0DO
4872      IF (AXST.GT.T1) GO TO 190
4873      BETT=0.0DO
4874      BETTP=0.0DO
4875      GO TO 240
4876      IF (AXST.GT.T5) GO TO 210
4877      IF (AD2.EQ.0.0DO) GO TO 200
4878      AN2=AXST-T1
4879      BETT=AN2/AD2
4880      BETTP=0.0DO
4881      GO TO 240
4882      BETT=1.0DO
4883      BETTP=0.0DO
4884      GO TO 240
4885      IF (AXST.GT.T3) GO TO 220
4886      BETT=1.0DO
4887      BETTP=0.0DO
4888      GO TO 240
4889      IF (AXST.GT.T6) GO TO 230
4890      IF (AD2.EQ.0.0DO) GO TO 230
4891      BETT=1.0DO
4892      AN2=AXST-T3
4893      BETTP=AN2/AD2
4894      GO TO 240
4895      BETTP=1.0DO
4896      BETT=1.0DO
4897      C
4898      C
4899      C          CALCULATE FORCES IN WEB FOR ELASTIC, YIELDING,
4900      C          AND STRAIN-HARDENING REGIONS
4901      C
4902      240      A1=0.25*HW*TW*(1.0DO-BETB)
4903      A2=2.0DO*(AXST+ERCM)*E
4904      A3=AD1*E*(1.0DO+BETB)
4905      FW1=A1*(A2-A3)
4906      A1=0.25DO*HW*TW*(BETB-BETTP)
4907      A2=2.0DO*(YSW*(AXST-T1)*E1)
4908      A3=AD1*E1*(BETB+BETTP)
4909      FW2=A1*(A2-A3)
4910      A1=0.25DO*HW*TW*BETBP
4911      A2=2.0DO*(YSW*(ESH-EYW)*E1+(AXST-T3)*E2)
4912      A3=AD1*E2*BETBP
4913      FW3=A1*(A2-A3)
4914      A1=0.25DO*HW*TW*BETTP
4915      A2=2.0DO*(YSW*(ESH-EYW)*E1+(AXST-T3)*E2)
4916      A3=AD2*E2*BETTP
4917      FW4=A1*(A2-A3)
4918      A1=0.25DO*HW*TW*(BETT-BETTP)
4919      A2=2.0DO*(YSW*(AXST-T1)*E1)
4920      A3=AD2*E1*(BETT+BETTP)
4921      FW5=A1*(A2-A3)
4922      A1=0.25DO*HW*TW*(1.0DO-BETT)
4923      A2=2.0DO*(AXST+ERCM)*E
4924      A3=AD2*E*(1.0DO+BETT)
4925      FW6=A1*(A2-A3)
4926      FW=FW1+FW2+FW3+FW4+FW5+FW6
4927      PY=YSFB*BF1+TF1+YSFT*BF2+TF2+YSW*HW*TW
4928      C
4929      C
4930      C          AXP      = AXIAL LOAD
4931      C          AXPY     = AXIAL LOAD/YIELD LOAD
4932      C          AXSAV    = AVERAGE AXIAL STRESS
4933      C
4934      C          AXP=F1+F2+FW
4935      C          AXPY=AXP/PY
4936      C          AXSAV=AXP/AREA
4937      C          RETURN
4938      C          END
4939      C
4940      C
4941      C
4942      C          SUBROUTINE 31
4943      C
4944      C
4945      C
4946      C          SUBROUTINE START
4947      C
4948      C
4949      C          THIS ROUTINE INITIALISES A STARTING
4950      C          VECTOR, SVEC, FOR USE IN MATRIX ITERATION FOR
4951      C          VARIOUS TYPES OF LOADS AND BOUNDARY CONDITIONS
4952      C
4953      C
4954      C          SUBROUTINE START(ABG,SVEC,SVAL)
4955      C          IMPLICIT REAL*8(A-H,D-Z)
4956      C          DIMENSION ABG(15,15),SVEC(15),STAR(15)
4957      C          COMMON /BLK1/E,V,NS,1TYPE,LTYPE,INEL,N,ICLAMP,1KIND
4958      C          BMAX=0.0DO
4959      C
4960      C
4961      C          INITIALISE A STARTING VECTOR EQUAL TO THE
4962      C          MAIN DIAGONAL OF THE MATRIX ABG(I,I)
4963      C          DO 10 J=1,N
4964      C             SVEC(I)=ABG(I,I)
4965      C             SIZE=SVEC(I)
4966      C             IF (DABS(SIZE).LT.1.0D-40) SVEC(I)=0.0DO
4967      C             IF (DABS(SIZE).LE.BMAX) GO TO 10
4968      C             BMAX=DABS(SIZE)
4969      C             KS=1
4970      C          CONTINUE
4971      C

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5085      GO TO 110
5086 70    IF (RSTEA GT T3) GO TO 90
5087      IF (AD1.EQ.O.ODO) GO TO 80
5088      BET1=(T6+EYW)/AD1
5089      BETP1=1.ODO
5090      GO TO 110
5091 80    BET1=1.ODO
5092      BETP1=1.ODO
5093      GO TO 110
5094 90    IF (AD1.EQ.O.ODO) GO TO 100
5095      BET1=(T6+EYW)/AD1
5096      BETP1=(T6+ESH)/AD1
5097      GO TO 110
5098 100   BET1=1.ODO
5099      BETP1=1.ODO
5100 110   CALL FWCALC(BCL,HW,TW,E2)
5101      C
5102      C
5103      C      INTEGRATION OVER LOWER HALF OF WEB
5104      C
5105      C      CALCULATE COEFFICIENTS OF GEOMETRIC STIFFNESS
5106      C      SUBMATRICES
5107      C
5108      FAC1=(YSW+(ESH-EYW)*E1-(ESH+T6)*E2)*FE*STR
5109      FAC2=(RST+T1+ERTB)*E2*FE*STR
5110      FAC3=(YSW-(EYW+T6)*E1)*FE*STR
5111      FAC4=AD1*(E1*FE*STP
5112      FAC5=T6*E*FE*STR
5113      FAC6=AD1*(E*FE*STR
5114      FAC7=(YSW+(T6-EYW)*E1)*FE*STR
5115      FAC8=(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)*FE*STR
5116      C
5117      C
5118      C      INTEGRATE STIFFNESS SUBMATRICES OVER
5119      C      STRAIN-HARDENED REGION OF THE WEB
5120      C
5121      A=-1.ODO
5122      B=-BETP1
5123      IF (DABS(A-B).LT.1.OD-5) GO TO 130
5124      CALL PHI(B,A,F1,CW,7,9,17)
5125      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5126      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5127      CALL PHIY(B,A,FIY,CW,7,9,15)
5128      CALL EPHI(B,A,EFI,CW,7,9,18)
5129      DO 120 I=1,7
5130      DO 120 J=1,7
5131      WB1(I,J)=FA*F1(I,J)
5132      WB2(I,J)=FB*FIYY(I,J)
5133      WB3(I,J)=FC*FIFI(I,J)
5134      WB4(I,J)=FD*FIY(I,J)
5135      WB5(I,J)=FAC1*F1(I,J)+FAC2*EF1(I,J)
5136 120   A=-BETP1
5137 130   B=-BET1
5138      IF (DABS(A-B).LT.1.OD-5) GO TO 150
5139      CALL FWCALC(BCL,HW,TW,E1)
5140      CALL PHI(B,A,F1,CW,7,9,17)
5141      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5142      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5143      CALL PHIY(B,A,FIY,CW,7,9,15)
5144      CALL EPHI(B,A,EFI,CW,7,9,18)
5145      DO 140 I=1,7
5146      DO 140 J=1,7
5147      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5148      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5149      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5150      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5151      WB5(I,J)=WB5(I,J)+FAC3*F1(I,J)+FAC4*EF1(I,J)
5152      C
5153      C
5154      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5155      C      OF THE WEB
5156      C
5157 150   A=-BET1
5158      B=-BET3
5159      IF (DABS(A-B).LT.1.OD-5) GO TO 170
5160      CALL FWCALC(BCL,HW,TW,E)
5161      CALL PHI(B,A,F1,CW,7,9,17)
5162      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5163      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5164      CALL PHIY(B,A,FIY,CW,7,9,15)
5165      CALL EPHI(B,A,EFI,CW,7,9,18)
5166      DO 160 I=1,7
5167      DO 160 J=1,7
5168      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5169      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5170      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5171      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5172      WB5(I,J)=WB5(I,J)+FAC5*F1(I,J)+FAC6*EF1(I,J)
5173      C
5174      C
5175      C      INTEGRATE STIFFNESS SUBMATRICES OVER YIELDED REGION
5176      C      OF THE WEB
5177      C
5178 170   A=-BET3
5179      B=-BETP3
5180      IF (DABS(A-B).LT.1.OD-5) GO TO 190
5181      CALL FWCALC(BCL,HW,TW,E1)
5182      CALL PHI(B,A,F1,CW,7,9,17)
5183      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5184      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5185      CALL PHIY(B,A,FIY,CW,7,9,15)
5186      CALL EPHI(B,A,EFI,CW,7,9,18)
5187      DO 180 I=1,7
5188      DO 180 J=1,7
5189      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5190      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5191      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5192      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5193      WB5(I,J)=WB5(I,J)+FAC7*F1(I,J)+FAC4*EF1(I,J)
5194      C
5195      C
5196      C      INTEGRATE STIFFNESS SUBMATRICES OVER
5197      C      STRAIN-HARDENED REGION OF THE WEB

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5198 C
5199 190 A:=BETP3
5200 B=0.0DO
5201 IF (DABS(A-B).LT.1.0D-5) GO TO 210
5202 CALL FWCALC(BCL,HW,TW,E2)
5203 CALL PHI(B,A,F1,CW,7,9,17)
5204 CALL PHIYY(B,A,F1YY,CW,7,9,13)
5205 CALL PHIPHI(B,A,F1FI,CW,7,9,15)
5206 CALL PH1Y(B,A,F1Y,CW,7,9,15)
5207 CALL EPHI(B,A,EF1,CW,7,9,18)
5208 DO 200 I=1,7
5209 DO 200 J=1,7
5210 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5211 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
5212 WB3(I,J)=WB3(I,J)+FC*F1FI(I,J)
5213 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
5214 200 WB5(I,J)=WB5(I,J)+FAC6*F1(I,J)+FAC2*EF1(I,J)
5215 C
5216 C
5217 C SET LIMITS OF INTEGRATION FOR TOP HALF OF WEB FOR
5218 C ELASTIC, YIELDED, AND STRAIN HARDENED REGIONS.
5219 C
5220 C CASE I - MIDDLE ELASTIC
5221 C
5222 210 IF (TE.GT.EYW) GO TO 330
5223 IF (STEA.GT.T4) GO TO 220
5224 BET2=1.0DO
5225 BETP2=1.0DO
5226 GO TO 260
5227 220 IF (STEA.GT.T5) GO TO 240
5228 IF (AD2.EQ.0.0DO) GO TO 230
5229 BET2=(EYW-T6)/AD2
5230 BETP2=1.0DO
5231 GO TO 260
5232 230 BET2=0.0DO
5233 BETP2=1.0DO
5234 GO TO 260
5235 240 IF (AD2.EQ.0.0DO) GO TO 250
5236 BET2=(EYW-T6)/AD2
5237 BETP2=(ESH-T6)/AD2
5238 GO TO 260
5239 250 BET2=0.0DO
5240 BETP2=0.0DO
5241 C
5242 C
5243 C CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
5244 C STIFFNESS SUBMATRICES
5245 C
5246 260 CALL FWCALC(BCL,HW,TW,E)
5247 FAC1=T6+E*FE*STR
5248 FAC2=AD2+E*FE*STR
5249 FAC3=(YSW+(T6-EYW)*E1)*FE*STR
5250 FAC4=AD2+E1*FE*STR
5251 FAC5=(YSW+(ESH-EYW)*E1+(T6+ESH)*E2)*FE*STR
5252 FAC6=AD2+E2*FE*STR
5253 C
5254 C
5255 C INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5256 C OF THE WEB
5257 C
5258 A=0.0DO
5259 B=BET2
5260 IF (DABS(A-B).LT.1.0D-5) GO TO 280
5261 CALL PHI(B,A,F1,CW,7,9,17)
5262 CALL PHIYY(B,A,F1YY,CW,7,9,13)
5263 CALL PHIPHI(B,A,F1FI,CW,7,9,15)
5264 CALL PH1Y(B,A,F1Y,CW,7,9,15)
5265 CALL EPHI(B,A,EF1,CW,7,9,18)
5266 DO 270 I=1,7
5267 DO 270 J=1,7
5268 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5269 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
5270 WB3(I,J)=WB3(I,J)+FC*F1FI(I,J)
5271 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
5272 270 WB5(I,J)=WB5(I,J)+FAC1*F1(I,J)+FAC2*EF1(I,J)
5273 C
5274 C
5275 C INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5276 C OF THE WEB
5277 C
5278 280 A=BET2
5279 B=BETP2
5280 IF (DABS(A-B).LT.1.0D-5) GO TO 300
5281 CALL FWCALC(BCL,HW,TW,E1)
5282 CALL PHI(B,A,F1,CW,7,9,17)
5283 CALL PHIYY(B,A,F1YY,CW,7,9,13)
5284 CALL PHIPHI(B,A,F1FI,CW,7,9,15)
5285 CALL PH1Y(B,A,F1Y,CW,7,9,15)
5286 CALL EPHI(B,A,EF1,CW,7,9,18)
5287 DO 290 I=1,7
5288 DO 290 J=1,7
5289 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
5290 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
5291 WB3(I,J)=WB3(I,J)+FC*F1FI(I,J)
5292 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
5293 290 WB5(I,J)=WB5(I,J)+FAC3*F1(I,J)+FAC4*EF1(I,J)
5294 C
5295 C
5296 C INTEGRATE STIFFNESS SUBMATRICES OVER
5297 C STRAIN-HARDENED REGION OF THE WEB
5298 C
5299 300 A=BETP2
5300 B=1.0DO
5301 IF (DABS(A-B).LT.1.0D-5) GO TO 320
5302 CALL FWCALC(BCL,HW,TW,E2)
5303 CALL PHI(B,A,F1,CW,7,9,17)
5304 CALL PHIYY(B,A,F1YY,CW,7,9,13)
5305 CALL PHIPHI(B,A,F1FI,CW,7,9,15)
5306 CALL PH1Y(B,A,F1Y,CW,7,9,15)
5307 CALL EPHI(B,A,EF1,CW,7,9,18)
5308 DO 310 I=1,7
5309 DO 310 J=1,7
5310 WB1(I,J)=WB1(I,J)+FA*F1(I,J)

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5311      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5312      WB3(I,J)=WB3(I,J)-FC*FIFI(I,J)
5313      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5314      WB5(I,J)=WB5(I,J)+FAC5*FI(I,J)+FAC6*EFI(I,J)
5315      220  TW=TSAVE
5316      RETURN
5317      C
5318      C
5319      C      SET LIMITS OF INTEGRATION FOR TOP HALF OF WEB FOR
5320      C      ELASTIC, YIELDED, AND STRAIN HARDENED REGIONS
5321      C
5322      C      CASE 1) - MIDDLE YIELDED
5323      C
5324      330  IF (T6 GT.ESH) GO TO 450
5325      IF (STEALGT.T4) GO TO 350
5326      IF (AD2 EQ.O.ODO) GO TO 340
5327      BET2=(EYW-T6)/AD2
5328      BETP2=1.ODO
5329      GO TO 380
5330      340  BET2=1.ODO
5331      BETP2=1.ODO
5332      GO TO 380
5333      350  IF (STEALGT.T5) GO TO 360
5334      BET2=1.ODO
5335      BETP2=1.ODO
5336      GO TO 380
5337      360  IF (AD2.EQ.O.ODO) GO TO 370
5338      BETP2=(ESH-T6)/AD2
5339      BET2=BETP2
5340      GO TO 380
5341      370  BETP2=1.ODO
5342      BET2=BETP2
5343      C
5344      C
5345      C      CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
5346      C      STIFFNESS SUBMATRICES
5347      C
5348      380  CALL FWCALC(BCL,HW,TW,E1)
5349      FAC1=(YSW-(T6-EYW)*E1)*FE*STR
5350      FAC2=AD2*E1*FE*STR
5351      FAC3=T6*E*FE*STR
5352      FAC4=AD2*E*FE*STR
5353      FAC5=(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)*FE*STR
5354      FAC6=AD2*E2*FE*STR
5355      C
5356      C
5357      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5358      C      OF THE WEB
5359      C
5360      A=O.ODO
5361      B=BET2
5362      IF (DABS(A-B).LT.1.OD-5) GO TO 400
5363      CALL PHI(B,A,FI,CW,7,9,17)
5364      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5365      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5366      CALL PHIY(B,A,FIY,CW,7,9,16)
5367      CALL EPHI(B,A,EFI,CW,7,9,18)
5368      DO 390 I=1,7
5369      DO 390 J=1,7
5370      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5371      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5372      WB3(I,J)=WB3(I,J)-FC*FIFI(I,J)
5373      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5374      WB5(I,J)=WB5(I,J)+FAC1*FI(I,J)+FAC2*EFI(I,J)
5375      C
5376      C
5377      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5378      C      OF THE WEB
5379      C
5380      400  A=BET2
5381      B=BETP2
5382      IF (DABS(A-B).LT.1.OD-5) GO TO 420
5383      CALL FWCALC(BCL,HW,TW,E)
5384      CALL PHI(B,A,FI,CW,7,9,17)
5385      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5386      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5387      CALL PHIY(B,A,FIY,CW,7,9,16)
5388      CALL EPHI(B,A,EFI,CW,7,9,18)
5389      DO 410 I=1,7
5390      DO 410 J=1,7
5391      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5392      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5393      WB3(I,J)=WB3(I,J)-FC*FIFI(I,J)
5394      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5395      WB5(I,J)=WB5(I,J)+FAC3*FI(I,J)+FAC4*EFI(I,J)
5396      C
5397      C
5398      C      INTEGRATE STIFFNESS SUBMATRICES OVER
5399      C      STRAIN-HARDENED REGION OF THE WEB
5400      C
5401      420  A=BETP2
5402      B=1.ODO
5403      IF (DABS(A-B).LT.1.OD-5) GO TO 440
5404      CALL FWCALC(BCL,HW,TW,E2)
5405      CALL PHI(B,A,FI,CW,7,9,17)
5406      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5407      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5408      CALL PHIY(B,A,FIY,CW,7,9,16)
5409      CALL EPHI(B,A,EFI,CW,7,9,18)
5410      DO 430 I=1,7
5411      DO 430 J=1,7
5412      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5413      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5414      WB3(I,J)=WB3(I,J)-FC*FIFI(I,J)
5415      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
5416      WB5(I,J)=WB5(I,J)+FAC5*FI(I,J)+FAC6*EFI(I,J)
5417      440  TW=TSAVE
5418      RETURN
5419      C
5420      C
5421      C      SET LIMITS OF INTEGRATION FOR TOP HALF OF WEB FOR
5422      C      ELASTIC, YIELDED, AND STRAIN HARDENED REGIONS
5423      C

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5537 COMMON /BLK4/FA,FB,FC,FD,FE
5538 COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
5539 COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
5540 COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
5541 COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
5542
5543 C
5544 C INITIALISE STIFFNESS SUBMATRICES TO ZERO
5545 C
5546 DO 10 I=1,7
5547 DO 10 J=I,7
5548 WB1(I,J)=0.0DO
5549 WB2(I,J)=0.0DO
5550 WB3(I,J)=0.0DO
5551 WB4(I,J)=0.0DO
5552 WB5(I,J)=0.0DO
5553
5554 C
5555 C CALCULATE STRAINS AND STRAIN LIMITS
5556 C
5557 STR=1.0DO/ST
5558 TSAVE=TW
5559 TW=2.0DO*TW+HW/(2.0DO*HW-TF1-TF2-2.0DO*TW)
5560 T1=EYW-ERCM
5561 T2=EYW+ERTB
5562 T3=ESH-ERCM
5563 T4=ESH+ERTB
5564 T5=EYW+ERTT
5565 T6=ESH+ERTT
5566 AD1=ERTB+ERCM
5567 AD2=ERTT+ERCM
5568
5569 C
5570 C SET LIMITS OF INTEGRATION FOR ELASTIC, YIELDED, AND
5571 C STRAIN-HARDENED PORTIONS OF PLATE IN LOWER HALF OF
5572 C WEB
5573 C
5574 IF (ST.GT.T1) GO TO 20
5575 BETB=0.0DO
5576 BETBP=0.0DO
5577 GO TO 70
5578 20 IF (ST.GT.T2) GO TO 40
5579 IF (AD1.EQ.0.0DO) GO TO 30
5580 AN1=ST-T1
5581 BETB=AN1/AD1
5582 BETBP=0.0DO
5583 GO TO 70
5584 30 BETB=1.0DO
5585 BETBP=0.0DO
5586 GO TO 70
5587 40 IF (ST.GT.T3) GO TO 50
5588 BETB=1.0DO
5589 BETBP=0.0DO
5590 GO TO 70
5591 50 IF (ST.GT.T4) GO TO 60
5592 IF (AD1.EQ.0.0DO) GO TO 60
5593 AN1=ST-T3
5594 BETB=1.0DO
5595 BETBP=AN1/AD1
5596 GO TO 70
5597 60 BETB=1.0DO
5598 BETBP=1.0DO
5599 IF (ST.GT.T1) GO TO 80
5600 BETTP=0.0DO
5601 BETTP=0.0DO
5602 GO TO 130
5603 80 IF (ST.GT.T5) GO TO 100
5604 IF (AD2.EQ.0.0DO) GO TO 90
5605 AN2=ST-T1
5606 BETT=AN2/AD2
5607 BETTP=0.0DO
5608 GO TO 130
5609 90 BETT=1.0DO
5610 BETTP=0.0DO
5611 GO TO 130
5612 100 IF (ST.GT.T3) GO TO 110
5613 BETT=1.0DO
5614 BETTP=0.0DO
5615 GO TO 130
5616 110 IF (ST.GT.T6) GO TO 120
5617 IF (AD2.EQ.0.0DO) GO TO 120
5618 BETT=1.0DO
5619 AN2=ST-T3
5620 BETTP=AN2/AD2
5621 GO TO 130
5622 120 BETTP=1.0DO
5623 BETT=1.0DO
5624 130 CALL FWCALC(BCL,HW,TW,E)
5625
5626 C
5627 C CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
5628 C STIFFNESS SUBMATRICES
5629 C
5630 FAC1=(ST+ERCM)*E*FE
5631 FAC2=AD1*E*FE
5632 FAC3=(YSW+(ST-T1)*E1)*FE
5633 FAC4=AD1*E1*FE
5634 FAC5=(YSW+(ESH-EYW)*E1+(ST-T3)*E2)*FE
5635 FAC6=AD1*E2*FE
5636 FAC7=AD2*E2*FE
5637 FAC8=AD2*E1*FE
5638 FAC9=AD2*E*FE
5639
5640 C
5641 C INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5642 C OF THE WEB
5643 C
5644 A=1.0DO
5645 B=-BETB
5646 IF (DABS(A-B).LT.1.0D-5) GO TO 150
5647 CALL PHIB(A,B,F1,CW,7,9,17)
5648 CALL PHIY1(B,A,F1Y,CW,7,9,13)
5649 CALL PHIPHI(B,A,F1F1,CW,7,9,15)

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5763      C
5764      C
5765      C
5766      C
5767      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
5768      C
5769      C
5770      C
5771      C
5772      C
5773      C
5774      C
5775      C
5776      C
5777      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
5778      C
5779      C
5780      C
5781      C
5782      C
5783      C
5784      C
5785      C
5786      C
5787      C
5788      C
5789      C
5790      C
5791      C
5792      C
5793      C
5794      C
5795      C
5796      C
5797      C
5798      C
5799      C
5800      C
5801      C
5802      C
5803      C
5804      C
5805      C
5806      C
5807      C
5808      C
5809      C
5810      C
5811      C
5812      C
5813      C
5814      C
5815      C
5816      C
5817      C
5818      C
5819      C
5820      C
5821      C
5822      C
5823      C
5824      C
5825      C
5826      C
5827      C
5828      C
5829      C
5830      C
5831      C
5832      C
5833      C
5834      C
5835      C
5836      C
5837      C
5838      C
5839      C
5840      C
5841      C
5842      C
5843      C
5844      C
5845      C
5846      C
5847      C
5848      C
5849      C
5850      C
5851      C
5852      C
5853      C
5854      C
5855      C
5856      C
5857      C
5858      C
5859      C
5860      C
5861      C
5862      C
5863      C
5864      C
5865      C
5866      C
5867      C
5868      C
5869      C
5870      C
5871      C
5872      C
5873      C
5874      C
5875      C

SUBROUTINE 34

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

SUBROUTINE WBINBE

THIS ROUTINE FORMULATES THE STIFFNESS
SUBMATRICES WB1, WB2, WB3, WB4 AND WB5 FOR THE
WEB IN THE INELASTIC AND STRAIN-HARDENING RANGES
WHEN THE SPECIMEN IS SUBJECTED TO BENDING
STRESSES.

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE WBINBE(ST)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION F1(7,7),FIYY(7,7),F1F1(7,7),FIY(7,7),EF1(7,7)
COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
COMMON /BLK4/FA,FB,FC,FD,FE
COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT

INITIALISE STIFFNESS SUBMATRICES TO ZERO

DO 10 I=1,7
DO 10 J=1,7
WB1(I,J)=0.0DO
WB2(I,J)=0.0DO
WB3(I,J)=0.0DO
WB4(I,J)=0.0DO
WB5(I,J)=0.0DO
10 C

CALCULATE STRAINS AND STRAIN LIMITS.

STR=1.0DO/ST
RST=R*ST
TSAVE:TW
TW=2.0DO*TW+HW/(2.0DO+HW-TF1-TF2-2.0DO*TW)
EQ=(2.0DO*Y1/(2.0DO*Y1+HW))*ST
T1=ERCM+EC
T2=EYW-ERTB
T3=ESH-ERTB
T4=EYW+ERTT
T5=ESH+ERTT
AD1=RST+T1+ERTB
AD2=ST-T1-ERTT

SET LIMITS OF INTEGRATION FOR ELASTIC, YIELDED, AND
STRAIN-HARDENED PORTIONS OF PLATE IN LOWER HALF OF
WEB.

IF (T1.GT.EYW) GO TO 20
BETP3=0.0DO
BETP3=0.0DO
GO TO 60
20 IF (T1.GT.ESH) GO TO 40
IF (AD1.EQ.0.0DO) GO TO 30
BET3=(T1-EYW)/AD1
BETP3=0.0DO
GO TO 60
30 BET3=1.0DO
BETP3=0.0DO
GO TO 60
40 IF (AD1.EQ.0.0DO) GO TO 50
BET3=(T1-EYW)/AD1
BETP3=(T1-ESH)/AD1
GO TO 60
50 BET3=1.0DO
BETP3=1.0DO
60 IF (RST.GT.T2) GO TO 70
BETP1=1.0DO
GO TO 110
70 IF (RST.GT.T3) GO TO 80
IF (AD1.EQ.0.0DO) GO TO 80
BET1=(T1+EYW)/AD1
BETP1=1.0DO
GO TO 110
80 BET1=1.0DO
BETP1=1.0DO
GO TO 110
90 IF (AD1.EQ.0.0DO) GO TO 100
BET1=(T1+EYW)/AD1
BETP1=(T1+ESH)/AD1
GO TO 110
100 BET1=1.0DO
BETP1=1.0DO
110 CALL FWCALC(BCL,HW,TW,E2)

INTEGRATION OVER LOWER HALF OF WEB

CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
STIFFNESS SUBMATRICES.

FAC1=(YSW+(ESH-EYW)*E1+(ESH+T1)*E2)*FE
FAC2=(RST+T1+ERTB)*E2*FE
FAC3=(YSW-(EYW+T1)*E1)*FE
FAC4=AD1+E1*FE
FAC5=T1*E*FE
FAC6=AD1*E*FE
FAC7=(YSW+(T1-EYW)*E1)*FE
FAC8=(YSW+(ESH-EYW)*E1+(T1-ESH)*E2)*FE
A=-1.0DO
B=-BETP1
IF (DABS(A-B).LT.1.0D-5) GO TO 130

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5876      CALL PHI(B,A,FI,CW,7,9,17)
5877      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5878      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5879      CALL PHIY(B,A,FIY,CW,7,9,15)
5880      CALL EPHI(B,A,EFI,CW,7,9,18)
5881      DO 120 I=1,7
5882      DO 120 J=1,7
5883      WB1(I,J)=FA*FI(I,J)
5884      WB2(I,J)=FB*FIYY(I,J)
5885      WB3(I,J)=FC*FIFI(I,J)
5886      WB4(I,J)=FD*FIY(I,J)
120      WB5(I,J)=FAC1*FI(I,J)+FAC2*EFI(I,J)
5888      C
5889      C
5890      C      INTEGRATE STIFFNESS SUBMATRICES OVER
5891      C      STRAIN-HARDENED REGION OF THE WEB
5892      C
5893      130      A=-BETP1
5894      B=-BET1
5895      IF (DABS(A-B).LT.1.0D-5) GO TO 150
5896      CALL FWCALC(BCL,HW,TW,E1)
5897      CALL PHI(B,A,FI,CW,7,9,17)
5898      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5899      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5900      CALL PHIY(B,A,FIY,CW,7,9,15)
5901      CALL EPHI(B,A,EFI,CW,7,9,18)
5902      DO 140 I=1,7
5903      DO 140 J=1,7
5904      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5905      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5906      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5907      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
140      WB5(I,J)=WB5(I,J)+FAC3*FI(I,J)+FAC4*EFI(I,J)
5909      C
5910      C
5911      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
5912      C      OF THE WEB.
5913      C
5914      150      A=-BET1
5915      B=-BET3
5916      IF (DABS(A-B).LT.1.0D-5) GO TO 170
5917      CALL FWCALC(BCL,HW,TW,E)
5918      CALL PHI(B,A,FI,CW,7,9,17)
5919      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5920      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5921      CALL PHIY(B,A,FIY,CW,7,9,15)
5922      CALL EPHI(B,A,EFI,CW,7,9,18)
5923      DO 160 I=1,7
5924      DO 160 J=1,7
5925      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5926      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5927      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5928      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
160      WB5(I,J)=WB5(I,J)+FAC5*FI(I,J)+FAC6*EFI(I,J)
5930      C
5931      C
5932      C      INTEGRATE STIFFNESS SUBMATRICES OVER YIELDED REGION
5933      C      OF THE WEB.
5934      C
5935      170      A=-BET3
5936      B=-BETP2
5937      IF (DABS(A-B).LT.1.0D-5) GO TO 190
5938      CALL FWCALC(BCL,HW,TW,E1)
5939      CALL PHI(B,A,FI,CW,7,9,17)
5940      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5941      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5942      CALL PHIY(B,A,FIY,CW,7,9,15)
5943      CALL EPHI(B,A,EFI,CW,7,9,18)
5944      DO 180 I=1,7
5945      DO 180 J=1,7
5946      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5947      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5948      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5949      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
180      WB5(I,J)=WB5(I,J)+FAC7*FI(I,J)+FAC4*EFI(I,J)
5951      C
5952      C
5953      C      INTEGRATE STIFFNESS SUBMATRICES OVER
5954      C      STRAIN-HARDENED REGION OF THE WEB
5955      C
5956      190      A=-BETP3
5957      B=C ODO
5958      IF (DABS(A-B).LT.1.0D-5) GO TO 210
5959      CALL FWCALC(BCL,HW,TW,E2)
5960      CALL PHI(B,A,FI,CW,7,9,17)
5961      CALL PHIYY(B,A,FIYY,CW,7,9,13)
5962      CALL PHIPHI(B,A,FIFI,CW,7,9,15)
5963      CALL PHIY(B,A,FIY,CW,7,9,15)
5964      CALL EPHI(B,A,EFI,CW,7,9,18)
5965      DO 200 I=1,7
5966      DO 200 J=1,7
5967      WB1(I,J)=WB1(I,J)+FA*FI(I,J)
5968      WB2(I,J)=WB2(I,J)+FB*FIYY(I,J)
5969      WB3(I,J)=WB3(I,J)+FC*FIFI(I,J)
5970      WB4(I,J)=WB4(I,J)+FD*FIY(I,J)
200      WB5(I,J)=WB5(I,J)+FAC8*FI(I,J)+FAC2*EFI(I,J)
5972      C
5973      C
5974      C      SET LIMITS OF INTEGRATION FOR TOP HALF OF WEB FOR
5975      C      ELASTIC, YIELDED, AND STRAIN HARDENED REGIONS
5976      C
5977      C      CASE I - MIDDLE ELASTIC
5978      C
5979      210      IF (T1 GT EYW) GO TO 330
5980      IF (ST GT T4) GO TO 220
5981      BET2=1.0D0
5982      BETP2=1.0D0
5983      GO TO 260
5984      220      IF (ST GT T5) GO TO 240
5985      IF (AD2 EQ 0.0D0) GO TO 230
5986      BET2=(EYW-T1)/AD2
5987      BETP2=1.0D0
5988      GO TO 260

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5885 230 BET2=1.0DO
5886 BETP2=1.0DO
5887 GO TO 260
5888
5889 240 IF (AD2 EQ 0.0DO) GO TO 250
5890 BET2=(EYW-T1)/AD2
5891 BETP2=(ESW-T1)/AD2
5892 GO TO 260
5893
5894 250 BET2=0.0DO
5895 BETP2=0.0DO
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6102 C          CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
6103 C          STIFFNESS SUBMATRICES
6104 C
6105 380 CALL FWCALC(BCL,HW,TW,E1)
6106 FAC1=(YSW*(T1-EYW)*E1)*FE*STR
6107 FAC2=AD2*E1*FE*STR
6108 FAC3=T1*E*FE*STR
6109 FAC4=AD2*E*FE*STR
6110 FAC5=(YSW*(ESH-EYW)*E1+(T1-ESH)*E2)*FE*STR
6111 FAC6=AD2*E2*FE*STR
6112 C
6113 C
6114 C          INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
6115 C          OF THE WEB
6116 C
6117 A=0.0DO
6118 B=BET2
6119 IF (DABS(A-B).LT.1.0D-5) GO TO 400
6120 CALL PHI(B,A,F1,CW,7,9,17)
6121 CALL PHIYY(B,A,F1YY,CW,7,9,13)
6122 CALL PHIPHI(B,A,F1F1,CW,7,9,15)
6123 CALL PH1Y(B,A,F1Y,CW,7,9,15)
6124 CALL EPHI(B,A,E1,CW,7,9,16)
6125 DO 390 I=1,7
6126 DO 390 J=1,7
6127 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6128 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6129 WB3(I,J)=WB3(I,J)-FC*F1F1(I,J)
6130 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6131 WB5(I,J)=WB5(I,J)+FAC1*F1(I,J)+FAC2*E1(I,J)+FAC3*E1(I,J)
6132 C
6133 C
6134 C          INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC REGION
6135 C          OF THE WEB
6136 C
6137 400 A=BET2
6138 B=BETP2
6139 IF (DABS(A-B).LT.1.0D-5) GO TO 420
6140 CALL FWCALC(BCL,HW,TW,E)
6141 CALL PHI(B,A,F1,CW,7,9,17)
6142 CALL PHIYY(B,A,F1YY,CW,7,9,13)
6143 CALL PHIPHI(B,A,F1F1,CW,7,9,15)
6144 CALL PH1Y(B,A,F1Y,CW,7,9,15)
6145 CALL EPHI(B,A,E1,CW,7,9,16)
6146 DO 410 I=1,7
6147 DO 410 J=1,7
6148 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6149 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6150 WB3(I,J)=WB3(I,J)-FC*F1F1(I,J)
6151 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6152 WB5(I,J)=WB5(I,J)+FAC3*F1(I,J)+FAC4*E1(I,J)+FAC5*E1(I,J)
6153 C
6154 C
6155 C          INTEGRATE STIFFNESS SUBMATRICES OVER
6156 C          STRAIN-HARDENED REGION OF THE WEB
6157 C
6158 420 A=BETP2
6159 B=1.0DO
6160 IF (DABS(A-B).LT.1.0D-5) GO TO 440
6161 CALL FWCALC(BCL,HW,TW,E2)
6162 CALL PHI(B,A,F1,CW,7,9,17)
6163 CALL PHIYY(B,A,F1YY,CW,7,9,13)
6164 CALL PHIPHI(B,A,F1F1,CW,7,9,15)
6165 CALL PH1Y(B,A,F1Y,CW,7,9,15)
6166 CALL EPHI(B,A,E1,CW,7,9,16)
6167 DO 430 I=1,7
6168 DO 430 J=1,7
6169 WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6170 WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6171 WB3(I,J)=WB3(I,J)-FC*F1F1(I,J)
6172 WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6173 WB5(I,J)=WB5(I,J)+FAC5*F1(I,J)+FAC6*E1(I,J)+FAC3*E1(I,J)
6174 TW=TSAVE
6175 RETURN
6176 C
6177 C
6178 C          SET LIMITS OF INTEGRATION FOR TOP HALF OF WEB FOR
6179 C          ELASTIC, YIELDED, AND STRAIN HARDENED REGIONS
6180 C
6181 C          CASE II - MIDDLE STRAIN-HARDENED
6182 C
6183 450 IF (ST.GT.T4) GO TO 470
6184 IF (AD2.EQ.0.0DO) GO TO 460
6185 BET2=(EYW-T1)/AD2
6186 BETP2=(ESH-T1)/AD2
6187 GO TO 480
6188 460 BET2=1.0DO
6189 BETP2=1.0DO
6190 GO TO 480
6191 470 IF (ST.GT.T5) GO TO 480
6192 IF (AD2.EQ.0.0DO) GO TO 480
6193 BET2=1.0DO
6194 BETP2=(ESH-T1)/AD2
6195 GO TO 480
6196 480 BET2=1.0DO
6197 BETP2=1.0DO
6198 C
6199 C
6200 C          CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
6201 C          STIFFNESS SUBMATRICES
6202 C
6203 490 CALL FWCALC(BCL,HW,TW,E2)
6204 FAC1=(YSW*(ESH-EYW)*E1+(T1-ESH)*E2)*FE*STR
6205 FAC2=AD2*E2*FE*STR
6206 FAC3=(YSW*(T1-EYW)*E1)*FE*STR
6207 FAC4=AD2*E1*FE*STR
6208 FAC5=T1*E*FE*STR
6209 FAC6=AD2*E*FE*STR
6210 C
6211 C
6212 C          INTEGRATE STIFFNESS SUBMATRICES OVER
6213 C          STRAIN-HARDENED REGION OF THE WEB
6214 C

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6215      A=1.0D0
6216      B=BETP2
6217      IF (DABS(A-B).LT.1.0D-5) GO TO 510
6218      CALL PH1(B,A,F1,CW,7,5,17)
6219      CALL PH1YY(B,A,F1YY,CW,7,9,13)
6220      CALL PH1PH1(B,A,F1F1,CW,7,9,15)
6221      CALL PH1Y(B,A,F1Y,CW,7,9,16)
6222      CALL EPH1(B,A,EF1,CW,7,9,18)
6223      DO 500 I=1,7
6224      DO 500 J=1,7
6225      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6226      WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6227      WB3(I,J)=WB3(I,J)+FC*F1F1(I,J)
6228      WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6229      WB5(I,J)=WB5(I,J)+FAC1*F1(I,J)+FAC2*EF1(I,J)
6230
6231      C
6232      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC
6233      C      REGION OF THE WEB
6234      C
6235      510      A=BETP2
6236      B=BET2
6237      IF (DABS(A-B).LT.1.0D-5) GO TO 530
6238      CALL FWCALC(BCL,HW,TW,E1)
6239      CALL PH1(B,A,F1,CW,7,9,17)
6240      CALL PH1YY(B,A,F1YY,CW,7,9,13)
6241      CALL PH1PH1(B,A,F1F1,CW,7,9,15)
6242      CALL PH1Y(B,A,F1Y,CW,7,9,16)
6243      CALL EPH1(B,A,EF1,CW,7,9,18)
6244      DO 520 I=1,7
6245      DO 520 J=1,7
6246      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6247      WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6248      WB3(I,J)=WB3(I,J)+FC*F1F1(I,J)
6249      WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6250      WB5(I,J)=WB5(I,J)+FAC3*F1(I,J)+FAC4*EF1(I,J)
6251
6252      C
6253      C      INTEGRATE STIFFNESS SUBMATRICES OVER ELASTIC
6254      C      REGION OF THE WEB
6255      C
6256      530      A=BET2
6257      B=1.0D0
6258      IF (DABS(A-B).LT.1.0D-5) GO TO 550
6259      CALL FWCALC(BCL,HW,TW,E)
6260      CALL PH1(B,A,F1,CW,7,9,17)
6261      CALL PH1YY(B,A,F1YY,CW,7,9,13)
6262      CALL PH1PH1(B,A,F1F1,CW,7,9,15)
6263      CALL PH1Y(B,A,F1Y,CW,7,9,16)
6264      CALL EPH1(B,A,EF1,CW,7,9,18)
6265      DO 540 I=1,7
6266      DO 540 J=1,7
6267      WB1(I,J)=WB1(I,J)+FA*F1(I,J)
6268      WB2(I,J)=WB2(I,J)+FB*F1YY(I,J)
6269      WB3(I,J)=WB3(I,J)+FC*F1F1(I,J)
6270      WB4(I,J)=WB4(I,J)+FD*F1Y(I,J)
6271      WB5(I,J)=WB5(I,J)+FAC5*F1(I,J)+FAC6*EF1(I,J)
6272      550      TW=TSAVE
6273      RETURN
6274      END
6275      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6276      C
6277      C      SUBROUTINE 35
6278      C
6279      C
6280      C
6281      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6282      C
6283      C      SUBROUTINE WBFORM
6284      C
6285      C      THIS ROUTINE CALCULATES BENDING AND
6286      C      GEOMETRIC STIFFNESS SUBMATRICES WB1, WB2, WB3,
6287      C      WB4, AND WB5 FOR THE WEB IN THE ELASTIC RANGE
6288      C
6289      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6290      C      SUBROUTINE WBFORM
6291      C      IMPLICIT REAL*8(A-H,O-Z)
6292      C      DIMENSION F1(7,7),F1YY(7,7),F1F1(7,7),F1Y(7,7),EF1(7,7)
6293      C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
6294      C      COMMON /BLK4/FA,FB,FC,FD,FE
6295      C      COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
6296      C      COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
6297      C      COMMON /BLK9/CW(7,9),WB1(7,7),WB2(7,7),WB3(7,7),WB4(7,7),WB5(7,7)
6298      C      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
6299
6300      C
6301      C      ADJUST WEB THICKNESS TO ACCOUNT FOR WEB-TO-FLANGE
6302      C      FILLETS
6303      C
6304      C      IF (ITYPE.EQ.7) TSAVE=TW
6305      C      IF (ITYPE.EQ.7) TW=2.0D0*TW*HW/(2.0D0*HW-TF1-TF2+2.0D0*TW)
6306      C      CALL FWCALC(BCL,HW,TW,E)
6307      C      IF (ITYPE.EQ.7) TW=TSAVE
6308      C      IF (ITYPE.NE.7) SCRW=0.0D0
6309
6310      C
6311      C      CALCULATE FACTORS FOR MULTIPLYING GEOMETRIC
6312      C      STIFFNESS SUBMATRICES
6313      C
6314      C      FAC1=E*ERCM*FE
6315      C      FAC2=E*FE*(ERTB+ERCM)
6316      C      FAC3=E*FE*(ERTT+ERCM)
6317
6318      C
6319      C      INTEGRATE STIFFNESS SUBMATRICES OVER WEB DEPTH
6320      C      FOR W-SHAPE
6321      C
6322      B=1.0D0
6323      A=1.0D0
6324      CALL PH1(B,A,F1,CW,7,9,17)
6325      CALL PH1YY(B,A,F1YY,CW,7,9,13)
6326      CALL PH1PH1(B,A,F1F1,CW,7,9,15)
6327      CALL PH1Y(B,A,F1Y,CW,7,9,16)

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6441 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6442 C
6443 C SUBROUTINE RAYLE C
6444 C
6445 C THIS SUBROUTINE CALCULATES THE EIGENVALUE C
6446 C USING THE RAYLEIGH QUOTIENT WHEN THE EIGEN-SHAPE C
6447 C IS KNOWN C
6448 C
6449 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6450 SUBROUTINE RAYLE(SVEC,ABG,EVAL) C
6451 IMPLJCT REAL*8(A-H,O-Z) C
6452 DIMENSION SVEC(15),TVEC(15),ABG(15,15) C
6453 COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND C
6454 C
6455 C
6456 C SVEC = EIGENVECTOR C
6457 C ABG = INVERSE STIFFNESS TIMES GEOMETRIC STIFFNESS C
6458 C MATRICES C
6459 C EVAL = EIGENVALUE C
6460 C
6461 N1=1 C
6462 IF (ITYPE NE.7) GO TO 10 C
6463 DA1=DABS(SVEC(11)) C
6464 DA2=DABS(SVEC(9)) C
6465 TLL=1.0D-6 C
6466 IF (DA2 LT.TLL AND DA2.LT.TLL) N1=11 C
6467 10 DO 30 J=N1,N C
6468 SUM1=0.0D0 C
6469 DO 20 J=N1,N C
6470 SUM1=SUM1+ABG(I,J)*SVEC(J) C
6471 20 TVEC(I)=SUM1 C
6472 SUMT=0.0D0 C
6473 SUMB=0.0D0 C
6474 DO 40 J=N1,N C
6475 SUMT=SUMT+SVEC(I)*TVEC(J) C
6476 40 SUMB=SUMB+SVEC(I)*SVEC(J) C
6477 EVAL=SUMB/SUMT C
6478 RETURN C
6479 END C
6480 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6481 C
6482 C SUBROUTINE 37 C
6483 C
6484 C
6485 C
6486 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6487 C
6488 C SUBROUTINE GQUAD C
6489 C THIS SUBROUTINE USES GAUSSIAN QUADRATURE C
6490 C TO INTEGRATE THE FOURIER SHAPE FUNCTION ALONG C
6491 C THE LENGTH OF A PLATE A SIX POINT GAUSS C
6492 C INTEGRATION IS USED FOR EACH SUBDIVISION OF C
6493 C LENGTH. THE NUMBER OF SUBDIVISIONS EQUALS THE C
6494 C NUMBER OF CURRENT HALF WAVELENGTHS ALONG THE C
6495 C MEMBER. C
6496 C
6497 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6498 SUBROUTINE GQUAD(M,RM1,RM2,RM3,RM4) C
6499 IMPLICIT REAL*8(A-H,O-Z) C
6500 COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND C
6501 COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2 C
6502 DIMENSION P(21),F1(6),F2(6),F3(6),F4(6),SP(6) C
6503 C
6504 C
6505 C SET GAUSS SAMPLING POINTS FOR INTERVAL -1 TO +1 C
6506 C
6507 G1=0.932469514203152D0 C
6508 G2=-G1 C
6509 G3=0.661209386466265D0 C
6510 G4=-G3 C
6511 G5=0.238619186083197D0 C
6512 G6=-G5 C
6513 C
6514 C
6515 C SET WEIGHTING FACTORS FOR INTERVAL -1 TO +1. C
6516 C
6517 B1=0.171324492379170D0 C
6518 B2=0.360761573048139D0 C
6519 B3=0.467913934572691D0 C
6520 C
6521 C
6522 C CALCULATE NUMBER OF ELEMENTS AND DETERMINE END C
6523 C COORDINATES P(I) OF EACH ELEMENT OF THE LENGTH C
6524 C
6525 ELEM=1.0D0/M C
6526 NEL=M C
6527 NP=NEL+1 C
6528 P(1)=G.CDO C
6529 DO 10 I=2,NP C
6530 IM1=I-1 C
6531 10 P(I)=P(IM1)+ELEM C
6532 RM1=0.0D0 C
6533 RM2=RM1 C
6534 RM3=RM2 C
6535 RM4=RM3 C
6536 C
6537 C
6538 C LOOP OVER NUMBER OF ELEMENTS C
6539 C
6540 DO 50 I=1,NEL C
6541 IP1=I+1 C
6542 C
6543 C
6544 C CALCULATE SAMPLING POINTS AND WEIGHTING FACTORS FOR C
6545 C ELEMENT FROM A TO B C
6546 C
6547 AB=0.5D0*(P(I)+P(IP1)) C
6548 BA=0.5D0*(P(IP1)-P(I)) C
6549 SP(1)=AB+BA*G1 C
6550 SP(2)=AB+BA*G2 C
6551 SP(3)=AB+BA*G3 C
6552 SP(4)=AB+BA*G4 C
6553 SP(5)=AB+BA*G5 C

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6554      SP(6)=AB+BA*G6
6555      W1=BA*B1
6556      W2=BA*B2
6557      W3=BA*B3
6558
6559      C
6560      C
6561      C      DETERMINE THE VALUE OF THE INTEGRAND FUNCTIONS AT
6562      C      EACH OF THE 6 SAMPLING POINTS
6563
6564      CALL FUNCT(SP,F1,F2,F3,F4,M)
6565
6566      C
6567      C      LOOP OVER 6 SAMPLING POINTS MULTIPLYING EACH BY ITS
6568      C      GAUSS WEIGHT AND ADD TO THE TOTAL INTEGRAL.
6569
6570      DO 40 J=1,6
6571      IF (J.GT.2) GO TO 20
6572      RM1=RM1+F1(J)*W1
6573      RM2=RM2+F2(J)*W1
6574      RM3=RM3+F3(J)*W1
6575      RM4=RM4+F4(J)*W1
6576      GO TO 40
6577 20  IF (J.GT.4) GO TO 30
6578      RM1=RM1+F1(J)*W2
6579      RM2=RM2+F2(J)*W2
6580      RM3=RM3+F3(J)*W2
6581      RM4=RM4+F4(J)*W2
6582      GO TO 40
6583 30  RM1=RM1+F1(J)*W3
6584      RM2=RM2+F2(J)*W3
6585      RM3=RM3+F3(J)*W3
6586      RM4=RM4+F4(J)*W3
6587 40  CONTINUE
6588 50  CONTINUE
6589      RETURN
6590      END
6591
6592      C
6593      C      SUBROUTINE 38
6594      C
6595      C
6596      C
6597      C
6598      C      SUBROUTINE FUNCT
6599      C
6600      C      THIS SUBROUTINE EVALUATES THE INTEGRAND
6601      C      FUNCTION AT EACH OF THE 6 GAUSS POINTS AND
6602      C      RETURNS TO GAUSS TWELVE DIFFERENT LONGITUDINAL
6603      C      END CONDITIONS ARE USED.
6604
6605      C
6606      C      SUBROUTINE FUNCT(SP,F1,F2,F3,F4,M)
6607      C      IMPLICIT REAL*8(A-H,O-Z)
6608      C      COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
6609      C      COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
6610      C      DIMENSION F1(6),F2(6),F3(6),F4(6),SP(6)
6611      C      P=DARCOS(-1.0D0)
6612
6613      C
6614      C
6615      C      LOOP OVER THE SIX SAMPLING POINTS EVALUATING THE
6616      C      FUNCTION AT EACH POINT.
6617
6618      DO 140 I=1,6
6619      S1=SP(I)
6620      S2=SP(I)**2
6621      S3=SP(I)**3
6622      S4=SP(I)**4
6623      S5=SP(I)**5
6624      ARG=M*P)*SP(I)
6625
6626      C
6627      C      SELECT WHICH FUNCTION IS TO BE EVALUATED ACCORDING
6628      C      TO THE VALUE OF IKIND.
6629
6630      C
6631      C      GO TO (10,20,30,40,50,60,70,80,90,100,110,120),IKIND
6632 10  F1=16.0D0*S4-32.0D0*S3+16.0D0*S2
6633      F2=(64.0D0*S3-96.0D0*S2+32.0D0*S1)/BCL
6634      FPP=(192.0D0*S2-192.0D0*S1+22.0D0)/BCL**2
6635      GO TO 130
6636 20  F1=S1
6637      F2=1.0D0/BCL
6638      FPP=0.0D0
6639      GO TO 130
6640 30  F1=S2
6641      F2=2.0D0*S1/BCL
6642      FPP=2.0D0/BCL**2
6643      GO TO 130
6644 40  AA=-111.80339889D0
6645      F1=AA*(S5-2.5D0*S4+2.0D0*S3-0.5D0*S2)
6646      F2=AA*(15.0D0*S4-10.0D0*S3+6.0D0*S2-S1)/BCL
6647      FPP=AA*(20.0D0*S3-30.0D0*S2+12.0D0*S1-1.0D0)/BCL**2
6648      GO TO 130
6649 50  F1=(-27.0D0/4.0D0)*(S3-S2)
6650      F2=(-27.0D0/4.0D0)*(3.0D0*S2-2.0D0*S1)
6651      FPP=(-27.0D0/4.0D0)*(6.0D0*S1-2.0D0)
6652      GO TO 130
6653 60  F1=-6.0D0*S3+9.0D0*S2-3.0D0*S1
6654      F2=(-18.0D0*(S2-S1)-3.0D0)/BCL
6655      FPP=(-36.0D0*S1+16.0D0)/BCL**2
6656      GO TO 130
6657 70  F1=-2.0D0*S3+3.0D0*S2
6658      F2=(-6.0D0*(S2-S1))/BCL
6659      FPP=(-12.0D0*S1+6.0D0)/BCL**2
6660      GO TO 130
6661 80  F1=-52+2.0D0*S1
6662      F2=-2.0D0*(S1-1.0D0)/BCL
6663      FPP=-2.0D0/BCL**2
6664      GO TO 130
6665 90  F1=-4.0D0*(S2-S1)
6666      F2=(-8.0D0*S1+4.0D0)/BCL
6667      FPP=-8.0D0/BCL**2
6668      GO TO 130

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6667      100  F=-64 ODO*S3=(S1-1.ODO)**3
6668      FPP=-152.ODO*S2=(S1-1.ODO)**2*(2.ODO*S1-1.ODO)/BCL
6669      FPP=-384 ODO*S1=(S1-1.ODO)*(5.ODO*S2-5.ODO*S1+1.ODO)/BCL**2
6670      GO TO 130
6671      110  F=S3
6672      FPP=3.ODO*S2/BCL
6673      FPP=6 ODO*S1/BCL**2
6674      GO TO 130
6675      120  F=S4
6676      FPP=4.ODO*S3/BCL
6677      FPP=12 ODO*S2/BCL**2
6678      130  G=DSIN(ARG)
6679      GP=M=P)*DCOS(ARG)/BCL
6680      GPP=-M**2*P1**2*G/BCL**2
6681      F1(1)=BCL*(FPP+G+2.ODO*FP*GP+F*GPP)**2
6682      F2(1)=BCL*(F+G)*F*G
6683      F3(1)=BCL*(FPP+G+2.ODO*FP*GP+GPP*F)*F*G
6684      F4(1)=BCL*(FP+G+GP*F)**2
6685      140  CONTINUE
6686      RETURN
6687      END
6688      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6689      C
6690      C
6691      C SUBROUTINE 39
6692      C
6693      C
6694      C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6695      C
6696      C SUBROUTINE SWEEP
6697      C
6698      C THIS SUBROUTINE IS CALLED FROM EIGEN, AND
6699      C AND IT SWEEPS OUT A GIVEN MODE FROM THE GIVEN
6700      C EIGENSYSTEM BEING SOLVED.
6701      C
6702      C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6703      C SUBROUTINE SWEEP(ABG,SG,SVEC,ICH)
6704      C IMPLICIT REAL*8(A-H,O-Z)
6705      C DIMENSION ABG(15,15),SG(15,15),SVEC(15)
6706      C DIMENSION S(15,15),P(15,15),FVEC(15)
6707      C COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
6708      C IF (ICH.EQ.1) GO TO 10
6709      C GO TO 30
6710      C
6711      C
6712      C FOR THE FIRST SWEEP SET S(I,J) TO THE IDENTITY
6713      C MATRIX.
6714      C
6715      10  DO 20 I=1,N
6716      DO 20 J=1,N
6717      S(I,J)=0 ODO
6718      IF (I.EQ.J) S(I,J)=1.ODO
6719      20  CONTINUE
6720      C
6721      C
6722      C FOR SUBSEQUENT SWEEPS USE PREVIOUS VALUE OF S(I,J)
6723      C
6724      30  DO 50 I=1,N
6725      SUM=0.ODO
6726      DO 40 J=1,N
6727      SUM=SUM+SVEC(J)*SG(J,I)
6728      FVEC(I)=SUM
6729      40  CONTINUE
6730      SUMB=0.ODO
6731      DO 60 I=1,N
6732      SUMB=SUMB+FVEC(I)*SVEC(I)
6733      DO 60 J=1,N
6734      P(I,J)=FVEC(J)*SVEC(I)
6735      60  CONTINUE
6736      RSUMB=1.ODO/SUMB
6737      DO 70 I=1,N
6738      DO 70 J=1,N
6739      S(I,J)=S(I,J)-P(I,J)*RSUMB
6740      P(I,J)=ABG(I,J)
6741      70  CONTINUE
6742      C
6743      C
6744      C MULTIPLY THE CURRENT MATRIX ABG(I,J) WHICH IS NOW
6745      C STORED IN P(I,J) INTO THE CURRENT SWEEPING MATRIX
6746      C S(I,J) AND STORE THE RESULT IN ABG(I,J) AND RETURN
6747      C TO EIGEN TO GET THE NEXT MODE
6748      C
6749      CALL MULT(ABG,P,S,N)
6750      RETURN
6751      END
6752      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6753      C
6754      C
6755      C SUBROUTINE 40
6756      C
6757      C
6758      C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6759      C
6760      C SUBROUTINE FORCE
6761      C
6762      C THIS SUBROUTINE IS CALLED FROM SATEO IN
6763      C ORDER TO EVALUATE THE SUM SF OF THE FORCES
6764      C ACTING ON A CROSS-SECTION IN THE LONGITUDINAL
6765      C DIRECTION AT A GIVEN COMPRESSIVE STRAIN, ESB.
6766      C
6767      C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
6768      C SUBROUTINE FORCE(ESB,EAPP,AXLO,SF)
6769      C IMPLICIT REAL*8(A-H,O-Z)
6770      C COMMON /BLK1/E,V,NS,ITYPE,LTYPE,INEL,N,ICLAMP,IKIND
6771      C COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYWF,EYTF
6772      C COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
6773      C COMMON /BLK10/HW,TW,BCL,BF1,BF2,BF2,TF2
6774      C COMMON /BLK11/E1,E2,ESH,YSW,YSFB,YSFT,SRTB,SRCM,SRTT,SRCB,SRCT
6775      C
6776      C
6777      C EC = STRAIN AT THE MIDDLE OF THE WEB
6778      C EAPP = APPLIED AXIAL STRAIN
6779      C ESB = BENDING STRAIN IN THE COMPRESSION FLANGE

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6780      C      REB = BENDING STRAIN IN THE TENSION FLANGE
6781      C
6782      EC=(2.0DO*Y1/(2.0DO*Y1-HW))*ESB
6783      EAPB=ESB+EAPP
6784      EB=ESB
6785      REB=R*ESB
6786      EARST=EAPP-REB
6787      RSTEA=REB-EAPP
6788      T1=EYTF-ERCT
6789      T2=EYTF+ERTT
6790      T3=ESH-ERCT
6791      T4=ESH+ERTT
6792      ADEN=ERCT+ERTT
6793      IF (EAPB.GT.T1) GO TO 10
6794      ALFT=1.0DO
6795      ALFP=1.0DO
6796      GO TO 60
6797      10  IF (EAPB.GT.T2) GO TO 30
6798      IF (ADEN.EQ.0.0DO) GO TO 20
6799      ANUM=T2-EAPB
6800      ALFT=ANUM/ADEN
6801      IF (ALFT.GT.1.0DO) ALFT=1.0DO
6802      IF (ALFT.LT.0.0DO) ALFT=0.0DO
6803      ALFP=1.0DO
6804      GO TO 60
6805      20  ALFT=0.0DO
6806      ALFP=1.0DO
6807      GO TO 60
6808      30  IF (EAPB.GT.T3) GO TO 40
6809      ALFT=0.0DO
6810      ALFP=1.0DO
6811      GO TO 60
6812      40  IF (EAPB.GT.T4) GO TO 50
6813      IF (ADEN.EQ.0.0DO) GO TO 50
6814      ANUM=T4-EAPB
6815      ALFP=ANUM/ADEN
6816      IF (ALFP.GT.1.0DO) ALFP=1.0DO
6817      IF (ALFP.LT.0.0DO) ALFP=0.0DO
6818      ALFT=0.0DO
6819      GO TO 60
6820      50  ALFP=0.0DO
6821      ALFT=0.0DO
6822      C
6823      C
6824      C      EVALUATE FORCES IN THE COMPRESSION FLANGE
6825      C
6826      60  A1=0.5DO*BF2*TF2*ALFT
6827      A2=2.0DO*(EAPB-ERTT)*E
6828      A3=ADEN+E*ALFT
6829      F21=A1*(A2+A3)
6830      A1=0.5DO*BF2*TF2*(ALFP+ALFT)
6831      A2=2.0DO*(YSFT+(EAPB-T2)*E1)
6832      A3=ADEN+E1*(ALFP+ALFT)
6833      F22=A1*(A2+A3)
6834      A1=0.5DO*BF2*TF2*(1.0DO+ALFP)
6835      A2=2.0DO*(YSFT+(ESH-EYTF)*E1+(EAPB-T4)*E2)
6836      A3=ADEN+E2*(1.0DO+ALFP)
6837      F23=A1*(A2+A3)
6838      F2=F21+F22+F23
6839      IF (EARST.GT.0.0DO) GO TO 130
6840      T1=EYBF-ERTB
6841      T2=EYBF+ERCB
6842      T3=ESH-ERTB
6843      T4=ESH+ERCB
6844      AD1=ERTB+ERCB
6845      IF (RSTEA.GT.T1) GO TO 70
6846      ALFB=0.0DO
6847      ALFBP=0.0DO
6848      GO TO 120
6849      70  IF (RSTEA.GT.T2) GO TO 90
6850      IF (AD1.EQ.0.0DO) GO TO 80
6851      AN1=RSTEA-T1
6852      ALFB=AN1/AD1
6853      IF (ALFB.GT.1.0DO) ALFB=1.0DO
6854      IF (ALFB.LT.0.0DO) ALFB=0.0DO
6855      ALFBP=0.0DO
6856      GO TO 120
6857      80  ALFB=1.0DO
6858      ALFBP=0.0DO
6859      GO TO 120
6860      90  IF (RSTEA.GT.T3) GO TO 100
6861      ALFB=1.0DO
6862      ALFBP=0.0DO
6863      GO TO 120
6864      100 IF (RSTEA.GT.T4) GO TO 110
6865      IF (AD1.EQ.0.0DO) GO TO 110
6866      AN1=RSTEA-T3
6867      ALFB=AN1/AD1
6868      IF (ALFBP.GT.1.0DO) ALFBP=1.0DO
6869      IF (ALFBP.LT.0.0DO) ALFBP=0.0DO
6870      ALFB=1.0DO
6871      GO TO 120
6872      110 ALFB=1.0DO
6873      ALFBP=1.0DO
6874      C
6875      C
6876      C      EVALUATE FORCES IN TENSION FLANGE WHEN FLANGE IS
6877      C      IN TENSION
6878      C
6879      120 A1=0.5DO*BF1*TF1*ALFBP
6880      A2=2.0DO*(-YSFB-(ESH-EYBF)*E1+(T3-RSTEA)*E2)
6881      A3=AD1+E2*ALFBP
6882      F11=A1*(A2+A3)
6883      A1=0.5DO*BF1*TF1*(ALFP+ALFBP)
6884      A2=2.0DO*(-YSFB+(T1-RSTEA)*E1)
6885      A3=AD1+E1*(ALFB+ALFBP)
6886      F12=A1*(A2+A3)
6887      A1=0.5DO*BF1*TF1*(1.0DO+ALFB)
6888      A2=2.0DO*(EARST-ERTB)*E
6889      A3=AD1+E*(1.0DO+ALFB)
6890      F13=A1*(A2+A3)
6891      F1=F11+F12+F13
6892      GO TO 200

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6893      130      T1=EYBF-ERCB
6894              T2=EYBF-ERTB
6895              T3=ESH-ERCB
6896              T4=ESH-ERTB
6897              ADEN=ERCB+ERTB
6898              IF (EARST.GT.T1) GO TO 140
6899              ALFT=1.0DO
6900              ALFP=1.0DO
6901              GO TO 190
6902      140      IF (EARST.GT.T2) GO TO 160
6903              IF (ADEN.EQ.0.0DO) GO TO 150
6904              ANUM=T2-EARST
6905              ALFT=ANUM/ADEN
6906              IF (ALFT.GT.1.0DO) ALFT=1.0DO
6907              IF (ALFT.LT.0.0DO) ALFT=0.0DO
6908              ALFP=1.0DO
6909              GO TO 190
6910      150      ALFT=0.0DO
6911              ALFP=1.0DO
6912              GO TO 190
6913      150      IF (EARST.GT.T3) GO TO 170
6914              ALFT=0.0DO
6915              ALFP=1.0DO
6916              GO TO 190
6917      170      IF (EARST.GT.T4) GO TO 180
6918              IF (ADEN.EQ.0.0DO) GO TO 160
6919              ANUM=T4-EARST
6920              ALFP=ANUM/ADEN
6921              IF (ALFP.GT.1.0DO) ALFP=1.0DO
6922              IF (ALFP.LT.0.0DO) ALFP=0.0DO
6923              ALFT=0.0DO
6924              GO TO 190
6925      180      ALFP=0.0DO
6926              ALFT=0.0DO
6927      C
6928      C
6929      C          EVALUATE FORCES IN TENSION FLANGE WHEN FLANGE IS IN
6930      C          COMPRESSION
6931      C
6932      190      A1=0.5DO*BF1*TF1*ALFT
6933              A2=2.0DO*(EARST-ERTB)*E
6934              A3=ADEN+E*ALFT
6935              F11=A1*(A2+A3)
6936              A1=0.5DO*BF1*TF1*(ALFP+ALFT)
6937              A2=2.0DO*(YSFB+(EARST-T2)*E1)
6938              A3=ADEN+E1*(ALFP+ALFT)
6939              F12=A1*(A2+A3)
6940              A1=0.5DO*BF1*TF1*(1.0DO-ALFP)
6941              A2=2.0DO*(YSFB+(ESH-EYBF)*E1+(EARST-T4)*E2)
6942              A3=ADEN+E2*(1.0DO+ALFP)
6943              F13=A1*(A2+A3)
6944              F1=F11+F12+F13
6945      200      T1=ERCM+EC
6946              T2=EYW-ERTB
6947              T3=ESH-ERTB
6948              T4=EYW-ERTT
6949              T5=ESH-ERTT
6950              T6=T1+EAPP
6951              AD1=REB+T1+ERTB
6952              AD2=EB-T1-ERTT
6953              IF (T6.GT.EYW) GO TO 210
6954              BET3=0.0DO
6955              BETP3=0.0DO
6956              GO TO 250
6957      210      IF (T6.GT.ESH) GO TO 230
6958              IF (AD1.EQ.0.0DO) GO TO 220
6959              BET3=(T6-EYW)/AD1
6960              IF (BET3.GT.1.0DO) BET3=1.0DO
6961              IF (BET3.LT.0.0DO) BET3=0.0DO
6962              BETP3=0.0DO
6963              GO TO 250
6964      220      BET3=1.0DO
6965              BETP3=0.0DO
6966              GO TO 250
6967      230      IF (AD1.EQ.0.0DO) GO TO 240
6968              BET3=(T6-EYW)/AD1
6969              BETP3=(T6-ESH)/AD1
6970              IF (BET3.GT.1.0DO) BET3=1.0DO
6971              IF (BET3.LT.0.0DO) BET3=0.0DO
6972              IF (BETP3.GT.1.0DO) BETP3=1.0DO
6973              IF (BETP3.LT.0.0DO) BETP3=0.0DO
6974              GO TO 250
6975      240      BET3=1.0DO
6976              BETP3=1.0DO
6977      250      IF (RSTEA.GT.T2) GO TO 260
6978              BET1=1.0DO
6979              BETP1=1.0DO
6980              GO TO 300
6981      260      IF (RSTEA.GT.T3) GO TO 280
6982              IF (AD1.EQ.0.0DO) GO TO 270
6983              BET1=(T6+EYW)/AD1
6984              IF (BET1.GT.1.0DO) BET1=1.0DO
6985              IF (BET1.LT.0.0DO) BET1=0.0DO
6986              BETP1=1.0DO
6987              GO TO 300
6988      270      BET1=1.0DO
6989              BETP1=1.0DO
6990              GO TO 300
6991      280      IF (AD1.EQ.0.0DO) GO TO 290
6992              BET1=(T6+EYW)/AD1
6993              BETP1=(T6+ESH)/AD1
6994              IF (BET1.GT.1.0DO) BET1=1.0DO
6995              IF (BET1.LT.0.0DO) BET1=0.0DO
6996              IF (BETP1.GT.1.0DO) BETP1=1.0DO
6997              IF (BETP1.LT.0.0DO) BETP1=0.0DO
6998              GO TO 300
6999      290      BET1=1.0DO
7000              BETP1=1.0DO
7001      C
7002      C
7003      C          EVALUATE FORCES IN LOWER HALF OF WEB
7004      C
7005      300      B1=0.25DO*HW*TW*(1.0DO-BETP1)

```



```

7006      B2=2 ODO*(-YSW-(ESH-EYW)*E1+(ESH+T6)*E2)
7007      B3=(REB+T1+ERTB)*E2*(1.ODO+BETP1)
7008      FW1=B1*(B2-B3)
7009      B1=0.25DO*HW*TW*(BETP1-BET1)
7010      B2=2 ODO*(-YSW+(EYW+T6)*E1)
7011      B3=(REB+T1+ERTB)*E1*(BETP1+BET1)
7012      FW2=B1*(B2-B3)
7013      B1=0.25DO*HW*TW*(BET1-BET3)
7014      B2=2 ODO*T6=E
7015      B3=(REB+T1+ERTB)*E*(BET1+BET3)
7016      FW3=B1*(B2-B3)
7017      B1=0.25DO*HW*TW*(BET3-BETP3)
7018      B2=2 ODO*(YSW+(T6-EYW)*E1)
7019      B3=(REB+T1+ERTB)*E1*(BET3+BETP3)
7020      FW4=B1*(B2-B3)
7021      B1=0.25DO*TW*HW=BETP3
7022      B2=2 ODO*(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)
7023      B3=(REB+T1+ERTB)*E2*BETP3
7024      FW5=B1*(B2-B3)
7025      IF (T6.GT.EYW) GO TO 360
7026      IF (EAPEB.GT.T4) GO TO 310
7027      BET2=1.ODO
7028      BETP2=1.ODO
7029      GO TO 350
7030      310 IF (EAPEB.GT.T5) GO TO 330
7031      IF (AD2.EQ.0.ODO) GO TO 320
7032      BET2=(EYW-T6)/AD2
7033      IF (BET2.GT.1.ODO) BET2=1.ODO
7034      IF (BET2.LT.0.ODO) BET2=0.ODO
7035      BETP2=1.ODO
7036      GO TO 350
7037      320 BET2=0.ODO
7038      BETP2=1.ODO
7039      GO TO 350
7040      330 IF (AD2.EQ.0.ODO) GO TO 340
7041      BET2=(EYW-T6)/AD2
7042      BETP2=(ESH-T6)/AD2
7043      IF (BET2.GT.1.ODO) BET2=1.ODO
7044      IF (BET2.LT.0.ODO) BET2=0.ODO
7045      IF (BETP2.GT.1.ODO) BETP2=1.ODO
7046      IF (BETP2.LT.0.ODO) BETP2=0.ODO
7047      GO TO 350
7048      340 BET2=0.ODO
7049      BETP2=0.ODO
7050      C
7051      C
7052      C      EVALUATE FORCES IN UPPER HALF OF WEB WHEN MIDDLE OF
7053      C      WEB IS STILL ELASTIC.
7054      C
7055      350 B1=0.25DO*HW*TW=BET2
7056      B2=2 ODO*T6=E
7057      B3=(ESB-T1-ERTT)*E*BET2
7058      FW6=B1*(B2+B3)
7059      B1=0.25DO*HW*TW*(BETP2-BET2)
7060      B2=2 ODO*(YSW+(T6-EYW)*E1)
7061      B3=(ESB-T1-ERTT)*E1*(BETP2+BET2)
7062      FW7=B1*(B2+B3)
7063      B1=0.25DO*HW*TW=(1.ODO-BETP2)
7064      B2=2 ODO*(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)
7065      B3=(ESB-T1-ERTT)*E2*(1.ODO+BETP2)
7066      FW8=B1*(B2+B3)
7067      GO TO 470
7068      360 IF (T6.GT.ESH) GO TO 420
7069      IF (EAPEB.GT.T4) GO TO 380
7070      IF (AD2.EQ.0.ODO) GO TO 370
7071      BET2=(EYW-T6)/AD2
7072      IF (BET2.GT.1.ODO) BET2=1.ODO
7073      IF (BET2.LT.0.ODO) BET2=0.ODO
7074      BETP2=1.ODO
7075      GO TO 410
7076      370 BET2=1.ODO
7077      BETP2=1.ODO
7078      GO TO 410
7079      380 IF (EAPEB.GT.T5) GO TO 390
7080      BET2=1.ODO
7081      BETP2=1.ODO
7082      GO TO 410
7083      390 IF (AD2.EQ.0.ODO) GO TO 400
7084      BET2=(ESH-T6)/AD2
7085      IF (BETP2.GT.1.ODO) BETP2=1.ODO
7086      IF (BETP2.LT.0.ODO) BETP2=0.ODO
7087      BET2=BETP2
7088      GO TO 410
7089      400 BETP2=1.ODO
7090      BET2=BETP2
7091      C
7092      C
7093      C      EVALUATE FORCES IN UPPER HALF OF WEB WHEN MIDDLE OF
7094      C      WEB IS YIELDED.
7095      C
7096      410 B1=0.25DO*HW*TW=BET2
7097      B2=2 ODO*(YSW+(T6-EYW)*E1)
7098      B3=(ESB-T1-ERTT)*E1*BET2
7099      FW6=B1*(B2+B3)
7100      B1=0.25DO*HW*TW*(BETP2-BET2)
7101      B2=2 ODO*T6=E
7102      B3=(ESB-T1-ERTT)*E*(BETP2+BET2)
7103      FW7=B1*(B2+B3)
7104      B1=0.25DO*HW*TW=(1.ODO-BETP2)
7105      B2=2 ODO*(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)
7106      B3=(ESB-T1-ERTT)*E2*(1.ODO+BETP2)
7107      FW8=B1*(B2+B3)
7108      GO TO 470
7109      420 IF (EAPEB.GT.T4) GO TO 440
7110      IF (AD2.EQ.0.ODO) GO TO 430
7111      BET2=(EYW-T6)/AD2
7112      BETP2=(ESH-T6)/AD2
7113      IF (BET2.GT.1.ODO) BET2=1.ODO
7114      IF (BET2.LT.0.ODO) BET2=0.ODO
7115      IF (BETP2.GT.1.ODO) BETP2=1.ODO
7116      IF (BETP2.LT.0.ODO) BETP2=0.ODO
7117      GO TO 460
7118      430 BET2=1.ODO

```





```

7120 BETP2=1.0DO
7120 GO TO 460
7121 440 IF (EAPB.GT.T5) GO TO 450
7122 IF (AD2.EQ.O.ODO) GO TO 450
7123 BET2=1.0DO
7124 BETP2=(ESH-T6)/AD2
7125 IF (BETP2.GT.1.0DO) BETP2=1.0DO
7126 IF (BETP2.LT.O.ODO) BETP2=O.ODO
7127 GO TO 460
7128 450 BET2=1.0DO
7129 BETP2=1.0DO
7130 C
7131 C
7132 C EVALUATE FORCES IN UPPER HALF OF WEB WHEN MIDDLE OF
7133 C WEB IS STRAIN-HARDENED.
7134 C
7135 460 B1=O.25DO*HW*TW*BETP2
7136 B2=2.ODO*(YSW+(ESH-EYW)*E1+(T6-ESH)*E2)
7137 B3=(ESB-T1-ERTT)*E2*BETP2
7138 FW6=B1*(B2+B3)
7139 B1=O.25DO*HW*TW*(BET2-BETP2)
7140 B2=2.ODO*(YSW*(T6-EYW)*E1)
7141 B3=(ESB-T1-ERTT)*E1*(BET2+BETP2)
7142 FW7=B1*(B2+B3)
7143 B1=O.25DO*HW*TW*(1.ODO-BET2)
7144 B2=2.ODO*T6*E
7145 B3=(ESB-T1-ERTT)*E*(1.ODO+BET2)
7146 FW8=B1*(B2+B3)
7147 FF=F1+FF2
7148 FWA=FW1+FW2+FW3+FW4
7149 FWW=FWW+FW5+FW6+FW7+FW8
7150 SF=AXLD-FF-FWW
7151 RETURN
7152 END
7153 C
7154 C
7155 C
7156 C SUBROUTINE 41
7157 C
7158 C
7159 C
7160 C
7161 C SUBROUTINE SATEO
7162 C
7163 C THIS SUBROUTINE USES AN ITERATIVE TECHNIQUE
7164 C TO DETERMINE THE LOCATION OF THE NEUTRAL AXIS
7165 C DISTANCE Y1 FROM THE CENTER OF THE COMPRESSION
7166 C FLANGE IN ORDER TO SATISFY EQUILIBRIUM AT A
7167 C CROSS-SECTION. THE RATIO OF STRAIN IN THE TOP
7168 C FLANGE TO STRAIN IN THE BOTTOM FLANGE, R, IS ALSO
7169 C FOUND FOR A FIXED STRAIN, ESB, IN THE TOP FLANGE.
7170 C THIS SUBROUTINE IS CALLED FROM PLATE4 AND PLATES.
7171 C
7172 C
7173 C SUBROUTINE SATEO(ESB,EAPP,AXLD)
7174 C IMPLICIT REAL*8(A-H,O-Z)
7175 C COMMON /BLK5/ERTB,ERCM,ERTT,ERCB,ERCT,EYW,EYBF,EYTF
7176 C COMMON /BLK6/ERTBAR,YB,Y1,AREA,ECC,ALAR,R
7177 C COMMON /BLK10/HW,TW,BCL,BF1,TF1,BF2,TF2
7178 C NSTOP=O
7179 C
7180 C
7181 C DETERMINE THE VALUE OF Y1 SO THAT THE SUM OF THE
7182 C FORCES AT THE SECTION HAS A POSITIVE NET VALUE
7183 C (Y1D1)
7184 C
7185 C YD1=O.5DO*HW
7186 5 Y1=O.5DO*HW-YD1
7187 RDN=O.5DO*HW-Y1
7188 IF (RDN.EQ.O.ODO) GO TO 10
7189 RNUM=O.5DO*HW+Y1
7190 R=RNUM/RDN
7191 GO TO 20
7192 10 R=1.ODO
7193 20 YD2=YD1
7194 CALL FORCE(ESB,EAPP,AXLD,SF1)
7195 IF (SF1.LT.O.ODO) GO TO 21
7196 IF (SF1.EQ.O.ODO) RETURN
7197 GO TO 22
7198 21 NSTOP=NSTOP+1
7199 IF (NSTOP.GT.20) GO TO 110
7200 YD1=O.5DO*YD1
7201 GO TO 5
7202 C
7203 C
7204 C DETERMINE THE VALUE OF Y1 SO THAT THE SUM OF THE
7205 C FORCES AT THE SECTION HAS A NEGATIVE NET VALUE
7206 C (Y1D2)
7207 C
7208 C
7209 22 NSTOP=O
7210 30 YD2=1.3DO*YD2
7211 Y1=O.5DO*HW-YD2
7212 RDN=O.5DO*HW-Y1
7213 IF (RDN.EQ.O.ODO) GO TO 40
7214 RNUM=O.5DO*HW+Y1
7215 R=RNUM/RDN
7216 GO TO 50
7217 40 R=1.ODO
7218 50 CALL FORCE(ESB,EAPP,AXLD,SF2)
7219 NSTOP=NSTOP+1
7220 IF (NSTOP.GT.20) GO TO 110
7221 IF (SF2.GT.O.ODO) GO TO 60
7222 IF (SF2.EQ.O.ODO) RETURN
7223 NCOUNT=O
7224 GO TO 70
7225 60 YD1=YD2
7226 SF1=SF2
7227 GO TO 30
7228 C
7229 C CHOOSE A VALUE OF YD3 HALF WAY BETWEEN YD1 AND YD2
7230 C
7231 70 YD3=O.5DO*(YD1+YD2)

```



```

7232      Y1=0.5D0*HW-YD3
7233      RDEN=0.5D0*HW-Y1
7234      IF (RDEN EQ 0.0D0) GO TO 80
7235      RNUM=0.5D0*HW+Y1
7236      R=RNUM/RDEN
7237      GO TO 90
7238      80      R=1.0D0
7239      90      CALL FORCE(ESB,EAPP,AXLO,SF3)
7240              NCOUNT=NCOUNT+1
7241              DY1=DABS((YD1-YD3)/YD1)
7242              DY2=DABS((YD2-YD3)/YD2)
7243
7244      C
7245      C      WHEN YD3 IS SUCH THAT THE LOCATION Y1 OF THE
7246      C      NEUTRAL AXIS DOES NOT CHANGE BY MORE THAN 1% STOP
7247      C      ITERATION AND RETURN. OTHERWISE CONTINUE WITH THE
7248      C      METHOD OF BISECTION.
7249      C
7250      TL=0.01D0
7251      IF (DY1 LT TL OR DY2 LT TL) RETURN
7252      IF (NCOUNT GT 50) GO TO 120
7253      SF13=SF1*SF3
7254      IF (SF13 LT 0.0D0) GO TO 100
7255      YD1=YD3
7256      SF1=SF3
7257      GO TO 70
7258      100      YD2=YD3
7259      SF2=SF3
7260      GO TO 70
7261      110      WRITE (6,130)
7262              STOP
7263      120      WRITE (6,140)
7264              STOP
7265      130      FORMAT (' ',5X,'SF2 FAILED TO BECOME NEGATIVE IN 20 ITERATIONS IN
7266      &SUBROUTINE SATEO.')
7267      140      FORMAT (' ',5X,'SUBROUTINE SATEO FAILED TO CONVERGE IN 50 ITERATIO
7268      &NS ')
7269      END
END OF FILE

```



## APPENDIX D

### SAMPLE PROBLEMS

#### D.1 Introduction

The results of two sample problems were obtained using the computer program listed in Appendix C. These results are presented below for the purpose of illustrating the use of the computer program. In sample problem number one a W shape is subjected to pure bending and the critical local buckling moment as well as the buckled configuration are determined (by an inelastic analysis if necessary). In sample problem number two the same W shape is subjected to a given axial load and analysed for the critical superimposed moment which causes local buckling. As part of the output of this problem, the critical axial load causing local buckling as well as the buckled configuration are presented. The superimposed local buckling moment and resulting buckled configuration are also given.

#### D.2 Example Number 1

The input data for this problem consists of 6 lines (one line per computer card). Referring to the first line of data, the total number of specimens to be analysed is 1, the modulus of elasticity is 29600 (ksi.) and Poisson's ratio is 0.3. The first integer of the second line of input indicates that this is the first specimen in this group to be analysed. The second, third and fourth



digits indicate that the configuration to be analysed is a W shape (ITYPE = 7), the loading is to be pure bending (LTYPE = 2), and an inelastic analysis is to be performed (INEL = 1). The fifth and sixth digits indicate that the ends of the specimen are pinned (ICLAMP = 0), and that an unmodified longitudinal sinewave shape is to be used (IKIND = 0).

In the third line of input, the first three members indicate that the yield stress of the compression flange, web, and tension flange are all 44.0 (ksi.). The last three numbers indicate that the ratio of residual strain to yield strain in the tension zone edge of web, middle of web, and compression zone edge of web are 0.3. In the fourth line there are seven numbers corresponding to the specimen length (20.0 in.), the distance between mid-planes of flanges (10.0 in.), the web thickness (0.25 in.), the width and thickness of the tension flange (10.0 in. and 0.35 in.), and the width and thickness of the compression flange (10.0 in. and 0.35 in.), respectively.

The slope of the yielding portion of the stress-strain curve, the strain-hardening modulus, and the strain at the onset of strain-hardening are given in the fifth line as 800.0 ksi., 800.0 ksi., and 0.02 in./in., respectively. The integer zero in the sixth line is a signal to indicate the end of the data for this particular run.

In the output portion of this problem the data from the input are listed. In addition to these, various geometric properties of the section are listed as well as the critical bending moment, and the ratios of the critical bending moment to the yield and plastic moments. The final page of output shows the critical compression flange





strain and the half wavelength of the longitudinal local buckle. The buckled configuration is shown in the form of values of the displacement coordinates indicated on a computer reproduction of the cross-section shape.

### D-3 Example Number 2

In this problem the same W shape section as used in Example Number 1 is analysed here as a beam-column. The input data consists of 8 lines. Lines 1 to 5 are identical to those of the previous problem with the exception that an integer, 3, in the second line is used to indicate that the number is subjected to an axial and flexural load combined. The ratio of applied axial load to the yield load is 0.3 as shown in line 6. Negative unity in line 7 indicates that control is to be returned to the main program and zero in line 8 signals the end of this run.

During the output stage, the input data and various geometric properties of the section are listed. In the first block of output the critical axial load, critical axial strain and buckled configuration of the cross-section are also printed. In the second stage of the solution, the superimposed moment causing local buckling and the corresponding buckled configuration are determined. These, in addition to the ratios of critical moment to yield moment, reduced plastic moment, and plastic moment, form the major part of the second block of output. The remainder of the output is similar to that described previously for Example Number 1.



## Example Number 1

```

1      1,29600.0,0.3
2      1,7,2,1,0,0,
3      44.0,44.0,44.0,0.3,0.3,0.3,
4      20.0,10.0,0.25,10.0,0.35,10.0,0.35,
5      800.0,800.0,0.02
6      0,

```

Number of specimens to be analysed = 1

Poisson ratio = 0.3000

Elastic modulus = 29600.0 ksi



Critical Bending Moment = 1790.30 Kip-in

$\frac{\text{Critical Bending Moment}}{\text{Yield Moment}} = 1.07483$

$\frac{\text{Critical Bending Moment}}{\text{Plastic Moment}} = 0.99678$

Distance to the Neutral Axis  
From Center of Compression Flange = 5.6912 inches

RDNA = 0.5691 inches

Bottom Flange Slenderness Ratio = 14.2857

Top Flange Slenderness Ratio = 14.2857

Web Slenderness Ratio = 38.6000

Code Slenderness Ratio - Bottom = 94.7607

Code Slenderness Ratio - Top = 94.7607

Code Slenderness Ratio - Web = 256.0434



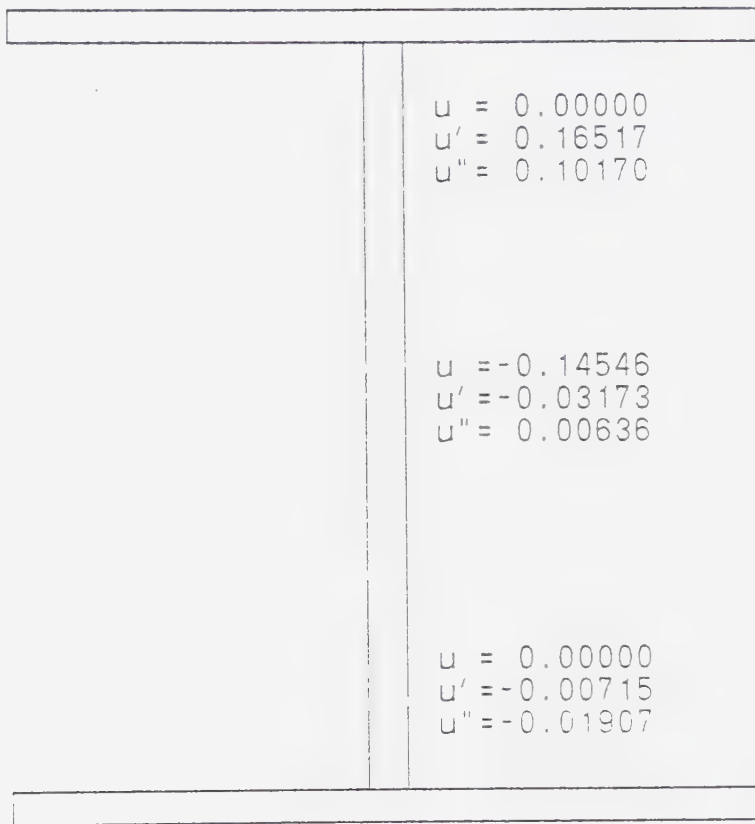
Critical strain = 0.002079

Half wavelength = 10.0000

$v = -1.00000$   
 $v' = 0.26955$

$v = 0.00000$   
 $v' = 0.16517$

$v = 1.00000$   
 $v' = 0.26955$



$v = 0.03020$   
 $v' = -0.00667$

$v = 0.00000$   
 $v' = -0.00715$

$v = -0.03020$   
 $v' = -0.00667$





## Example Number 1

```

1      1,29600.0,0.3,
2      1,7,3,1,0,0,
3      44.0,44.0,44.0,0.3,0.3,0.3
4      20.0,10.0,0.25,10.0,0.35,10.0,0.35,
5      800.0,800.0,0.02,
6      0.3,
7      -1.0,
8      0,

```

Number of specimens to be analysed = 1

Poisson ratio = 0.3000

Elastic modulus = 29600.0 ksi



## Wide Flange Section 1

Subjected to pure bending.

Inelastic analysis.

Length of Wide Flange	= 20.0000 inches
Web Depth	= 10.00000 inches
Web Thickness	= 0.25000 inches
Bottom Flange Width	= 10.00000 inches
Bottom Flange Thickness	= 0.35000 inches
Top Flange Width	= 10.00000 inches
Top Flange Thickness	= 0.35000 inches
Area of Cross-section	= 9.5000 sq.in.
Distance to Centroid From Center of Bottom Flange	= 5.0000 inches
Centroidal Moment of Inertia	= 195.9048 inches
Web Yield Stress	= 44.0000 Ksi
Bottom Flange Yield Stress	= 44.0000 Ksi
Top Flange Yield Stress	= 44.0000 Ksi
Res. Tens. Strain (bott.)	= 13.2000 in/in
Res. Comp. Strain (Mid. Web)	= 13.20000 in/in
Res. Tens. Strain (Top)	= 13.20000 in/in
Res. Comp. Strain (Bott.)	= 13.20000 in/in
Res. Comp. Strain (Top)	= 13.20000 in/in
Yield Modulus	= 800.0000 Ksi
Strain Hardening Modulus	= 800.0000 Ksi
Strain Hardening Strain	= 0.02000



## Wide Flange Section 1

Calculation of critical axial load.

Inelastic analysis.

P(Applied)/P(Yield)	=	0.3000
Length of Wide Flange	=	20.0000 inches
Web Depth	=	10.00000 inches
Web Thickness	=	0.25000 inches
Bottom Flange Width	=	10.00000 inches
Bottom Flange Thickness	=	0.35000 inches
Top Flange Width	=	10.00000 inches
Top Flange Thickness	=	0.35000 inches
Area of Cross-section	=	9.5000 sq.in.
Distance to Centroid From Center of Bottom Flange	=	5.0000 inches
Centroidal Moment of Inertia	=	195.9048 inches
Web Yield Stress	=	44.0000 Ksi
Bottom Flange Yield Stress	=	44.0000 Ksi
Top Flange Yield Stress	=	44.0000 Ksi
Res. Tens. Stress (bott.)	=	13.20000 Ksi.
Res. Comp. Stress (Mid. Web)	=	13.20000 Ksi.
Res. Tens. Stress (Top)	=	13.20000 Ksi.
Res. Comp. Stress (Bott.)	=	13.20000 Ksi.
Res. Comp. Stress (Top)	=	13.20000 Ksi.
Yield Modulus	=	800.0000 Ksi
Strain Hardening Modulus	=	800.0000 Ksi



Strain Hardening Strain	=	0.02000
Axial Load at Buckling	=	390.66 Kips
(Axial Load)/(Yield Load)	=	0.9346
Average Stress at Buckling	=	41.1220 Ksi
Bottom Flange Slenderness Ratio	=	14.2857
Top Flange Slenderness Ratio	=	14.2857
Web Slenderness Ratio	=	38.6000
Code Slenderness Ratio - Bottom	=	94.7607
Code Slenderness Ratio - Top	=	94.7607
Code Slenderness Ratio - Web	=	256.0434





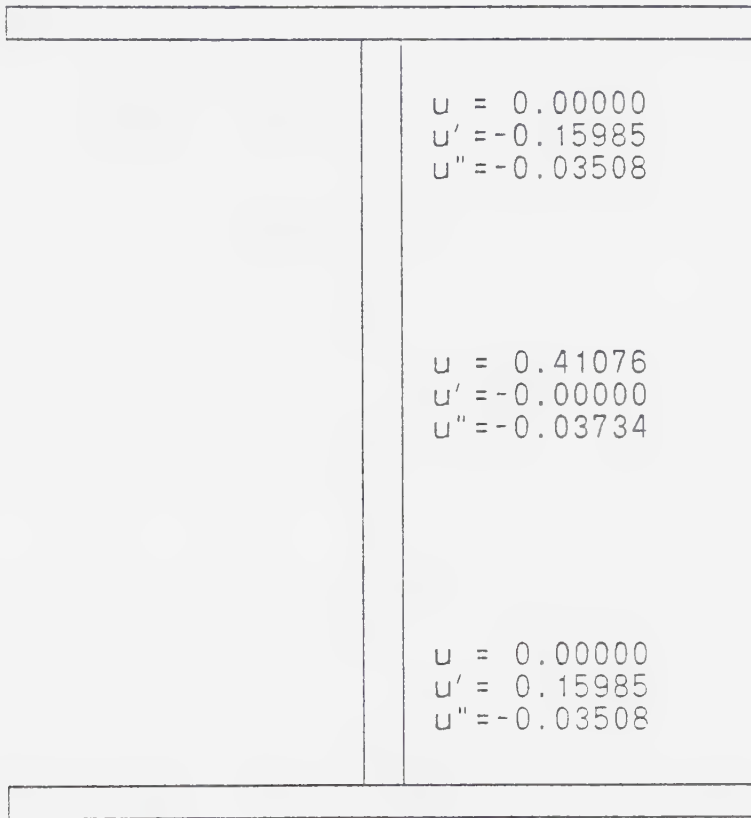
Critical strain = 0.001509

Half wavelength = 10.0000

$v = 1.00000$   
 $v' = -0.27582$

$v = 0.00000$   
 $v' = -0.15985$

$v = -1.00000$   
 $v' = -0.27582$



$v = -1.00000$   
 $v' = 0.27582$

$v = 0.00000$   
 $v' = 0.15985$

$v = 1.00000$   
 $v' = 0.27582$



Beam Column Case No 1

Subjected to an applied axial load of 125.400000 Kips plus bending.

P(Applied)/P(Yield)	=	0.3000
Critical Axial Strain	=	0.00150891
Applied Axial Strain	=	0.00044595
Critical Bending Moment	=	1163.74 Kip-in
$\frac{\text{Critical Bending Moment}}{\text{Yield Moment}}$	=	0.69866
$\frac{\text{Critical Bending Moment}}{\text{Red. Plast. Mom.}}$	=	0.80500
$\frac{\text{Critical Bending Moment}}{\text{Plastic Moment}}$	=	0.64793
Distance to the Neutral Axis From Center of Compression Flange	=	7.3309 inches
RDNA	=	0.7331
Bottom Flange Slenderness Ratio	=	14.2857
Top Flange Slenderness Ratio	=	14.2857
Web Slenderness Ratio	=	38.6000
Code Slenderness Ratio - Bottom	=	94.7607
Code Slenderness Ratio - Top	=	94.7607
Code Slenderness Ratio - Web	=	256.0434
Crit. Bend. Strain/Yield Strain	=	0.75463



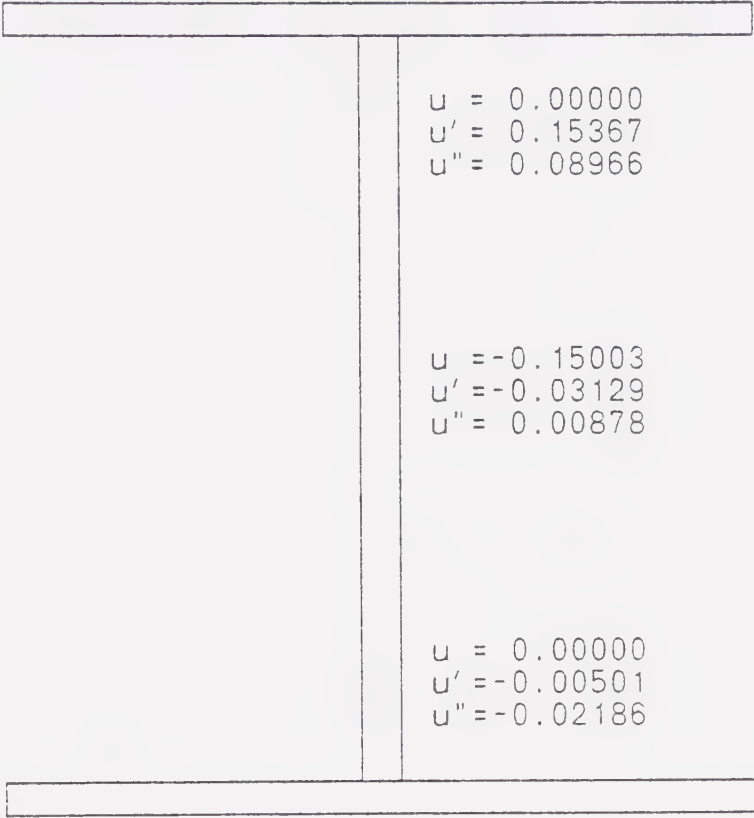
Critical strain = 0.001122

Half wavelength = 10.0000

$v = -1.00000$   
 $v' = 0.27928$

$v = 0.00000$   
 $v' = 0.15367$

$v = 1.00000$   
 $v' = 0.27928$



$v = 0.01497$   
 $v' = -0.00195$

$v = 0.00000$   
 $v' = -0.00501$

$v = -0.01497$   
 $v' = -0.00195$

















**B30290**